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Single Valued Neutrosophic Sub Implicative Ideals of KU-Algebras

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Abstract – We consider the concepts single valued neutrosophic of sub-implicative ideals in KU-algebras, and investigate some related properties. We give conditions for a single valued neutrosophic ideal to be a single valued neutrosophic sub-implicative ideal. We show that any single valued neutrosophic sub-implicative ideal is a single valued neutrosophic ideal, but the converse is not true. Using a level set of a single valued neutrosophic set in a KU-algebra, we give a characterization of single valued neutrosophic sub-implicative ideal.

Keywords – *Single valued neutrosophic sub-algebra, Single valued neutrosophic ku-ideal of KU-ideals.*

1. Introduction

Prabpayak and Leerawat [10,11] introduced a new algebraic structure which is called KU-algebras. They studied ideals and congruences in KU-algebras. Also, they introduced the concept of homomorphism of KU-algebra and investigated some related properties. Moreover, they derived some straightforward consequences of the relations between quotient KU-algebras and isomorphism. Mostafa et al. [4,5,13] introduced the notion of fuzzy KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Mostafa et al. [6] introduced the notions of *ku*-***s***ub implicative / *ku*-positive implicative and *ku*-sub-commutative ideals in KU–algebras and investigated some their related properties. Fuzzy set theory was introduced by Zadeh since 1965 [14]. Immediately, it became a useful method to study the problems of imprecision and uncertainty. Since, a lot of new theories treating imprecision and uncertainty have been introduced. For instance, Intuitionistic fuzzy sets were introduced in1986, by Atanassov [2], which is a generalization of the notion of a fuzzy set. When fuzzy set give the degree of membership of an element in a given set, Intuitionistic fuzzy sets give a degree of membership and a degree of non-membership of an element in a given set. In 1998 [7,8], Smarandache gave the concept of neutrosophic set which generalized fuzzy set and

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intuitionistic fuzzy set. This new concept is difficult to apply in the real appliction. It is a set in which each proposition is estimated to have a degree of truth (T), adegree of indeterminacy (I) and a degree of falsity (F). Agboola and Davvaz introduced the concept of neutrosophic BCI/BCK-algebras in [1]. Davvaz et al. [3] introduce a neutrosophic KU-algebra and KU-ideal and investigate some related properties. Recently Wang et al. [12] introduced an instance of neutrosophic set known as single valued neutrosophic set which was motivated from the practical point of view and that can be used in real scientific and engineering applications. In this paper, we establish the concept of single valued neutrosophication sub-implicative ideals on KU-algebras, and investigate some of their properties.

2. Preliminaries

Now we will recall some known concepts related to KU-algebra from the literature which will be helpful in further study of this article.

Definition 2.1. [10,11] Algebra (X, $*$, 0) of type (2, 0) is said to be a KU-algebra, if it satisfies the following axioms:

(ku_1) $(x*y)*[(y*z))*(x*z)]=0$,
(ku_2) $x*0=0$,
(ku_3) $0*x=x$,
(ku_4) $x*y=0$ and $y*x=0$ implies $x=y$,
(ku_5) $x*x=0$, for all $x,y,z\in X$.

On a KU-algebra $(X,*,0)$ we can define a binary relation $\leq$ on X by putting:

$x\leq y \Leftrightarrow y*x=0$.

Thus a KU - algebra X satisfies the conditions:

$(ku_{1^\backslash})$: $(y*z)*(x*z)\leq (x*y)$
$(ku_{2^\backslash})$: $0\leq x$
$(ku_{3^\backslash})$: $x\leq y, y\leq x$ implies $x=y$,
$(ku_{4^\backslash})$: $y*x\leq x$.

Remark 2.2. Substituting $z*x$ for x and $z*y$ for y in ku_1, we get

$$[(z*x)*(z*y)]*[(z*y)*z))*[(z*x)*z)]\leq [(z*x)*(z*y)]*[(z*x)*(z*y)]=0$$

by (ku_1), hence $(x*y)*[(z*x)*(z*y)]=0$ that mean the condition (ku_1) and $(x*y)*[(z*x)*(z*y)]=0$ are equivalent.

For any elements x and y of a KU-algebra, $y*x^n$ denotes by $\overbrace{(y*x)*x)......*x}^{n\,times}$

Theorem 2.3. [5] In a KU-algebra X, the following axioms are satisfied: For all $x, y, z \in X$,

(1) $x \leq y$ imply $y * z \leq x * z$,
(2) $x * (y * z) = y * (x * z)$,for all $x, y, z \in X$,
(3) $((y * x) * x) \leq y$.
(4) $(y * x^3) = (y * x)$

We will refer to X is a KU-algebra unless otherwise indicated.

Definition 2.4. [10,11] Let I be a non empty subset of a KU-algebra X. Then I is said to be an ideal of X, if

(I_1) $0 \in I$
(I_2) $\forall y, z \in X$, if $(y * z) \in I$ and $y \in I$, imply $z \in I$.

Definition 2.5. [5] Let I be a non empty subset of a KU-algebra X. Then I is said to be an KU- ideal of X, if

(I_1) $0 \in I$
(I_3) $\forall x, y, z \in X$, if $x * (y * z) \in I$ and $y \in I$, imply $x * z \in I$.

Definition 2.6. [6] KU- algebra X is said to be implicative if it satisfies

$$(x * y^2) = (x * y) * (y * x^2)$$

Definition 2.7. [6] KU- algebra X is said to be commutative if it satisfies $x \leq y$ *implies* $(x * y^2) = x$

Lemma 2.8. [6] Let X be a KU-algebra. X is *ku*-implicative iff X is *ku*-positive implicative and *ku*–commutative.

Definition 2.9 [6] A non empty subset A of a KU-algebra X is called a *ku-sub* implicative ideal of X, if $\forall x, y, z \in X$,

(1) $0 \in A$
(2) $z * ((x * y) * ((y * x^2)) \in A$ and $z \in A$, imply $(x * y^2) \in A$.

Definition 2.10. [6] Let $(X, *, 0)$ be a KU-algebra, a nonempty subset A of X is said to be a *ku*-positive implicative ideal if it satisfies, for all x, y, z in X,

(1) $0 \in A$,
(2) $z * (x * y) \in A$ and $z * x \in A$ imply $z * y \in A$.

Definition 2.11 [6] A non empty subset A of a KU-algebra X is called a *ku*–sub commutative ideal of X, if

(1) $\mathbf{0} \in A$
(2) $z * \{((y * x^2)) * y^2)\} \in A$ and $z \in A$, imply $(y * x^2) \in A$.

Definition 2.12.[6] A nonempty subset A of a KU-algebra X is called a kp-ideal of X if it satisfies

(1) $\mathbf{0} \in A$,

(2) $(z * y) * (z * x) \in A$, $y \in A \Rightarrow x \in A$.

3. Single Valued Neutrosophic Sub Implicative Ideals of KU-Algebras

Let X be a non-empty set. A neutrosophic set (NS) in X (see [8,9]) is a structure of the form: $A := \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$, where $T_A(x) : X \to [0,1]$ is a truth membership function, $I_A(x) : X \to [0,1]$ is an indeterminate membership function and $F_A(x) : X \to [0,1]$ is a false membership , we shall use the symbol $A := \{\langle x, T_A, I_A, F_A\rangle \mid x \in X\}$ for the neutrosophic set $A := \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$.

Definition 3.1. Let X be a KU-algebra, a neutrosophic set

$$A := \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$$

in X is called a single valued neutrosophic- ideal (briefly NF- ideal) of X if it satisfies the following conditions:

(F_1) $\mu(0) \geq \mu(x)$ $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$, $F_A(0) \leq F_A(x)$ for all $x \in X$.
(F2) $\forall x, y \in X$, $T_A(y) \geq \min\{T_A(x * y), T_A(x)\}$.
(F_3) $\forall x, y \in X$, $I_A(y) \geq \min\{I_A(x * y), I_A(x)\}$
(F_4) $\forall x, y \in X$, $F_A(y) \leq \max\{F_A(x * y), F_A(x)\}$

Definition3.2. A non empty subset $A := \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ of a KU-algebra X is called a single valued neutrosophic *sub* implicative ideal (briefly NFSI - ideal) of X , if $\forall x, y, z \in X$,

(F_1) $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$, $F_A(0) \leq F_A(x)$
$(NFSI_1)$ $T_A(x * y^2) \geq \min\{T_A(z * ((x * y) * ((y * x^2)), T_A(z)\}$
$(NFSI_2)$ $I_A(x * y^2) \geq \min\{I_A(z * ((x * y) * ((y * x^2)), I_A(z)\}$
$(NFSI_3)$ $F_A(x * y^2) \leq \max\{F_A(z * ((x * y) * ((y * x^2)), F_A(z)\}$

Example.3.3. Let X = {0,1,2,3} be a set with a binary operation $*$ defined by the following table:

*	0	1	2	3
0	0	1	2	3
1	0	0	0	2
2	0	2	0	1
3	0	0	0	0

Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be an neutrosophic Set in X defined by $T_A(0) = T_A(2) = 0.7, T_A(3) = T_A(1) = 0.2$, $I_A(0) = I_A(2) = 0.6, I_A(3) = I_A(1) = 0.2$ and $F_A(0) = F_A(2) = 0, F_A(3) = F_A(1) = 0.3$, By routine calculations we know that $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ is NFSI - ideal of algebra of *X*.

Proposition 3.4. Every NFSI- ideal of a KU-algebra X is order reversing.

Proof. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be NFSI -ideal of X and let x, y, z $\in$ X be such that x $\leq$ z, then z$*$ x = 0 . Let y = x in $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$, we get

$$T_A(x) \geq \min\{T_A(z*x), T_A(z)\} = \min\{T_A(0), T_A(z)\} = T_A(z),$$
$$I_A(x) \geq \min\{I_A(z*x), I_A(z)\} = \min\{I_A(0), I_A(z)\} = I_A(z) \text{ and}$$
$$F_A(x) \leq \max\{F_A(z*x), F_A(z)\} = \max\{F_A(0), F_A(z)\} = F_A(z).$$

This completes the proof

Lemma 3.5. let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be a N FSI - ideal of KU - algebra X , if the inequality y $*$ x $\leq$ z hold in X , Then

$$T_A(y) \geq \min\{T_A(x)), T_A(z)\},\ I_A(y) \geq \min\{I_A(x)), I_A(z)\} \text{ and } F_A(y) \leq \max\{F_A(x)), F_A(z)\}.$$

Proof. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be NFSI -ideal of X and let x, y, z $\in$ X be such that y*x $\leq$ z, then z$*$ (y*x)= 0 or y$*$ (z*x)= 0 i.e $z * x \leq y$, we get **(**by *Proposition 3.4),*

$$T_A(z*x) \geq T_A(y)\ ,\ I_A(z*x) \geq I_A(y) \text{ and } F_A(z*x) \leq F_A(y) \qquad \text{(a).}$$

Put in $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$, x=y, we get:

$$T_A(x*x^2) \geq \min\left\{T_A(z*(\overbrace{(x*x)}^{0}*\overbrace{((x*x^2)}^{x})), T_A(z)\right\} = \min\{T_A(z*x), T_A(z)\}, \text{ i.e}$$

$$T_A(x) \geq \min\{T_A(z*x), \mu(z)\} \geq \min\{T_A(y), T_A(z)\} \text{ by (a)}$$

$$I_A(x*x^2) \geq \min\left\{I_A(z*(\overbrace{(x*x)}^{0}*\overbrace{((x*x^2)}^{x})), I_A(z)\right\} = \min\{I_A(z*x), I_A(z)\}, \text{ i.e}$$

$I_A(x) \geq \min\{I_A(z*x), I(z)\} \geq \min\{I_A(y), I_A(z)\}$ by (a) . and

$$F_A(x*x^2) \leq \max\left\{ F_A(z*(\overbrace{(x*x)}^{0}*\overbrace{((x*x^2)}^{x})), F_A(z) \right\} = \max\{F_A(z*x), F_A(z)\}, \text{i.e}$$

$F_A(x) \leq \max\{F_A(z*x), F(z)\} \leq \max\{F_A(y), F_A(z)\}$ by (a). This completes the proof.

Lemma 3.6. If X is implicative KU-algebra, then every NF ideal of X is an NFSI-ideal of X.

Proof. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be NF ideal of X. Substituting $x*y^2$ for y in (F_2), (F_3) and (F_4), we get

$T_A(x*y^2) \geq \min\{T_A(z*(x*y^2)), T_A(z)\}$, but KU- algebra is implicative i.e
$I_A(x*y^2) \geq \min\{I_A(z*(x*y^2)), I_A(z)\}$ and
$F_A(x*y^2) \leq \max\{F_A(z*(x*y^2)), F_A(z)\}$, but KU- algebra is implicative i.e
$(x*y^2) = (x*y)*(y*x^2)$, hence
$T_A(x*y^2) \geq \min\{T_A(z*(x*y^2)), \mu(z)\} = \min\{T_A(z*(x*y)*(y*x^2)), T_A(z)\}$,
$I_A(x*y^2) \geq \min\{I_A(z*(x*y^2)), I(z)\} = \min\{I_A(z*(x*y)*(y*x^2)), I_A(z)\}$, and
$F_A(x*y^2) \leq \max\{F_A(z*(x*y^2)), F(z)\} = \max\{F_A(z*(x*y)*(y*x^2)), F_A(z)\}$,

which shows that $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ is NFSI-ideal of X. This completes the proof.

Theorem 3.7. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be N F set of KU - algebra X satisfying the conditions $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$ then $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ satisfies the following inequalities ;

$(NFSI_4)$ $T_A(x*y^2) \geq T_A((x*y)*((y*x^2))$
$(NFSI_5)$ $I_A(x*y^2) \geq I_A((x*y)*(y*x^2))$
$(NFSI_6)$ $F_A(x*y^2) \leq F_A((x*y)*(y*x^2))$

Proof. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ satisfying conditions $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$ i.e.

$(NFSI_1)$ $T_A(x*y^2) \geq \min\{T_A(z*((x*y)*((y*x^2)), T_A(z)\}$
$(NFSI_2)$ $I_A(x*y^2) \geq \min\{I_A(z*((x*y)*((y*x^2)), I_A(z)\}$
$(NFSI_3)$ $F_A(x*y^2) \leq \max\{F_A(z*((x*y)*((y*x^2)), F_A(z)\}$

then by taking z = 0 in $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$ and using (F_1) $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$, $F_A(0) \leq F_A(x)$ and (ku_3) we get

$$T_A(x*y^2) \geq \min\{T_A(0*((x*y)*((y*x^2)),T_A(0)\} = T_A((x*y)*(y*x^2)) \ .$$
$$I_A(x*y^2) \geq \min\{I_A(0*((x*y)*((y*x^2)),I_A(0)\} = I_A((x*y)*(y*x^2)) \ .$$
$$F_A(x*y^2) \leq \max\{F_A(0*((x*y)*((y*x^2)),F_A(0)\} = F_A((x*y)*(y*x^2)).$$

This completes the proof

Theorem 3.8. Every NFSI- ideal of a KU-algebra X is a NF - ideal, but the converse does not hold.

Proof. Let $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ be N FSI-ideal of X; put x=y in $(NFSI_1)$, $(NFSI_2)$ and $(NFSI_3)$, we get

$$T_A(\overbrace{x*x^2}^{x}) \geq \min\{T_A(z*((x*x)*((x*x^2)),T_A(z)\}, \ then$$
$$T_A(x) \geq \min\left\{T_A(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),T_A(z)\right\} = \min\{T_A(z*x),T_A(z)\} \quad ,$$

$$I_A(\overbrace{x*x^2}^{x}) \geq \min\{I_A(z*((x*x)*((x*x^2)),I_A(z)\}, \ therefore$$
$$I_A(x) \geq \min\left\{I_A(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),I_A(z)\right\} = \min\{I_A(z*x),I_A(z)\} \quad , and$$

$$F_A(\overbrace{x*x^2}^{x}) \geq \min\{F_A(z*((x*x)*((x*x^2)),F_A(z)\}, \ we \ \ get$$
$$F_A(x) \leq \max\left\{F_A(z*((\overbrace{x*x}^{0})*((\overbrace{x*x^2}^{x})),F_A(z)\right\} = \max\{F_A(z*x),F_A(z)\} \ .$$

Hence $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ is a a NF - ideal of X . This completes the proof

The following example shows that the converse of Theorem 3.8 may not be true.

Example 3.9. Let $X = \{0,1,2,3,4\}$ in which the operation $*$ is given by the table

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	1	3	4
2	0	0	0	3	4
3	0	0	0	0	4
4	0	0	0	0	0

Then $(X,*,0)$ is a KU-Algebra. Define a fuzzy set T_A: X→ [0,1] by T_A (0) = 0.7 , T_A (1) = T_A (2) = T_A (3) = T_A (4) = 0.2 , we get for z=0 , x=1 and y=2 L.H.S of $(NFSI_1)$ $T_A((1*2)*2)=T_A(1)=0.2$.

$R.H.S\ of\ (NFSI_1)\ \min\left\{T_A(0*((\overbrace{1*2}^{1})*((\overbrace{2*1}^{0})*1),T_A(0)\right\}=T_A(0)=0.7$, i.e in this case

$$T_A(x*y^2)\ngeq\ \min\{T_A(z*((x*y)*((y*x^2)),T_A(z)\}$$

We now give a condition for a NF- ideal to be a NFSI-ideal.

Theorem 3.10. Every a NF - ideal $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$of X satisfying the condition $(FSI_4),(FSI_5),(FSI_6)$ is a NFSI-ideal ideal of X .

Proof. Let $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$be NF ideal of X satisfying the conditions $(NFSI_4),(NFSI_5),(NFSI_6)$. we get $T_A(x*y^2)\geq\{T_A(((x*y)*((y*x^2))\}$,

$I_A(x*y^2)\geq\{I_A(((x*y)*((y*x^2))\}$ and $F_A(x*y^2)\leq\{F_A(((x*y)*((y*x^2))\}$

Therefore

$$T_A(x*y^2)\geq\min\{T_A(z*((x*y)*((y*x^2)),T_A(z)\},$$
$$I_A(x*y^2)\geq\min\{I_A(z*((x*y)*((y*x^2)),I_A(z)\}, \text{ and}$$
$$F_A(x*y^2)\leq\max\{F_A(z*((x*y)*((y*x^2)),F_A(z)\}$$

by (Definition of NF-ideal (F_2), (F_3) , (F_4)), we get

$$T_A(x*y^2)\geq\ T_A(((x*y)*((y*x^2))\ \geq\min\{T_A(z*((x*y)*((y*x^2)),T_A(z)\},$$
$$I_A(x*y^2)\geq\ I_A(((x*y)*((y*x^2))\ \geq\min\{I_A(z*((x*y)*((y*x^2)),I_A(z)\}, \text{ and}$$
$$F_A(x*y^2)\leq\ F_A(((x*y)*((y*x^2))\ \leq\max\{F_A(z*((x*y)*((y*x^2)),F_A(z)\},$$

which proves the condition $(NFSI_1),(NFSI_2),(NFSI_3)$. This completes the proof.

Theorem 3.11. Let $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$ be NF ideal of X. Then the following are equivalent:

(i) $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$is an NFSI-ideal of X,

(ii) $T_A(x*y^2)\geq T_A(z*((x*y)*((y*x^2))$, $I_A(x*y^2)\geq I_A(z*((x*y)*((y*x^2))$ and $F_A(x*y^2)\leq F_A(z*((x*y)*((y*x^2))$

(iii) $T_A(x*y^2)\geq T_A((x*y)*((y*x^2))$, $I_A(x*y^2)\geq I_A((x*y)*((y*x^2))$ and $F_A(x*y^2)\leq F_A((x*y)*((y*x^2))$

Proof. (i) $\Rightarrow$(ii) Suppose that $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$ be NFSI ideal of X. By $(NFSI_1),(NFSI_2),(NFSI_3)$ and (F_1) we have

$T_A(x*y^2)\geq \min\{T_A(0*((x*y)*((y*x^2)),T_A(0)\}=T_A(0*((x*y)*((y*x^2)$ *i.e*
$T_A(x*y^2)\geq T_A(((x*y)*((y*x^2))$,
$I_A(x*y^2)\geq \min\{I_A(0*((x*y)*((y*x^2)),I_A(0)\}=I_A(0*((x*y)*((y*x^2)$ *i.e*
$I_A(x*y^2)\geq I_A(((x*y)*((y*x^2))$ *and*
$F_A(x*y^2)\leq \max\{F_A(0*((x*y)*((y*x^2)),F_A(0)\}=F_A(0*((x*y)*((y*x^2)$ *i.e*
$F_A(x*y^2)\leq F_A(((x*y)*((y*x^2))$.

(ii) $\Rightarrow$ (iii) Since $(x*y)*(y*x^2)\leq x*y^2$, by Lemma 3.5 we obtain,

$T_A(x*y^2)\geq T_A((x*y)*((y*x^2))$, $I_A(x*y^2)\geq I_A((x*y)*((y*x^2))$ and
$F_A(x*y^2)\leq F_A((x*y)*((y*x^2))$

Combining (ii) we have

$T_A(x*y^2)\geq T_A((x*y)*((y*x^2))$, $I_A(x*y^2)\geq I_A((x*y)*((y*x^2))$ and
$F_A(x*y^2)\leq F_A((x*y)*((y*x^2))$

(iii) $\Rightarrow$ (i) Since $[(z*((x*y)*((y*x^2))]*[(x*y)*((y*x^2)]=$

$$=[(x*y)*(z*((y*x^2))]*[(x*y)*((y*x^2)]\leq$$
$$[(z*((y*x^2))]*[(y*x^2)]=[(z*((y*x^2))]*[0*(y*x^2)]$$
$$\leq 0*z=z\,.$$

By (Lemma 3.5) we obtain

$T_A((x*y)*((y*x^2)\geq \min\{\ T_A((x*y)*((y*x^2),T_A(z)\}$.
$I_A((x*y)*((y*x^2)\geq \min\{\ I_A((x*y)*((y*x^2),I_A(z)\}$, and
$F_A((x*y)*((y*x^2)\leq \max\{\ F_A((x*y)*((y*x^2),F_A(z)\}$.

From (iii), we have $T_A(x*y^2)\geq \min\{T_A(z*((x*y)*((y*x^2)),T_A(z)\}$,
$I_A(x*y^2)\geq \min\{I_A(z*((x*y)*((y*x^2)),I_A(z)\}$,and
$F_A(x*y^2)\leq \max\{F_A(z*((x*y)*((y*x^2)),F_A(z)\}$,

Hence $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$is an NFSI-ideal of X . The proof is complete.

Theorem 3.12. A single valued neutrosophic set $A:=\{\langle x,T_A,I_A,F_A\rangle \mid x\in X\}$ of a KU-algebra X is a NFSI-ideal of X if and only if $A_{t,s,m}=\{\langle x\in X \mid T_A\geq t,I_A\geq s,F_A\leq m\rangle\}\neq\Phi$, is a sub-implicative ideal of X.

Proof: Suppose that $A := \{\langle x, T_A, I_A, F_A\rangle \mid x \in X\}$ is a A single valued neutrosophic sub-implicative ideal of X and $A_{t,s,m} \neq \Phi$ for any $t, s, m \in (0,1]$, there exists $x \in A_{t,s,m}$ so that $T_A \geq t, I_A \geq s, F_A \leq m$. It follows from (F_1) that $T_A(0) \geq T_A(x) \geq t$, $I_A(0) \geq I_A(x) \geq s$, $F_A(0) \leq F_A(x) \leq m$ so that $0 \in A_{t,s,m}$. Let $x, y, z \in X$ be such that

$z * ((x * y) * ((y * x^2) \in A_{t,s,m}$ and $z \in A_{t,s,m}$. Using $(NFSI_1), (NFSI_2), (NFSI_3)$,

we know that

$$T_A(x * y^2) \geq \min\{T_A(z * ((x * y) * ((y * x^2)), T_A(z)\} = \min\{t, t\} = t \ ,$$
$$I_A(x * y^2) \geq \min\{I_A(z * ((x * y) * ((y * x^2)), I_A(z)\} = \min\{s, s\} = s \ , \text{ and}$$
$$F_A(x * y^2) \leq \max\{F_A(z * ((x * y) * ((y * x^2)), F_A(z)\} = \max\{m, m\} = m$$

thus $x * y^2 \in A_{t,s,m}$. Hence $A_{t,s,m}$ is a sub-implicative ideal of X. Conversely, suppose that $A_{t,s,m} \neq \Phi$ is a sub-implicative ideal of X ,for every $t, s, m \in (0,1]$. and any $x \in X$, let $T_A(x) = t$, $I_A(x) = s$ and $F_A(x) = m$. Then $x \in A_{t,s,m}$. Since $0 \in A_{t,s,m}$, it follows that $T_A(0) \geq t = T_A(x)$, $I_A(0) \geq s = I_A(x)$, $F_A(0) \leq m = F_A(x)$ so that $T_A(0) \geq T_A(x)$, $I_A(0) \geq I_A(x)$, $F_A(0) \leq F_A(x)$ for all $x \in X$. Now, we need to show that

$A := \{\langle x, T_A, I_A, F_A\rangle \mid x \in X\}$ satisfies $(NFSI_1), (NFSI_2), (NFSI_3)$

If not, then there exist $a, b, c \in X$ such that $T_A(a * b^2) \leq \min\{T_A(c * ((a * b) * ((b * a^2)), T_A(c)\}$

$$I_A(a * b^2) \leq \min\{I_A(c * ((a * b) * ((b * a^2)), I_A(c)\} \ , \text{ and}$$
$$F_A(a * b^2) \geq \max\{F_A(c * ((a * b) * ((b * a^2)), F_A(c)\}.$$

Taking

$$t_0 = \frac{1}{2}(T_A(a * b^2) + \{T_A(c * ((a * b) * ((b * a^2)), T_A(c)\}),$$
$$s_0 = \frac{1}{2}(I_A(a * b^2) + \{I_A(c * ((a * b) * ((b * a^2)), I_A(c)\}) \text{ and}$$
$$m_0 = \frac{1}{2}(F_A(a * b^2) + \{F_A(c * ((a * b) * ((b * a^2)), F_A(c)\}),$$

then we have

$$T_A(a * b^2) < t_0 < \{T_A(c * ((a * b) * ((b * a^2)), T_A(c)\}$$
$$I_A(a * b^2) < s_0 < \{I_A(c * ((a * b) * ((b * a^2)), I_A(c)\}$$
$$F_A(a * b^2) > m_0 > \{F_A(c * ((a * b) * ((b * a^2)), F_A(c)\}$$

Hence $c*((a*b)*(b*a^2)) \in A_{t,s,m}$ and $c \in A_{t,s,m}$, but $a*b^2 \notin A_{t,s,m}$ which means that $A_{t,s,m}$ is not a sub-implicative ideal of X, this is contradiction. Therefore $A := \{\langle x, T_A, I_A, F_A \rangle \mid x \in X\}$ is a A single valued neutrosophic sub-implicative ideal of X **.**

Conclusions

In the present paper, we have introduced the concept of single valued neutrosophic sub-implicative ideal KU-algebras and investigated some of their useful properties. In our opinion, these definitions and main results can be similarly extended to some other fuzzy algebraic systems such as hyper groups, hyper semigroups, hyper rings, hyper. It is our hope that this work would other foundations for further study of the theory of BC K/BC I- KU -algebras. Our obtained results can be perhaps applied in engineering, soft computing or even in medical diagnosis. In our future study of single valued neutrosophic sub commutative ideal structure of KU -algebras, may be the following topics should be considered:

(1) To establish single valued neutrosophic (s-weak–strong) hyper KU-ideals in hyper KU-algebras;

(2) To get more results in single valued neutrosophic ideals hyper KU-algebras and application.

(3) To consider the structure single valued neutrosophic dot (s-weak–strong) hyper KU-Ideals of hyper KU-Algebras.

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