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The Case of Time Axis Fallacy: 11th Grade Students' Intuitively-based Misconception in Probability and Teachers' Corresponding Practices*

Zaman Etkisi Örneği: 11. Sınıf Öğrencilerinin Olasılıkta Sezgi Temelli Kavram Yanılgısı ve Öğretmenlerin İlgili Uygulamaları

Mehmet Fatih ÖÇAL**

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Abstract. This case study aimed to investigate whether mathematics teachers' instructional practices were effective in resolving eleventh grade students' intuitively-based misconceptions regarding time axis fallacy. The participants were three mathematics teachers from different high schools and their students. Students were administered a diagnostic test comprising questions related to intuitively-based misconceptions in probability. The test was administered before and after students received teachers' instructions for probability subject. Teachers were interviewed about their knowledge of students' difficulties and misconceptions. Teachers' instructions for probability were observed and videotaped. Content analysis method was used in the data analysis. Considering the findings, it was observed that teachers did not give emphasis on unfamiliar situations related to time axis fallacy. Comparing the test results, there was slight increase in the number of students who fell into time axis fallacy. Based on the findings, it can be asserted that practitioners should be aware of possible intuitively-based misconceptions in probability and organize their instructions accordingly.

Keywords: Intuition, misconceptions, probability, time axis fallacy, mathematics teachers

Öz. Bu durum çalışmasının amacı, matematik öğretmenlerinin öğretim uygulamalarının on birinci sınıf öğrencilerinin sezgi temelli kavram yanılgıları bağlamında olasılık konusunda zaman etkisi yanılgısını gidermede etkili olup olmadığını incelemektir. Bu çalışmanın katılımcılarını farklı liselerde çalışan üç matematik öğretmeni ve bu öğretmenlerin öğrencileri oluşturmaktadır. Öğrencilere olasılıkta sezgi temelli kavram yanılgılarıyla ilgili sorular içeren bir tanı testi uygulanmıştır. Test, öğretmenlerin olasılık konusunun öğretiminin öncesi ve sonrasında uygulanmıştır. Ayrıca öğrencilerin zorlandıkları noktalar ve kavram yanılgılarına yönelik öğretmenler ile mülakatlar yapılmıştır. Öğretmenlerin olasılık öğretimleri sırasında hem gözlem yapılmış hem de dersler video kaydı altına alınmıştır. Verilerin analizinde içerik analiz yöntemi kullanılmıştır. Bulgular göz önüne alındığında, zaman etkisi kavram yanılgısıyla ilgili aşina olunmayan durumlara öğretmenlerin odaklanmadığı gözlemlenmiştir. Test sonuçları karşılaştırıldığında, zaman etkisi yanılgısına düşen öğrenci sayısında az bir artış olduğunu göstermiştir. Bulgular ışığında, uygulayıcıların olasılıkta sezgi temelli kavram yanılgılarından haberdar olmaları ve öğretimlerini bu bağlamda düzenlemeleri gerektiği söylenebilir.

Anahtar Kelimeler: Sezgi, kavram yanılgıları, olasılık, zaman etkisi yanılgısı, matematik öğretmenleri

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** *Correspondence:* Ağrı İbrahim Çeçen University, Turkey, e-mail: fatihocal@gmail.com ORCID: 0000-0003-0428-6176

Introduction

Probability is interconnected with the daily life situations, since people always experience uncertainties and chance factors (Andra, 2011; Way, 2003). When dealing with an event or a happening; people judge, made necessary decisions, and behave according to their judgments. From educational point of views, school mathematics curricula (e.g., Common Core State Standards [CCSS], 2010; Ministry of National Education [MoNE], 2017; National Council for Teachers of Mathematics [NCTM], 2000) give importance to teaching probability. It is stressed that probability is linked to other mathematical subjects such as counting techniques in numbers and operations, area concepts in geometry, binomial theorem, and the relationship between functions and the areas under their graphs in algebra (NCTM, 2000). Similar to NCTM (2000), the Common Core State Standards [CCSS] (2010) emphasizes to make connections between mathematics topics. In addition, students are expected to develop, use, and evaluate the probability models. In high school standards, the probability topic is considered under the statistics and probability strand. Instead of just teaching probability, CCSS (2010) emphasizes on using probability in order to make inferences and decisions, and to justify conclusions. The standards considered the probability as the tool to use in decision-making processes and in justifications of the situations encountered instead of just memorizing the rules in probability. Moreover, students are encouraged to use chance factor and probabilistic thinking in their daily lives (MoNE, 2017). For teaching probability, teachers need to take students' thinking into account and create an environment for students to discuss and analyze the events happening (e.g. fairness of the dice thrown), then, make inferences from the light of given facts (NCTM, 2000) instead of using traditional methods including clear and linear representations of mathematical knowledge without taking students' conceptions and misconceptions into account (Castro, 1998; Tarr & Lannin, 2005).

Despite the emphasized importance of teaching probability in mathematics curricula, students and even adults experience difficulty in understanding the probability (Kazak, 2008). Although students experience uncertainties and make judgments in their daily lives (Kahneman & Tversky, 1982), many misconceptions arise among students in probability. This situation directed mathematics education researchers' attention to search and identify the reasons for the misconceptions, how teachers and their instructions intervene the misconceptions, and what they do for resolving them experienced in the probability (e.g., Fischbein, 1975; Fischbein & Schnarch, 1997; Tirosh & Stavy, 1999a; 1999b; Stavy & Tirosh, 2000; Pratt, 2000; Polaki, 2002a; 2002b; Stohl & Tarr, 2002). Two reasons which are directly related to purpose of this study are teachers' potential role in teaching probability (Rubel, 2002) and students' intuitive thinking (Fischbein & Scharch, 1997).

Students use their intuitions while solving problem situations in their lives or while solving probability questions (Shaugnessy, 1992). Fischbein (1987) indicated that people gain their intuitions from two sources: from experience and from previous educational background gained via regular instructions. In fact, students can easily comprehend and practice mathematical knowledge by using their intuitions including different mathematics topics (Fischbein, 1987) and proofs of mathematical algorithms (Weber & Alcock, 2004). However, the intuitions that they are about to use during solving problems are not always correct. The misleading effect of students' intuitions may result in misconceptions in students' minds (Myers, 2002). From the perspective of probability topic, situations in probability questions may include disconnected or even conflicting intuitions in students' reasoning processes (Havill, 1998). Fischbein (1975) argued that "undeveloped probabilistic intuition is not able to follow procedures of sophisticated reasoning or to guide the selection of such procedures or evaluate the plausibility of obtained results" (p. 131). With undeveloped probabilistic intuitions, students encounter with difficulties in solving probability questions and develop intuitively based misconceptions in their minds. These misconceptions are observed even in the basic probability (Li, 2000). In fact, there are many intuitively based misconceptions found in different studies including availability and representativeness heuristics (e.g., Çelik & Güneş, 2007; Fischbein & Schnarch, 1997; Kahneman & Tversky, 1982), simple and compound events (e.g., Fischbein, Nello, & Marino, 1991; Rubel, 2002; Shaugnessy, 1992), conjunction fallacy (e.g., Tversky & Kahneman, 1983), and conditional probability misconceptions (e.g., Falk phenomenon) (Falk, 1979; Fischbein & Schnarch, 1997; Watson & Kelly, 2007).

Among all, investigation of students' misconceptions regarding conditional probability is important because it necessitate to establish solution on the previously determined criteria. Therefore, students need to think about the conditions then continue their judgment and reasoning accordingly (Fischbein & Schnarch, 1997).

With instructions, students may develop their thinking about conditional probability and reach to the upper levels in the classification; student may still experience difficulties and misconceptions even in high school and university years (Tarr & Lannin, 2005). One of the misconceptions related to conditional probability is known as the Falk phenomenon or fallacy of the time axis (Batanero, & Sanchez, 2005; Fischbein & Schnarch, 1997; Jones, Langrall, & Mooney, 2007; Shaugnessy, 1992). This misconception is thinking that "an event couldn't condition another event that occurs before it" (Batanero & Sanchez, 2005, p.251). In regular conditional probability questions, $P(A/B)$ denotes the conditional probability of event *A* given event *B* where *B* precedes *A*. Gras and Totohasina (1995; as cited in Savand, 2014) classified students' conceptions about conditional probability. However, these cognitions contradict with the characteristics of Falk phenomenon. According to cardinal conception that was generally imposed in regular instructions, students interpreted $P(A/B)$ as the ratio $\frac{card(A \cap B)}{card(B)}$. Causal conception is about the cause and effect relation between the events *A* and *B*, where the *B* is cause and *A* is its effect. From this type of conception, chronological conception appears where the event *B* always precedes the event *A* (Gras & Totohasina, 1995; as cited in Savand, 2014). If, however, the sequence of happening of the events changes, students' reasoning contradicts with their intuitions and they fall into this misconception (Falk, 1979; Jones, Langrall, & Money, 2007; Shaugnessy, 1992).

Falk (1979) revealed the existence of this misconception based on the following situation. There are two white and two black balls in an urn. Two balls are blindly drawn from the urn one after the other without replacement. The first question asked is a routine conditional probability question that can be found in any textbook: "If the first ball drawn is known to be white, then, what is the probability of getting white ball in the second draw?" The second question asked is the inverse: "if the second ball drawn is known to be white, then, what is the probability of getting white ball in the first ball draw?" This second question often leads students to fall into a problematic intuitive thinking as students experience confusion about the sequence of happening of the events. While in routine conditional probability questions, the dependent event occurs after the independent one, students' intuitions contradict with their thinking when the first event precedes the independent events (Kazak, 2009). Fischbein and Schnarch (1997) used a question similar to the one used by Falk (1979) with students from different age groups and grade levels to investigate the development of their intuitively based misconceptions. They reported that increase in the grade level also increased the number of students who correctly answered the part *a* of the question but fell into Falk phenomenon in the part *b*. Same question was asked by Watson and Kelly (2007) who also reported similar findings across different grade levels. They found that while the students correctly interpreted conditional statement in the context of the question, they did not appreciate the influence of time of events happening.

Another task explaining this misconception was used by Bar-Hillel and Falk (1982). The questions was as follows; **"**Mr. Smith is the father of two. We meet him walking along the street with a young boy whom he proudly introduced his son. What is the probability that Mr. Smith's other child is also a boy?" (p. 109). Bar-Hillel and Falk stated that even mathematicians' cognitions contradicted in the solution of this question. One mathematician suggested that the probability is one-half, while the other mathematician stated that it is one-third. The former suggested that being a boy (B) or a girl (G) is independent of the other. The latter stated that the sample size was four at the beginning (i.e., BB, BG, GB, GG). Since one of the children is stated as boy, then the sample size decrease to three.

Similar situations in conditional probability were studied in other studies (e.g., Fox & Levav, 2004; Granberg & Brown, 1995) reporting the existence of Falk phenomenon with college students. It is hard for students to change such incorrect intuitions which might lead to intuitively based misconceptions, since their intuitions resistant to change in their minds (Fischbein, 1987). Resolving students' incorrect intuitions, therefore their intuitively based misconceptions, requires intense effort which can be provided with regular instructions in schools (Shaugnessy, 1992). As school mathematics curricula emphasizes students' comprehension of knowledge and deeper understanding in probability (CCSS, 2010; NCTM, 2000; MoNE, 2017), the importance of teachers' role and instructions in teaching probability is obvious. Related to teachers' role, Stohl (2005) stated that "the success of any probability curriculum for developing students' probabilistic reasoning depends greatly on teachers' understanding of probability as well as much deeper understanding of issues such as students' misconceptions and use of representations and tools" (p. 351).

Therefore, teachers need to have the knowledge of content as well as the knowledge of students' cognition and misconceptions. In the instructions, teachers are recommended to provide students with familiarity with probability situations in line with the students' misconceptions (Riccomini, 2005). Students' difficulty in solving probability questions is not only about computational procedures. In fact, students need to understand the necessary cognitive processes in the questions, set up strategies, and base the solution on the appropriate solution method (Zahner, 2005). At this point, teachers should provide students with opportunities to experience unfamiliar tasks and methods for solving them to overcome intuitively based misconceptions such as the time axis fallacy or Falk phenomenon.

There have been many studies to determine and resolve probability misconceptions with different instruction methods including games (Chiese & Primi, 2008; 2009) and simulations (Polaki, 2002a; 2002b). Furthermore, specific instructional methods were developed and their effectiveness was presented in various studies to overcome what is known as Falk phenomenon (e.g., Babai et al., 2006; Ojeda, 1999; Polaki, 2002a). However, we need to understand to what extend teachers' regular instructional practices in traditional classrooms have impact on students' understanding of probability and resolve their intuitively based misconceptions. Today, the literature in this subject is generally based on what the probabilistic misconceptions are and what the reasons for these misconceptions (e.g., Fischbein, Nello, & Marino, 1991; Kennis, 2006). In addition, it is known that teachers and, therefore, students do not give particular attention to intuitively based misconceptions in probability. Therefore, a need to conduct an empiric study with students to get data for informing the mathematics education researchers about the situations of teacher practices and instructional strategies used during the probability teaching and resolving misconceptions arose that are based on students' intuitive thinking. Since this study is a part of more comprehensive one, one of the intuitively based misconceptions (known as time axis fallacy) was investigated in detail in this study.

From the Turkey's point of view, literature mentions about the possible materials to teach probability such as computer aided materials (Gürbüz, 2008), dramatization (Şengül & Ekinözü, 2004), concept map (Gürbüz, 2006). However, teachers do not search and apply these materials or teaching strategies. Especially in Turkey, teachers' teaching practices were parallel to the teacher book for the related course book. Memnun (2008) explains this situation by stating that teachers do not use common language that all students could understand to develop probabilistic thinking. This language stemmed from the use of course book. Therefore, teachers generally considered the course book as only necessary material to develop necessary probabilistic thinking for students. Here, there was a need for in-depth understanding of how teachers organized their teaching practices during teaching probability and whether they considered students' possible misconceptions during their instructions.

Correspondingly, the purpose of this study is to investigate how high school teachers' instructional practices affect students' intuitively based misconceptions in probability, particularly the misconception known as time axis fallacy or Falk phenomenon, in resolving it. Furthermore, since teachers' knowledge of students' conceptions and misconceptions shapes their instructional practices, this study also investigate teachers' knowledge and awareness of students' such misconceptions. In doing so, teachers' practices in teaching the concept of conditional probability, selecting related examples and questions, and organizing the instructions were taken into consideration.

Method

In the present study, case study as one of qualitative research technique was applied. In the case studies, one person, unit or group is investigated. There is an opportunity to do in-depth investigation on a specific case or phenomenon in the studies classified as case studies (Creswell, 2013). In addition, the case studies allow the researcher to evaluate the data gathered in the context of reason-result relations (Fraenkel & Wallen, 2009). Considering this study, the case to be investigated was 11th grade students and their teachers as a group. Based on the purpose of the study, they are following the same curriculum and the current status of students' particular intuitively-based misconception and teachers' corresponding practices were investigated in-depth. This qualitative case study was a part of larger one aiming to investigate teachers' knowledge of middle and high school students' intuitively based misconceptions in probability and effect of their instructional practices for resolving them.

Participants

The participants of this study were three mathematics teachers from different high schools and their eleventh grade students. The teachers were Adem, Buğra, and Cahit (pseudonyms) teaching in vocational, Anatolian, and science high schools and had 17, 21, and 21 students in their classrooms, respectively. Teachers' demographic information is given in the Table 1 below.

In Turkish educational context, the schools were separated according to the achievement levels. Students with high levels of achievements usually enroll in science high schools after taking high school entrance exam subjected to eight grade students nationwide. On the other hand, most of the students with middle level of achievements were enrolling in Anatolian high schools, while those with lower level of achievements were enrolling in vocational high schools (based on teachers' opinions). In this study, one teacher was selected from each school type conveniently according to their willingness to participate this study.

Table 1.

Demographic information of the cases

Teachers	Gender	Age	Experience (years in teaching)	Type of School that the Teacher Works	Graduation Year and Department
Adem	M	27	2	Vocational High School	2009 – Department of Mathematics 2010 – Teaching Certificate
Buğra	M	32	8	Anatolian High School	2004 – Department of Mathematics 2006 – Teaching Certificate
Cahit	M	39	15	Science High School	1998 – Department of Mathematics Education

Instruments

The students in the participating teachers' eleventh grade classroom were administered a test called the Probability Test of Intuition (PTI) comprising of seven open-ended items to determine their intuitively based misconceptions in probability. The test was developed by the author and the test items were adapted from the problems found in the literature (e.g., Kahneman & Tversky, 1982; Falk, 1979; Fischbein & Schnarch, 1997; Watson & Kelly, 2007). The purpose was to prepare a test involving common intuitively-based misconceptions in probability. Expert opinions were gathered from one mathematics teacher who had 5 years teaching experience and one academician who had PhD degree in mathematics education and have studied teaching and learning probability. The test was reorganized according to experts' opinions and subjected to high school students as a pilot study. According to the pilot study results, the last version of PTI was prepared and administered as pre-test just before students started the probability unit and as post-test just after the teachers' instruction was over. The instruction period differed among the teachers. One of the items in the test intended to determine the existence of time axis fallacy or Falk phenomenon with these students. In this study, only this situation was investigated. The corresponding item was adapted from the one used by Fischbein and Schnarch (1997) as follows,

> There are equal numbers of blue and red balls in an urn. The ball chosen is not put into the urn. For two balls chosen one by one, compare the probabilities of the situations stated below. Justify your answers.
>
> a) given that the first ball chosen is blue, the probability of the second ball to be blue
> b) given that the second ball chosen is blue, the probability of the first ball to be blue

While the part (*a*) was routine conditional probability question that can be found in any textbook, the part (*b*) was about time-axis fallacy or the Falk phenomenon. In the part (*a*), the probability is less than one-half. If there were *n* blue and *n* red balls, the probability could be presented as $\frac{n-1}{2n-1}$ in the formal solution. The probability in the part (*b*) was one-half, because the further event does not affect the preceding one. Students were given 45 minutes to answer all the questions in the test.

Students' responses to the part (*a*) were codes as "correct", "incorrect", or "unanswered". If students stated that the probability of getting blue is lower, it was considered as correct answers in the part (*a*).

If there was no answer, it was counted as "ununswered". The examples of students' justifications indicating the "correct" and "incorrect" codes were as follows;

> Let's say there are n blue and n red balls. The probability getting blue ball after picking a blue ball without replacement is less than one half (Correct)
>
> The probability is equal to ½, since there are equal number of blue and red balls (Incorrect)

Their responses to part (*b*) were coded as "correct", "misconception", "incorrect", or "unanswered". If students presented the Falk phenomenon or the fallacy of time axis into the expected misconception, they were tailed into "misconception" code. On the other hand, if their answers were irrelevant to the misconception, they were coded as "incorrect". Students' responses of equality of probabilities in the part (*b*) were considered as correct. If there was no answer, it was counted as "ununswered". The examples of students' answers indicating correct, incorrect and misconception codes were as follows;

> The previous event is independent of the further one, so the probability is equal to ½ (Correct)
>
> If the second ball chosen is blue, the probability of getting blue ball in first selection is lower (Misconception)
>
> I think it is ¼ (Incorrect)

Classroom Observations and Teacher Interviews

Participating teachers' all lessons for probability unit were observed and videotaped by the researcher. The time allotted to teach probability in high schools was eight class-hours in the national secondary school mathematics curriculum. The teachers had little flexibility in changing teaching times according to their workload and students' understanding of the topics. The total number of class hours dedicated to teach probability by Adem, Buğra, and Cahit were ten, seven, and six hours respectively. For the purposes of this study, our focus will be on their instructional practices regarding teaching conditional probability. Adem spent one and a half class-hours for teaching conditional probability, while Buğra and Cahit spent one class-hour. They also spend some extra time for solving conditional probability questions during the mixed problem solving sessions. They all allotted their last lessons for solving mixed questions related to probability topic at the end of the unit. In the observations, teachers' practices in teaching the concept of condition, their instruction methods, and selection of the tasks were taken into consideration.

Prior to the study, the teachers were interviewed about their knowledge of students' difficulties and misconceptions in probability. The findings related to students' difficulties in conditional probability were presented. After the teachers finished teaching the probability unit, teachers were interviewed again and asked to determine what misconceptions might appear in the questions asked in the PTI. The possible reasons for these misconceptions were also sought with "how" and "why" questions. The interviews lasted about 30 to 55 minutes.

The researcher observed teachers' all instructions while they were teaching probability in classroom. The researcher sat on the back row seat in the classroom and took field notes regarding the purpose of the study. The researcher had no inclusion into the instruction or got in contact with students or teachers. After getting permissions from teachers and school administratives, the researcher also recorded the instructions with camera recorder. In case of specific situation (e.g., why teacher chose a particular example), the researcher did unstructured interview with teacher regarding this situation. These were also taken as field notes.

Data Analysis

In the analysis of the pre- and post-tests of the PTI, the descriptive analysis method was used (Fraenkel & Wallen, 2009; Yıldırım & Şimşek, 2006). In the descriptive analysis, the data gathered were summarized and interpreted according to the themes which were already determined. The frequency tables were presenting the number of students who fell into intuitively-based misconceptions of time-axis fallacy. In order to reflect students' thoughts about the questions and their justifications for the intuitively-based misconceptions, the direct quotations were presented from students' responses to answers in the PTI and also in the interviews with students.

The data that were gathered from interviews and classroom observations were analyzed according to content analysis method (Fraenkel & Wallen, 2009; Yıldırım & Şimşek, 2006). The procedure in content analysis is firstly to conceptualize the data gathered, to organize the data logically based on concepts appeared and, then, to determine the general themes that explain the data gathered (Yıldırım & Şimşek, 2006). In the data analysis, the main themes were students' responses indicating intuitively-based misconception of time-axis phenomenon, teachers' practices towards confronding it, and their knowledge of students regarding this misconception. In order to increase the reliability of the study, inter-rater reliability can be calculated (Fraenkel & Wallen, 2009). In the study, a research assistant doing her PhD in the field of mathematics education who had experience in descriptive and content analysis method also studied over students' pre- and post-test results of a one randomly selected classroom, one randomly selected teacher's interviews which were administered before and after the instructions, and total of five randomly selected observations. The inter-rater reliability rates for students' responses to the test item, interviews with teachers regarding time-axis fallay and their practices were %92, %89 and %91, respectively. These rates were over acceptable level of inter-rater level, which is being above %80 (Marques & McCall, 2005). The findings were presented after getting consensus between the coders.

Validity and Reliability

For the validity of the study, triangulation method was utilized. the data were gathered from different sources which were the observations of the classrooms, test results, interviews with students and three teachers, and field notes. Quotes from students' responses to open-ended questions were also included in this study. So, the author tried to provide rich and thick description about natural settings and the participants of the study. For the reliability of the study, the researcher observed the participants at different times, and searched whether the same observation and interpretations were made. The same interview form and questionnaire were administered to all participants, and the interviewer for all the interviews was the same person, who was the researcher. In order to increase the reliability, the researcher made a record of all data collection procedures and activities.

Findings

Pre-Test Results

Although conditional probability was not a topic officially placed and thus students are expected to learn prior to eleventh grade, students' correct responses to the part (*a*) of the question were high (see Table 2) before they formally taught the subject. It was found that, students generally used verbal justifications to solve the question.

Table 2.
Frequencies of Students' Responses for the Time Axis Fallacy Question in the Pre-test

Parts of the question	Responses	Adem's Class (*n*=17)	Buğra's Class (*n*=21)	Cahit's Class (*n*=21)	Total (*n*=59)
Part (a)	**Correct***	**6**	**17**	**11**	**34**
	Incorrect	6	3	10	19
	Unanswered	5	1	0	6
Part (b)	Correct	4	9	6	19
	Misconception*	**6**	**8**	**10**	**24**
	Incorrect	2	3	3	8
	Unanswered	5	1	2	8

* Correct answers in part (a) and answers reflected time axis fallacy as an intuitively based misconception in part (b) are highlighted in bold

As it was observed in the Table 2, about 58% of all students across three classes gave correct answers to part (*a*) in pre-test. Students eliminated one ball from the sample size, used the formula for the question, and found the correct answer. On the other hand, about 32% gave incorrect answer to this question. Interestingly, almost all students in the Anatolian high school gave correct answer to the part (*a*) of the question.

Although there one third of the students who gave correct answer to the part (*b*), almost half of all students fell into the intuitively based misconception. In general, students who fell into this misconception correctly answered the part (*a*). Only one student from Anatolian high school could not correctly answer the part (*a*) and fell into this misconception. The ratio between students who fell into the misconception in the part (*b*) and those who gave correct answer in part (a) was 71%. Therefore, almost three fourth of all students who gave correct responses to part (*a*) could not realize the independence of the preceding event from the latter one asked in the part (b).

Students' responses for the time axis fallacy were similar. Some of their justifications were as follows.

> If the second ball chosen is blue, then, the probability that the first ball chosen is blue is lower. Simply, since the balls are equally distributed in each urn, the probability of getting red is higher in the first selection.
>
> If the second ball chosen is blue, the probability of getting blue ball in first selection is lower.

Some students simplified the questions and considered that there were two white and two blue balls. Some other considered that there were five balls for each color. One answer was as follows.

> Let's say there are two white and two blue balls. If the second ball is blue, there will be two red ball and one blue ball in the first selection. Therefore, the probability of getting blue ball in the first selection is 1/3.

Teachers' Awareness of and Teaching Practices regarding Time-Axis Fallacy

From teachers' points of views, students had difficulties in two situations regarding the conditional probability. First, they stated that students struggled to distinguish whether the events given belonged to conditional probability. Second, even after students decided the question asked was about conditional probability, they experienced difficulty in determining which event was conditioned or independent one. All teachers stated that they provided shortcuts or showed keywords in their instructional practices to diminish such difficulties. In the pre-study interviews, teachers also emphasized the correct use of formula for conditional probability. Therefore, teachers were imposing the solutions of routine conditional probability questions, not the non-routine ones including those related to time axis fallacy or Falk phenomenon. Post-study interviews, on the other hand, indicated that teachers were aware of the possibility of existence of students' misconceptions of time axis fallacy.

> Students may experience difficulty because the color of second ball taken was given. So, they may experience in calculating the probability (Adem, post-study interviews).
>
> Students need to know that the probability of further event happening did not affect that of the preceding one (Buğra, post-study interviews).
>
> Students may ignore that the second situation did not affect the first one (Cahit, post-study interviews).

Although all teachers emphasized the independence of preceding event from the further one, Adem and Buğra indicated that they generally solved questions similar to those asked in the standardized exams such as the university entrance exam during the instruction. Therefore, their focus was on solving routine conditional probability questions. Buğra also stated that this misconception could affect their real life and make mistakes in their real lives. Moreover, Adem stated that they should give examples from real lives instead of just using question-answer interaction between students and teachers in classrooms.

In the interviews, teachers mentioned about the reasons for possible misconceptions in probability. They both mentioned about the reasons for time-axis fallacy and other types of misconceptions in general. The Table 3 summarizes teachers' opinions.

Table 3.
Reasons for intuitively-based misconceptions according to teachers' opinions

	Teachers		
Reasons for Intuitively-based Misconceptions	**Adem***	**Buğra**	**Cahit**
Insufficiency in readiness	√**	√	√
Rote memorization	√	√	√
Unable to imagine the problem situation	√	√	√
Unable to relate with daily life	√		
Low level of students' understanding	√		√
Necessity of thinking		√	√
Not being open to interpretation			√
Unable to synthese the facts			√
Unable to understand the logic of the probability		√	√
University entrance exam	√	√	√

*: All names are pseudonyms.
**: "√" indicates that the teacher considered the issue as a reason for students' difficulty in probability

The Table 3 gives general opinions regarding intuitively-based misconceptions, which can also be particularly attributed to time-axis fallacy. All teachers agreed on the the codes "insufficiency in readiness", "rote memorization", "unable to imagine the problem situation" and "university entrance exam" as reasons for intuitively-based misconception. For example, for the firs code, Barış indicated that;

> Mathematics is ongoing lesson which the subjects are built on one another. Therefore, they should be aware of some subjects before beginning to the probability and also to other subject.

What is interesting is that all teachers saw "rote memorization" as a reason for misconception. However, all teachers provided various rote memorizations during their teaching practices. They related this situation with the existence of "university entrance exam."

Teachers were asked how to confront this misconceptions, all teachers suggested the use of "course books", "supplementary books" for university entrance exam and "visual materials". They mentioned about what they always did in their regular instructions. Solving as many problem as they could do was seen as a way to confront any kind of misconceptions in probability. In practice, none of the teachers used visual materials in teaching conditional probability.

After observing teachers' instructions, it was realized that teachers followed similar manners in teaching probability. They began with introducing and teaching basic concepts, then present and solve problems regarding what was taught. While doing so, they tried to enrich their instruction by choosing different types of questions regarding the topic. Therefore, teachers' instructional practices were investigated based on three focal points, which were teaching the concept of condition, methods of instruction in solving questions, and the selection of the questions solved in the classroom. For developing the concept of condition, all teachers gave formal definition of condition and conditional probability from textbooks. While Buğra and Cahit directly wrote the definitions on the board, Adem discussed the concept of condition with his students. He tried to have a consensus about the meaning of the condition. Then, they all provided the following formula for calculating the probability of the event *A* given that the event B, $P\left(A/B\right)=\dfrac{P(A\cap B)}{P(B)}, P(B)>0$. They all emphasized that the happening of event *A* depend on the happening of event *B*. In other words, they indicated the importance of the sequence of the events happening in the conditional probability. The focus was that the event *B* must happen before the event *A* happens. Contrary to the other two teachers, Adem simplified the formula by assuming that students could experience difficulty in using it to solve questions. He stated that the probability could also be found by dividing the number of elements of $A Ç B$ by the sample size for the event *B*. This information was provided theoretically without any example.

Teachers preferred to explain the meaning and applications of the formula by solving questions related to conditional probability. However, they expected to memorize the formula and lead students to use it properly while solving such types of questions. Teachers emphasized the importance of keywords to determine whether a question given is about conditional probability. For example, teachers stated that if the question included keywords like "*known to be*", students should know that it is related to conditional probability. It appeared that, although teachers directed students to rote memorizations in approaching conditional probability questions, it was observed that students had difficulty in memorization and applying them for the questions specific to conditional probability.

> If there are statements like "known to be", it is conditional probability.

> The sample size of the conditional probability is the sample size of the event B.
>
> The expected elements of the conditional probability are the intersection of the events A and B.
>
> If the occurrence of the event A is dependent on the event B, it is conditional probability.

For this situation, all teachers agreed upon students' difficulty in remembering the appropriate statements while solving probability problems. They stated that this difficulty stemmed from the abundacy of types of probability problems. Buğra stated as follows;

> For each subject in probability, students need to remember various statements. When encountering a problem, firstly, they had to determine the type of problem. Whether it is inclusive-mutually exclusive, sample size, dependent-independent or other type of problems… That is why remembering the correct statement and applying it to the given problem is very hard for students.

Moreover, Cahit indicated a short conversation with students while solving hard problems. After presinting a probability problem (whether it is time-axis fallacy type of question or not), students were asking the type of question, so they could begin to solve it. Cahit also indicated that students had to memorize and correctly apply to the problems they encountered. He continued as follows;

> Students will take university entrance exam. There will be a time limitation to solve many questions and the questions are multiple-choice type. It would be better to meaningfully understand the topic and solve questions but we do not have enough time. They have to memorize and get familiar to them. They have to solve as many problem as they can, so they can be successful.

Teachers' instructional practices regarding conditional probability were mostly centered on solving standard questions selected from various supplementary books written for nationwide university entrance exams. The usual practice was to write the question on the board, wait few minutes for students to work on it and then apply standard algorithms to solve it. Teachers expected students to identify the event *A* and event B in the question text and directed students to the formula for conditional probability to find numeric values of $P(A \cap B)$ and $P(B)$. Once they found the right answer, they moved to another question. While Buğra and Cahit usually asked students whether they understood the solution, Adem went through the solution one more time for each question.

Adem and Buğra solved only four questions related to conditional probability in their classrooms. They chose the questions randomly from the supplementary books for university entrance exams instead of using the course textbook. These questions were routine questions that were in line with those appeared previous nationwide university entrance exams. Such questions were useful for solving part *a* of conditional probability question asked in this study. However, teachers did not solve non-routine questions that might reveal if students have any intuitively based misconceptions. Only one such question was asked in Buğra's classroom as follows.

> There are two yellow and three red balls in the first urn and three yellow and four red balls in the second urn. It is known that a ball taken form is red in the second urn, what is the probability that the red ball is taken from the first urn?

Both Buğra and his students experienced difficulty in solving this question. Before starting the solution, Buğra checked the solution from the book the question was taken. He then put the facts/givens into the formula and found the numeric value. He did not explain the time factor in the question even though a second event was provided in the question.

On the other hand, regarding conditional probability, Cahit solved ten questions some of which were routine questions asked by the other two teachers in this study. He tried to spend as little time as possible in solving them. However, his students needed to think more in the following three questions.

> A coin is thrown two times. It is known that one outcome is head. What is the probability of getting tail in the second outcome?
>
> A die is thrown two times. It is known that one outcome is four. What is the probability of getting odd number in the second outcome?
>
> There are five red and three white balls in an urn. It is known that a ball selected among two balls is red. What is the probability that the other ball is white?

Although these questions were not related to time axis fallacy, being as non-routine questions, understanding the logic behind these questions might have helped students to resolve such misconception. In solving these questions, Cahit first determined the sample sizes (i.e. {HH, HT, TH} in the first question) and the set of expected elements (i.e. {HT, TH} in the first question). He then found the answers by substituting the facts/givens into the formula for conditional probability.

Post-Test Results

After students received regular instruction, the frequencies for the correct answers and misconception changed (see Table 4).

Table 4.
Frequencies of Students' Responses for the Time Axis Fallacy Question in the Post-test

Parts of the question	Responses	Adem's Class (n=17)	Buğra's Class (n=21)	Cahit's Class (n=21)	Total (n=59)
Part (a)	**Correct***	**11**	**17**	**21**	**49**
	Incorrect	6	3	0	9
	Unanswered	0	1	0	1
Part (b)	Correct	3	6	15	24
	Misconception*	**8**	**12**	**6**	**26**
	Incorrect	6	3	0	9
	Unanswered	0	0	0	0

* Correct answers in part (a) and answers reflected time axis fallacy as an intuitively based misconception in part (b) are highlighted in bold

Almost 85% of the all students found the correct answer for the first part of the question in the post-test. Among them, all science high school students (Cahit's classroom) and almost all students in Anatolian high school (Buğra's classroom) found the correct answers. In general, students used the formal solution. Many of them stated that the probability in the first part was $\frac{n-1}{2n-1}$, where *n* is the number of white or blue balls. In addition, some students considered that there were two (or five) blue and two (or five) red balls in the urn.

On the other hand, while the correct answers increased from 15 to 24 students for the second part of the question, the number of students whom presented time axis fallacy slightly increased to about 45% of all students. Interestingly, while the number of students who fell into this misconception increased in both vocational and Anatolian high schools, the inverse observed for the students in the science high school where the number of students presented this misconception decreased from 10 to 6 students and the correct answers increased from 6 to 15 students.

Responses students provided for the second part of the question were similar in nature different than the pre-test, students tried to use the general probability formula for approaching this question in the post-test. However, they still ignored whether the latter event affected the probability of preceding event. They considered that one ball was drawn while calculating the probability. Only the students in the science high school were successful in solving the question correctly.

Discussion and Conclusion

As the literature suggests the misconception of time axis fallacy appears in different grade levels in high schools (Fischbein & Schnarch, 1997; Watson & Kelly, 2007) and the results of this study confirms that. In this study, there were 34 students who correctly answered the part (*a*) of the question related to Falk phenomenon, while the number of students who fell into the misconception of time axis fallacy was 24 as students' answers to the part (*b*) suggested. Although, the number of correct answers increased for the part (*b*) immediately after teachers' regular instruction regarding the conditional probability, there was slight increase in the number of incorrect intuitions. Most of the correct answers in the part (*b*) were observed in Cahit's classroom. On the other hand, the correct answers the part (*b*) decreased in Adem and Buğra's classrooms. In fact, Adem and Buğra did not provided unfamiliar situation related to conditional probability in their instructions. It is suggested that students should get familiarity with different situations and question types to reduce the risk of time axis fallacy (Fox & Levav, 2000). Otherwise, it is highly possible that unfamiliar situations in probability strain students' intuition and lead to misconceptions (Papaieronymou, 2009). With this respect, it was observed that Cahit provided unfamiliar situations in his teaching practices. At the end, almost three-fourth of his students was successful in solving the part (*b*). Even so, there were six students whom still exhibited time axis fallacy.

In comparing the results of pre- and post-tests, the Falk phenomenon stayed still among the high school students in this study. The number of students who fell into this misconception was almost the same after the teachers' instructional practices. In fact, teachers' teaching practices were developing cause and effect relations in students' minds. Considering Gras and Totohasina's (1995; as cited in Savand, 2014) classification of students' cognition for conditional probability, all teachers imposed the causal and chronological conceptions while solving related questions. Independent and conditioned events in the questions asked in the classrooms were presented as cause and its effect, respectively. In addition, all questions except the one presented in Adem and Buğra's classrooms were imposing the chronological order that the independent event (event *B*) always preceded the conditioned one (event *A*). Although one question asked in Buğra's classroom had similar logic and structure with the Falk phenomenon, Buğra based the solution on the formula. He also experienced difficulty in solving it. On the other hand, Adem directly provided the formula for cardinal conception as a shortcut in the concept development phase. Similarly, Cahit used and briefly explained this formula during the problem solution sessions. All in all, the conceptions teachers imposed in their teaching practices contradicted with the characteristics of Falk phenomenon due to time inversion between the independent and conditioned events. Therefore, it could be concluded that instructional practices of the high school teachers in this study were not effective in resolving time axis fallacy in students' minds, especially those in Adem and Buğra's classrooms. With teaching practices, students were better able to manage the causality and time inversion between the events especially in Cahit's classroom. However, students could not employ them by means of Adem and Buğra's instructional practices.

Continuing with students' intuitive thinking, students whose responses were consistent with the time axis fallacy in the pre- and post-test were in the informal quantitative level in Tarr and Jones's (1997) classification of students' thinking levels about conditional probability. They were distinguishing the independent and conditioned events. However, their differentiation of the events was imprecise. In addition, they were unsuccessful in producing the sample sizes for the events in the question. Especially Adem and Buğra's instructional practices were not enough to make students develop progress toward the numerical level suggested by Tarr and Jones (1997).

Although it is hard to change students' incorrect intuitions (Fischbein, 1987), teachers' instructional practices were not in line with determining or changing their intuitions regarding time axis fallacy in this study. The interviews with teachers revealed their surface knowledge about students' cognitions regarding conditional probability. They mentioned about the intuitively based misconception of the time axis fallacy only when the related question was provided to them. Correspondingly, their instructional practices were based on following the course textbook or supplementary books through providing formulas and rote memorization of procedures for solving certain kinds of problems. Many documents and studies mentioned about the importance of visual materials in teaching processes (e.g., CCSS, 2010; Gürbüz, 2008; Kazak, 2008; MoNE, 2017; Polaki, 2002a). In addition, teachers also mentioned about the importance of use of visual materials in teaching. However, they did not utilize any kind of visual material in teaching conditional probability. Therefore, students' incorrect intuitions stayed still at the end of teachers' instructional practices.

Another important finding was that all teachers saw the use of "rote memorization" as a main reason for intuitively-based misconceptions. For example, Rubel (2002) implied how abundancy of using *if-then* statements hardens learning probability. However, teachers also indicated that they had to learn them, select the correct one and apply to the problem in order to be successful problem solver. This situation was also seen in Demirci, Özkaya and Konyalıoğlu's (2017) study. They saw the "rote memorizations" as essentials of learning probability, and particular to this study, of learning conditional probability. Teachers advocated that the existence of "university entrance exam" became the use of "rote memorization" as mandatory part of learning process.

On the other hand, it was found that instructional practices helped the students to solve routine questions related conditional probability (Evans, 2006), while they encountered difficulty in solving a non-routine type (Çelik & Güneş, 2007). In line with this situation, CCSS (2010) also emphasizes the importance of solving non-routine questions and applying and adapting different kinds of appropriate strategies to solve problems. As the part (*a*) of the question was a routine one, the findings showed that the number of students who correctly answered the question increased from 34 to 49 after the instruction. However, the same situation was not valid for the part (*b*) revealing the intuitively based misconception of time axis fallacy as a non-routine one. This result can be attributed to several reasons. First, it was observed that all teachers used supplementary books for university entrance exams in their lessons. The nationwide university entrance exam was the focal point for the teachers' instructions. In general, they asked questions similar to those asked in previous university entrance exams. These questions could be considered as routine ones. The findings of Köğce and Baki's (2009) study supported this idea that, in the written exams, high school teachers generally ask questions relatively similar to those asked in university entrance exams. Thus in this study, students were rather successful in solving such questions. Second, the teachers were expecting students to memorize formulas, rules, shortcuts while solving probability questions. Although rote memorizations were useful for students to understand the topic, excessive number of rules and formulas influence students negatively (Gürbüz, 2008). According to Memnun (2008) students confuse which rules to use when determining the event types and solving questions. This situation was considered as a reason for possible misconceptions in probability. Especially in the session of solving mixed questions, students questioned teachers to learn the type of event in the questions in a few instances, so they could apply the appropriate formula to solve them.

Suggestions

This study investigated particular type of intuitively-based misconception (time-axis fallacy) in probability. As seen in the current literature (e.g., Batanero & Sanchez, 2005; Çelik & Güneş, 2007; Fischbein, 1987; Watson & Kelly, 2007), there are various intuitively-based misconceptions that students commonly fell into. Therefore, there might be further studies that investigates students' current status for other types of intuitively-based misconceptions in probability, teachers' knowledge regarding them and their teaching practices to confront them.

This study seeked for teachers' knowledge about students' misconceptions and the factors resulting in them for time-axis fallacy. However, teachers' beliefs about probability topic and students' thinking processes were missed in this study. It is another factor that influences teachers' teaching practices. If their beliefs are consistent with students' thinking processes and intuitions, students can benefit from appropriate teaching practices to get rid of incorrect intuitions. In line with this situation, teachers' beliefs about the topic and students' intuitive thinking can be investigated.

In order to resolve students' intuitively-based misconceptions, different teaching methods can be compared with the regular instruction in experimental studies. For example, Polaki (2002a) conducted an experimental study for sample size misconception. In this study, it was found that the regular instructions were not effective in resolving intuitively-based misconceptions. An instruction can be organized specially for different types of intuitively-based misconceptions. The effective methods in resolving such misconceptions can be determined via comparing the methods.

The findings of the study indicated that intuitively-based misconceptions existed among high school students. In addition, these misconceptions continued to exist after regular instructions. When asked to teachers, they indicated students' possible misconceptions by means of their experiences. Before teaching the probability, teachers should have knowledge about the possible misconceptions and prepare the lessons accordingly. In doing so, teacher training programs should provide pre-service teachers with the knowledge of possible misconceptions, the factors causing difficulties in learning the subject, and the methods to resolve these misconceptions.

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Author

Dr. Mehmet Fatih ÖÇAL is lecturing in the Department of Mathematics and Science Education at Ağrı İbrahim Çeçen University, Ağrı, Turkey. His research areas mainly focus on problem solving and students' misconceptions and mistakes in mathematics.

Contact

Dr. Mehmet Fatih ÖÇAL, Ağrı İbrahim Çeçen Üniversitesi, Matematik ve Fen Bilimleri Eğitimi Bölümü, 04100, Ağrı, Türkiye.
e-mail: fatihocal@gmail.com