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Geometric design, additive manufacturing, and investigation of contact characteristics of a logarithmic spiral spur gear pair

Logaritmik spiral düz dişli çiftinin geometrik tasarımı, katmanlı imalatı ve temas karakteristiklerinin incelenmesi

***Yazar(lar) (Author(s))**: Mehmet YAZAR[1]*

ORCID[1]: 0000-0003-3999-3233

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Geometric Design, Additive Manufacturing, and Investigation of Contact Characteristics of a Logarithmic Spiral Spur Gear Pair

Highlights

- *The logarithmic spiral gear profiles are generated using a specially developed parametric design algorithm*
- *The kinematic performance of the gear pair is analyzed in a CAD environment.*
- *This study demonstrates that the contact path is continuous in logarithmic spiral gears.*
- *This contributes to a more balanced load distribution.*

Graphical Abstract

The design, manufacturing, and contact analysis processes of logarithmic spiral gears have been addressed using a comprehensive approach.

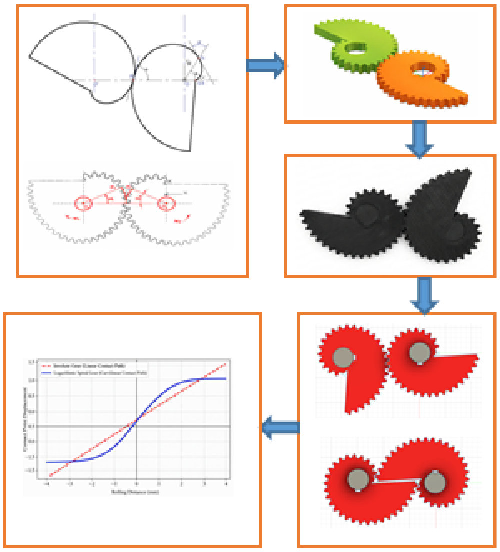

Figure. The design, manufacturing, and analysis process of a logarithmic spiral gear

Aim

The aim of this paper is to take a comprehensive approach to the design, manufacturing, and contact analysis processes of logarithmic spiral gears.

Design & Methodology

Logarithmic spiral gear profiles are generated using a parametric design algorithm developed specifically for this project; the resulting two-dimensional gear profiles are converted into three-dimensional solid models, and physical prototypes are manufactured using the FDM (Fused Deposition Modeling) additive manufacturing method with PLA (polylactic acid) filament.

Originality

The parametric design, additive manufacturability, and contact path characteristics of logarithmic spiral gear pairs in the non-circular gear category have been investigated.

Findings

This study demonstrates that the contact path is continuous in logarithmic spiral gears.

Conclusion

The results indicate that this leads to a wider contact area and a more balanced load distribution.

Declaration of Ethical Standards

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

Geometric Design, Additive Manufacturing, and Investigation of Contact Characteristics of a Logarithmic Spiral Spur Gear Pair

Araştırma Makalesi / Research Article

Mehmet YAZAR[1*]

[1]Çanakkale Onsekiz Mart University, Çanakkale Technical Sciences Vocational School, Department of Machinery and Metal Technologies, Türkiye

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ABSTRACT

In this study, the parametric design, additive manufacturing, and contact path characteristics of logarithmic spiral spur gear pairs, which fall under the category of non-circular gears, have been examined. Within the scope of this study, a parametric design algorithm has been developed to create logarithmic spiral spur gear profiles. The two-dimensional gear profiles obtained using the developed algorithm have been converted into three-dimensional solid models, and physical prototypes have been produced using the FDM (Fused Deposition Modeling) additive manufacturing method with PLA (polylactic acid) filament. The kinematic behavior of the gear pair was analyzed in a CAD program to determine the contact points formed at different angular positions, and the contact path was obtained by connecting these points. The analysis results have shown that the contact path in logarithmic spiral spur gears exhibits a curved rather than a linear character and that the contact is continuous along the tooth surface. This may contribute to a wider contact area and a more balanced load distribution. The findings indicate that the logarithmic spiral spur gear geometry offers distinct contact characteristics that could serve as an alternative to traditional involute gears and can be successfully manufactured using additive manufacturing methods. This study aims to contribute to the literature by adopting a comprehensive approach to the design, manufacturing, and contact analysis processes of logarithmic spiral spur gears.

Keywords: Non-circular gears, logarithmic spiral gear, parametric design, additive manufacturing, contact analysis

Logaritmik Spiral Düz Dişli Çiftinin Geometrik Tasarımı, Katmanlı İmalatı ve Temas Karakteristiklerinin İncelenmesi

ÖZ

Bu çalışmada, dairesel olmayan dişli kategorisinde yer alan logaritmik spiral düz dişli çiftlerinin parametrik tasarımı, katmanlı imalatı ve temas yolu karakteristikleri incelenmiştir. Çalışma kapsamında, logaritmik spiral düz dişli profillerinin oluşturulabilmesi amacıyla parametrik bir tasarım algoritması geliştirilmiştir. Geliştirilen algoritma yardımıyla elde edilen iki boyutlu dişli profilleri, üç boyutlu katı modellere dönüştürülmüş ve FDM (Fused Deposition Modeling) tipi katmanlı imalat yöntemi ile PLA (polilaktik asit) filament kullanılarak fiziksel prototipler üretilmiştir. Üretilen dişli çiftinin kinematik davranışı CAD ortamında analiz edilerek farklı açısal konumlarda oluşan temas noktaları belirlenmiş ve bu noktaların birleştirilmesiyle temas yolu elde edilmiştir. Analiz sonuçları, logaritmik spiral düz dişlilerde temas yolunun doğrusal değil eğrisel bir karakter sergilediğini ve temasın diş yüzeyi boyunca süreklilik gösterdiğini ortaya koymuştur. Bu durumun, temasın daha geniş bir alana yayılmasına ve yük dağılımının daha dengeli gerçekleşmesine katkı sağlayabileceği değerlendirilmiştir. Elde edilen bulgular, logaritmik spiral düz dişli geometrisinin geleneksel involüt dişlilere alternatif olabilecek farklı temas karakteristikleri sunduğunu ve katmanlı imalat yöntemleri ile başarılı bir şekilde üretilebildiğini göstermektedir. Bu çalışma, logaritmik spiral düz dişlilerin tasarım, üretim ve temas analizi süreçlerini bütüncül bir yaklaşımla ele alarak literatüre katkı sağlamayı amaçlamaktadır.

Anahtar Kelimeler: Dairesel olmayan dişliler, logaritmik spiral dişli, parametrik tasarım, katmanlı imalat, temas analizi.

1. INTRODUCTION

Gear mechanisms are among the fundamental machine elements widely used for power and motion transmission, and they have a broad range of applications in industrial settings. In traditional gear designs, the involute tooth profile is typically preferred; this profile is considered a standard solution due to its advantages, such as providing a constant speed ratio and ease of manufacturing [1]. However, in some engineering applications, the need for variable speed ratios, specific motion requirements, and different contact characteristics arises, necessitating the design of alternative gear geometries. Gears produced using these alternative geometries are known as non-circular gears. [2], [3].

Non-circular gears have been used in various mechanical systems for many years due to their ability to convert input motion into variable output motion as desired. Historically used in clock mechanisms, astronomical

*Sorumlu Yazar (Corresponding Author)
e-posta : mehmetyazar@comu.edu.tr

observation instruments, and automatic machines [4], [5], these gears continue to play an important role in various mechanical systems today. The literature indicates that non-circular gears offer simpler and more economical solutions as alternatives to cam mechanisms [6], [7]. Furthermore, the ability of these gears to control variable speed and torque characteristics makes them a preferred choice in various engineering applications [3], [6], [8], [9].

However, the complex geometric structures of non-circular gears complicate the design and manufacturing processes and limit their widespread use. The use of traditional methods (hobbing, planing, etc.) in the production of these gears requires specialized cutting tools and complex processing steps [10], [11], [12], [13]. While methods such as hobbing and gear shaping (Fellows method) offer advantages for mass production, they are limited in the production of gears with variable pitch curves due to their reliance on fixed cutting tool geometry [14]. Additionally, the mathematical modeling of the tooth profile and issues such as undercutting are important factors that must be considered during the design process [15]. To overcome these limitations, more advanced manufacturing techniques have been developed. CNC-based machining methods and wire electrical discharge machining (WEDM) technologies enable the direct production of complex gear geometries from a digital model. While these methods offer high precision, they are typically preferred for low-volume production due to their long processing times and high costs [13].

In recent years, advancements in computer-aided design (CAD/CAM) and numerically controlled manufacturing technologies have made it possible to design and manufacture gear systems with complex geometries more precisely and cost-effectively [16]. In particular, additive manufacturing technologies enable the low-cost production of complex curved geometries [17], [18], [19], [20], [21]. In this context, the effect of production parameters and print orientation on part performance in additive manufacturing processes has become a significant area of research [22], [23], [24].

Although studies have been conducted on non-circular gears and their tooth profiles, the contact characteristics of non-circular gears have not yet been sufficiently investigated. Therefore, conducting research on a new gear geometry to examine the effects of the contact characteristics of non-circular gears is of great importance. Logarithmic spiral curves are one of the geometric structures frequently found in nature and used in biomimetic design approaches. Applying these curves to gear profiles can offer variable transmission ratios and unique contact characteristics, unlike traditional involute gears. In a study [25], the effects of logarithmic spiral parameters on gear design were examined, revealing that this geometry has the potential to enhance mechanical performance. Similarly, in another study [26], the design processes and dynamic behavior of asymmetric logarithmic spiral spur gear mechanisms were examined, demonstrating that this gear type can be utilized in alternative mechanical systems.

It is evident that there is a limited body of research in the literature on logarithmic spiral spur gear profiles, particularly regarding their contact characteristics. Furthermore, it is noteworthy that most existing studies do not address the design, manufacturing, and contact analysis processes through a holistic approach. This situation prevents the full potential of logarithmic spiral spur gears in engineering applications from being realized.

In this study, the parametric design, additive manufacturing, and contact path characteristics of logarithmic spiral spur gear pairs were investigated using a holistic approach. Within this scope, a novel design algorithm capable of parametrically generating logarithmic spiral spur gear profiles was developed. The resulting gear profiles were modeled in a CAD environment, physical prototypes were produced using additive manufacturing, and the meshing behavior of the gear pair was analyzed for different angular positions. The aim is for the obtained results to contribute to the understanding of the meshing characteristics of logarithmic spiral spur gear and to facilitate the evaluation of this gear geometry as an alternative design approach.

2. MATERIAL and METHOD

2.1. Rack Cutter Design

A widely adopted approach for generating tooth profiles of both circular and non-circular gears is based on the principle of rolling a rack cutter over the gear surface without slipping. In this study, the geometry of the rack cutter was mathematically defined using complex numbers in order to generate the logarithmic spiral spur gear profile.

The coordinates of the rack cutter tooth profile are expressed in the complex plane using Equations (1)–(5).

$$p_{c_0} = m\left(-\frac{\pi}{4} - h\tan\left(\alpha\right) - Ih\right) \tag{1}$$

$$p_{c_1} = m\left(-\frac{\pi}{4} + \left(h+c\right)\tan(\alpha) - I\left(h+c\right)\right) \tag{2}$$

$$p_{c_2} = m\left(\frac{\pi}{4} - \left(h+c\right)\tan\left(\alpha\right) + I\left(h+c\right)\right) \tag{3}$$

$$p_{c_3} = m\left(\frac{\pi}{4} + h\tan\left(\alpha\right) - Ih\right) \tag{4}$$

$$p = p_{i-4} + m\pi \quad i = 4,5,\ldots \tag{5}$$

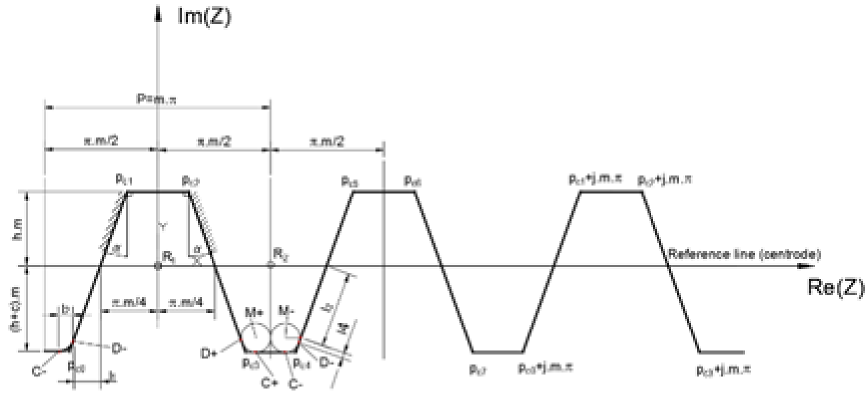

Figure 1. Reference profile of the rack cutter

Figure 1 illustrates the reference profile of the rack cutter. Here, i denotes the imaginary unit, α represents the tooth profile angle, m is the module, h+c denotes the dedendum height, and h represents the addendum height. Furthermore, the corner fillet radius of the rack cutter is assumed to be zero. This profile is considered as the primary reference in the creation of gear geometry.

The generation of the tooth profile is based on the principle of the rack cutter moving over the gear blank without slipping, i.e., through pure rolling motion. Under this condition, the pitch line of the rack cutter is tangent to the rolling center line of the gear at point P. This contact point plays a critical role in the formation of the tooth profile.

In order to ensure the geometric continuity of the rack cutter and to prevent sharp transitions that may occur at the corner points, the corner points must satisfy Equations (6) and (7).

$$w_i = \left(p_{c_i} - L\right)e^{\left(I(\phi+\mu)\right)} + \mathrm{Re}^{(I\phi)} \tag{6}$$

$$w_{i+1} = \left(p_{c_{i+1}} - L\right)e^{\left(I(\phi+\mu)\right)} + \mathrm{Re}^{(I\phi)} \tag{7}$$

The transition of the rack cutter from its initial position to the working position is defined by three fundamental motion components:

- Translation of the cutter along the axes by a distance of L
- Rotation of the cutter about point O, which is considered the gear center, by a specified angle
- Re-translation of the cutter to its new position

Through these transformation steps, the rolling motion of the rack cutter over the gear surface can be mathematically modeled.

Figure 2 illustrates the successive positions obtained as a result of the rack cutter rolling without slipping along the logarithmic spiral axis. These positions were calculated using Equations (6) and (7) and represent the process of tooth profile generation.

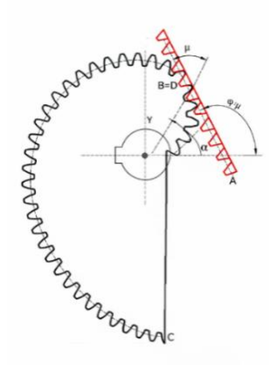

Figure 2. Rolling of the rack cutter without slipping along the logarithmic spiral axis

2.2. Logarithmic Spiral Spur Gear Design

The geometric design of the logarithmic spiral gear pair is based on the rolling principle of logarithmic spiral curves. Figure 3 illustrates the geometric configuration of a gear pair consisting of two logarithmic spirals with identical centroids. This configuration plays a fundamental role in determining the contact and motion characteristics of the gear pair.

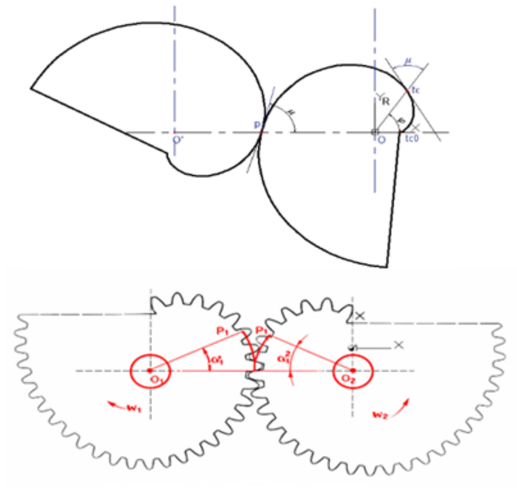

Figure 3. Geometric representation of a rolling pair formed by logarithmic spiral curves

The logarithmic spiral curve is defined in the polar coordinate system by Equation (8)

$$R = ae^{b\phi} \tag{8}$$

In this equation, the radius varies exponentially as a function of the angle, which constitutes the fundamental reason why logarithmic spiral gears provide a variable transmission ratio. The angle between the tangent line and the radius at any point on the spiral curve is expressed by Equation (9)

$$\mu = \arctan\left(\frac{1}{b}\right) \tag{9}$$

Furthermore, the arc length along the curve can be expressed in closed form by Equation (10), and this expression is utilized in the generation of the tooth profile.

$$L = \frac{a\sqrt{1+b^2}\left(e^{(b\phi)}-1\right)}{b} \tag{10}$$

The logarithmic spiral gear profile is obtained as a result of the rack cutter rolling without slipping along the spiral curve. In this context, the tooth surfaces are defined in the complex plane by considering their real and imaginary components.

Equations (11) and (12) are used for the right tooth surface.

$$\Delta x = \left(2h+c\right) m \tan\left(\alpha\right) \tag{11}$$

$$\Delta y = \left(2h+c\right) m \tag{12}$$

Equations (13) and (14) are used for the left tooth surface.

$$\Delta x = -\left(2h+c\right) m \tan\left(\alpha\right) \tag{13}$$

$$\Delta y = -\left(2h+c\right) m \tag{14}$$

The general tooth profile covering both surfaces is expressed by equation (15).

$$\Delta x^2 + \Delta y^2 = \left(2h+c\right)^2 m^2 \left(1+\tan\left(\alpha\right)^2\right) \tag{15}$$

This mathematical model enables the parametric generation of the logarithmic spiral spur gear profile.

In this study, a parametric design algorithm was developed based on the equations presented above. The developed software operates within the CAD environment and is capable of automatically generating the gear geometry depending on the parameters specified by the user.

The operating principle of the software is summarized by the flowchart presented in Figure 4.

When the program is executed, fundamental gear parameters such as module, number of teeth, and pressure angle are obtained from the user, and the coefficients of the logarithmic spiral curve are determined accordingly. Subsequently, the coordinates of the tooth profile are calculated by considering the rolling motion of the rack cutter along the spiral curve, and the geometric structure of all teeth is generated.

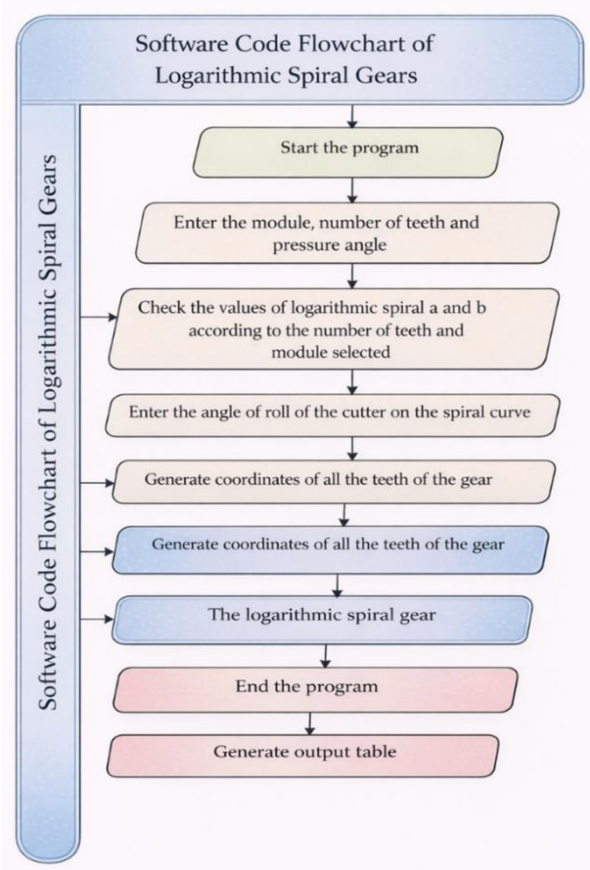

Figure 4. Flowchart of the logarithmic spiral spur gear design software

The design parameters used in this study are given in Table 1.

Table 1. Design parameters of the logarithmic spiral spur gear

Parameter	Value
Module	2
Number of teeth	30
Pressure angle	20°
Initial radius (a)	20 mm
Spiral rate of increase (b)	0.2
Center-to-center distance	73 mm
Material	PLA

The parameters listed in Table 1 constitute the basic input data used in the design of the logarithmic spiral spur gear geometry.

The gear profiles were first modeled in two dimensions (2D) and then converted into three-dimensional (3D) solid models. Two- and three-dimensional views of the resulting logarithmic spiral straight-tooth gear design are shown in Figure 5.

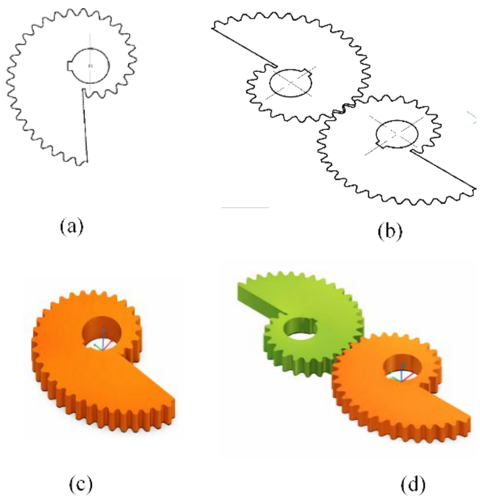

Figure 5. Geometric models of the logarithmic spiral spur gear: (a) 2D gear profile, (b) 2D view of the gear pair, (c) 3D model of a single gear, and (d) 3D model of the gear pair

The meshing condition for a logarithmic spiral spur gear pair is based on the principle that the points of contact share the same line of contact simultaneously. For this reason, the center distance of the gear pair is determined to ensure continuous contact.

In the gear pair used in this study, although the geometric parameters of both gears are identical, smooth meshing is achieved through the spatial variation of the contact points.

2.3 Additive Manufacturing

Additive manufacturing methods are used to obtain physical prototypes of logarithmic spiral gear pairs. The manufacturing processes were carried out using a Snapmaker 2.0 A350 three-dimensional printer. PLA (polylactic acid) filament, which is widely used and exhibits high dimensional stability, was selected as the manufacturing material.

During the additive manufacturing process, the printing parameters were optimized to ensure the accurate fabrication of the variable-radius geometry of the logarithmic spiral gear. The production parameters for additive manufacturing are given in Table 2.

During the manufacturing process, a layer height of 0.2 mm was selected to achieve a balance between surface quality and dimensional accuracy. Considering the loads to which the gears would be subjected during operation, the infill ratio was set at approximately 60%, and a “Grid” pattern providing isotropic strength was used as the internal structure. This approach enhanced the mechanical strength of the gears.

Table 2. 3D printing parameters for the fabrication of the logarithmic spiral gear

Parameter	Value
Printer model	Snapmaker 2.0 A350
Nozzle diameter	0.4 mm
Filament type	PLA
Filament diameter	1.75 mm
Extrusion temperature	210 °C
Print bed temperature	60 °C
Layer height	0.2 mm
Number of walls	3
Infill ratio	%60
Infill pattern	Grid
Print speed	50 mm/s
Fan cooling	%100

In this study, gear samples were produced in a horizontal orientation parallel to the build plate in order to achieve better surface quality and mechanical strength on the gear surfaces. In this orientation, the layer lines run parallel to the gear thickness, thereby minimizing the effects of layering that might occur on the gear surface. This reduces the concentration of stresses generated during contact in the weak interlayer regions.

It is well known that the printing orientation has a significant effect on the mechanical behavior of parts produced using the FDM method. For this reason, the printing orientation used in this study was specifically selected to avoid adversely affecting contact performance.

PLA material was preferred due to its advantages such as low cost, ease of processing, and suitability for additive manufacturing processes. The fundamental properties of the material used are presented in Table 3.

These values are based on typical PLA material properties found in manufacturer technical data sheets (TDS) and the literature [17], [23], [24].

The center distance in the logarithmic spiral spur gear pair was determined in a CAD environment to ensure continuous tooth contact. Rather than being defined by a fixed analytical expression, as is the case with circular gears, the appropriate center distance that satisfies the contact conditions was obtained geometrically.

Table 3. Properties of the PLA material used

Property	Value Range	Unit
Modulus of Elasticity	2.5 – 4.0	GPa
Tensile Strength	40 – 70	MPa
Yield Strength	50 – 60*	MPa
Compressive Strength	60 – 100*	MPa
Fatigue Strength	20 – 35*	MPa
Poisson Ratio	0.33 – 0.39	-
Density	1.21 – 1.25	g/cm³
Glass Transition Temperature (Tg)	55 – 65	°C
Melting Point (Tm)	150 – 180	°C
Decomposition Temperature	~180 – 200*	°C

(*) represent typical value ranges reported in the literature when they are not directly provided in the manufacturer's technical data sheet (TDS) for the PLA filament used.

An image of the generated logarithmic spiral spur gear pair is shown in Figure 6.

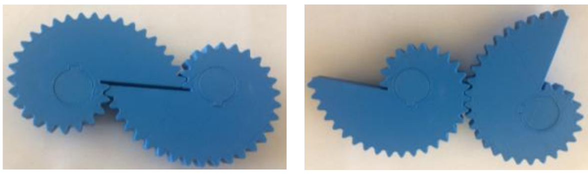

Figure 6. Logarithmic spiral gear pair fabricated using additive manufacturing

The gears produced were assembled for use in contact analysis, and experimental observations were conducted. The examinations revealed that the gears operated smoothly and that no mechanical issues arose. Furthermore, it was determined that the geometry of the produced gears closely matched the CAD model and that the dimensional characteristics specified in the design were preserved.

This demonstrates that the logarithmic spiral spur gears produced using the additive manufacturing method possess geometric accuracy and supports the reliability of the contact analyses performed.

3. RESULTS

In this section, the contact characteristics of the designed logarithmic spiral spur gear pair were investigated through both analyses conducted in a CAD environment and experimental studies. The analysis involved identifying the contact points formed at different angular positions of the gears during operation and determining the path of contact followed by these points.

3.1. Contact Analysis Method

As part of the contact analysis, the sequential positions in which the gear pair interacts with each other were considered, and the contact points between the gear surfaces were determined for each position. By examining the spatial variation of these points, the progression of contact along the gear surface was evaluated.

During the analysis process, the gear pair was positioned in a CAD environment, as shown in Figure 7, taking into account the appropriate center distance. While one gear was held stationary, the other gear was rotated in specific angular increments to obtain different contact positions.

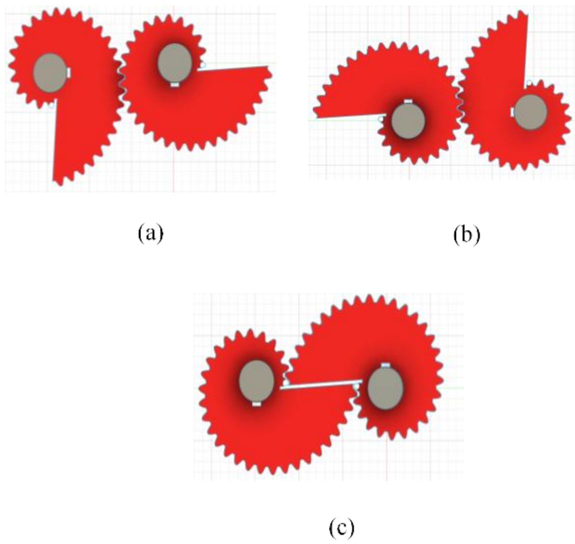

Figure 7. Different stages of contact in a logarithmic spiral spur gear pair: (a) start of contact, (b) midpoint of contact, (c) end of contact

Examining Figure 7 reveals that the contact point progresses gradually along the tooth surface. At the onset of contact, the contact formed in the lead-in region of the tooth surfaces advances toward the middle region as the gears rotate and reaches the lead-out region in the final stage of contact. This demonstrates that the contact is continuous.

Additionally, the contact points between the tooth surfaces were determined at each rotational position, and the coordinates of these points were recorded (Table 4). By connecting the sequential positions of the obtained contact points, the path of contact for the gear pair was established. The contact points were obtained by rotating the gear pair in 5° angular increments, and the coordinates of the contact points were determined for each position.

Examining Table 4, it can be seen that the contact point changes regularly and continuously as a function of the gear angular displacement. This indicates that, in logarithmic spiral spur gears, the contact progresses gradually along the tooth surface.

Table 4. Contact point coordinates for a logarithmic spiral spur gear (in 5° increments)

Angle (°)	X (mm)	Y (mm)
0	12.5	8.2
5	12.1	8.8
10	11.6	9.5
15	10.9	10.2
20	10.1	10.8
25	9.2	11.3
30	8.2	11.7
35	7.1	12.0
40	6.0	12.1
45	4.8	12.0
50	3.6	11.8
55	2.5	11.4
60	1.5	10.9
65	0.6	10.2
70	-0.2	9.5

3.2. Analysis and Comparison of Contact Characteristics

In order to evaluate the contact characteristics of a logarithmic spiral spur gear pair, the observed contact behavior was examined in comparison with a classical gear system featuring a circular centroid structure. The fundamental difference between the systems stems from the geometry of the centroid curve along which the rolling motion occurs.

In the case of a circular centroid, the rolling of the rack results in an involute tooth profile, and the contact path exhibits a linear character. In contrast, when a logarithmic spiral centroid is used, the path traced by the contact points exhibits a nonlinear structure. The contact paths corresponding to these two different centroid geometries are shown in Figure 8.

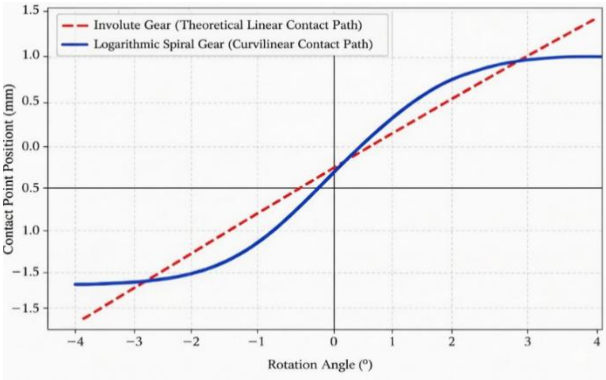

Figure 8. Comparison of contact paths for the circular centroid and the logarithmic spiral centroid

This indicates that contact in logarithmic spiral spur gears occurs more gradually and continuously along the tooth surface. Furthermore, the curved nature of the contact path exhibits behavior that could contribute to an increase in contact duration and a more balanced load transfer. This highlights the potential advantages of logarithmic spiral spur gears in terms of mechanical performance.

Table 5. Comparison of contact characteristics based on centroid geometry

Parameter	Circular Centroid	Logarithmic Spiral Centroid
Profile formation	Rack and pinion drive	Rack cutter roll
Centerline geometry	Circular	Logarithmic spiral
Contact path	Linear	Curved
Pressure angle	Constant	Variable
Speed ratio	Constant	Variable

An examination of Table 5 reveals that the centroid geometry has a direct effect on fundamental gear characteristics such as the contact path, pressure angle, and speed ratio.

3.3. Contact Force Analysis

A simplified force analysis was performed to evaluate the fundamental behavior of contact forces in a pair of logarithmic spiral spur gear. The force transmitted during gear contact is decomposed into its normal force component F_n and tangential force component F_t

The tangential force generated at the point of contact is related to the transmitted moment and is expressed by Equation (16).

$$F_t = \frac{T}{r} \tag{16}$$

Here, T represents the transmitted torque, and r represents the distance from the contact point to the gear center. The normal force is calculated using Equation (17) below, depending on the contact angle.

$$F_n = \frac{F_t}{\cos(\phi)} \tag{17}$$

In logarithmic spiral spur gears, since the contact point continuously shifts along the tooth surface, the r value is not constant but varies depending on the angular position. This results in a gradual and more balanced distribution of contact forces along the tooth surface.

This behavior suggests that it may provide lower stress concentrations and more stable load transfer compared to the sudden load changes observed in involute gears.

The results obtained reveal that contact forces in logarithmic spiral spur gears vary depending on position, and that this situation may positively affect mechanical performance by contributing to an increase in contact duration.

The variation of the tangential contact force with the gear rotation angle is shown in Figure 9.

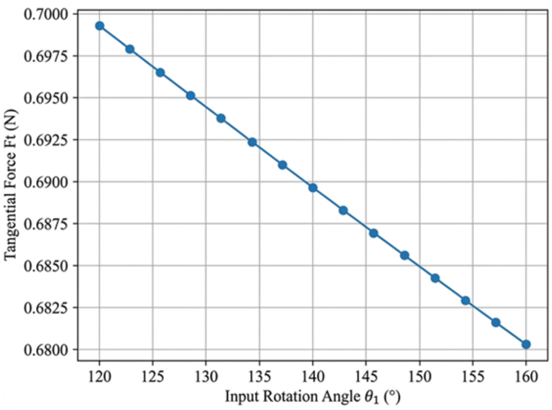

Figure 9. Variation of the tangential contact force in a logarithmic spiral spur gear as a function of the gear rotation angle

Examining Figure 9, it can be seen that the tangential force varies with the angular position as the distance from the contact point to the gear center changes in a logarithmic spiral spur gear. This indicates that the contact forces are distributed gradually along the tooth surface.

In logarithmic spiral straight gears, the transmission ratio is not constant due to the variable-radius structure of the gear geometry but varies depending on the angular position. This necessitates defining the transmission ratio as $i(\theta)$.

The transmission ratio can be expressed using Equation (18) based on the instantaneous radius values at the contact point:

$$i(\theta) = \frac{r_2(\theta)}{r_1(\theta)} \tag{18}$$

Here, r_1 and r_2 represent the instantaneous radius values at the contact point. In this study, rather than deriving an exact solution for the transmission ratio, it was decided to evaluate it indirectly based on the spatial variation of the contact points.

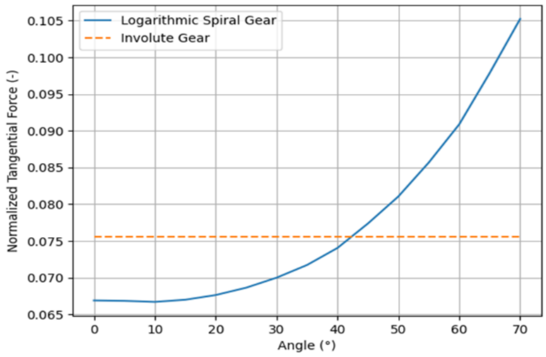

Figure 10. Comparison of normalized tangential force variations for logarithmic spiral and involute gears. The tangential force values have been normalized under the assumption of a unit torque (T = 1 Nm) to ensure comparability.

Examining Figure 10, it can be seen that the tangential force remains constant in involute gears because the pitch radius is constant. In contrast, it has been determined that in logarithmic spiral spur gears, the tangential force exhibits a variable character due to the distance from the contact point to the gear center changing depending on the angular position. This situation indicates that contact forces are distributed more gradually along the gear tooth surface, which may contribute to a more balanced load transfer.

3.4. Experimental Observations

Analyses conducted in a CAD environment have demonstrated that the contact behavior of logarithmic spiral spur gears differs from that of classical involute gears due to the variable-radius structure of the tooth profile. In this context, the aim was not only to identify the contact points but also to evaluate the geometric characteristics of the contact path formed by these points.

In the experimental study, the angular positions of the gear pair were determined using reference marks placed on the gears. The contact points formed at different angular positions were visually tracked, and the progression of contact along the gear tooth surface was recorded.

The different contact positions of the experimentally obtained gear model are shown in Figure 11.

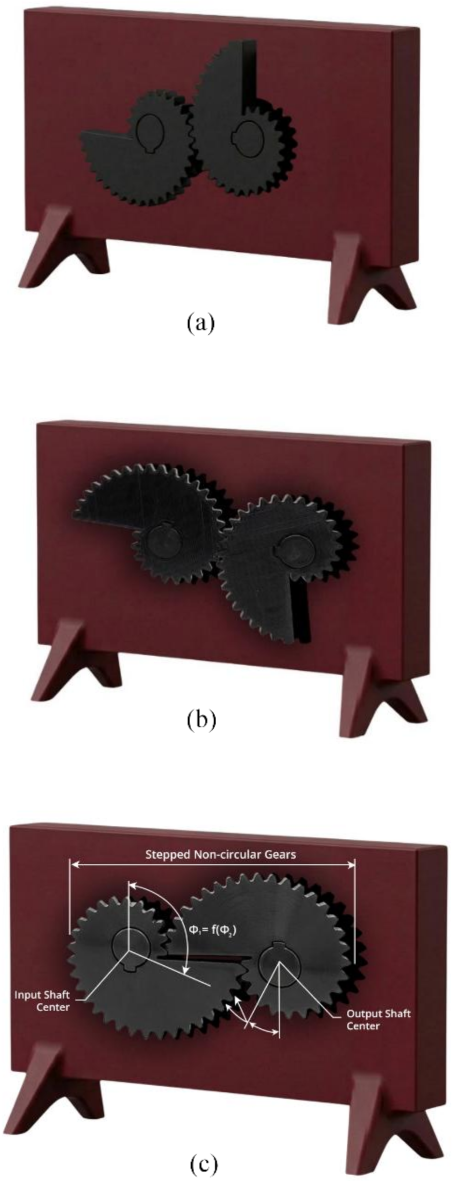

Figure 11. Experimental view of a pair of logarithmic spiral spur gears produced using an additive manufacturing method: (a) start of contact, (b) mid-contact position, (c) end of contact

Based on observations made using the experimental model shown in Figure 11, it was determined that contact progresses continuously and gradually along the tooth surface. Furthermore, the operational behavior of the gear pair was consistent with the results obtained in the CAD environment. Figure 12 illustrates the relationship between the angular positions of the gears. Upon examining Figure 12, it can be seen that there is a linear relationship between the lead-in and lead-out angular positions in involute gears, whereas in logarithmic spiral spur gears, this relationship is not linear and varies depending on the angular position.

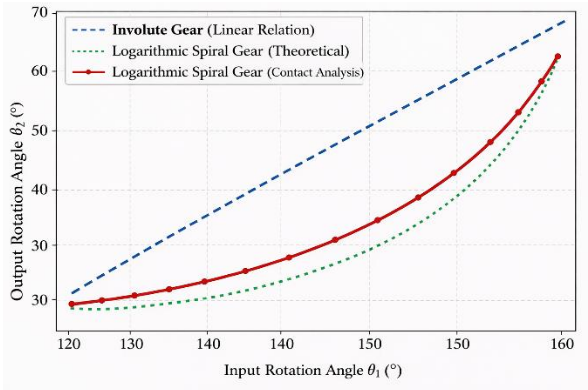

Figure 12. Comparison of the relationship between the input and output angular positions in logarithmic spiral and involute gear pairs

As shown in Figure 13, the contact point moves uniformly and continuously along the tooth surface as the gear rotation angle increases. This indicates that contact occurs gradually rather than abruptly in the geometry of a logarithmic spiral spur gear.

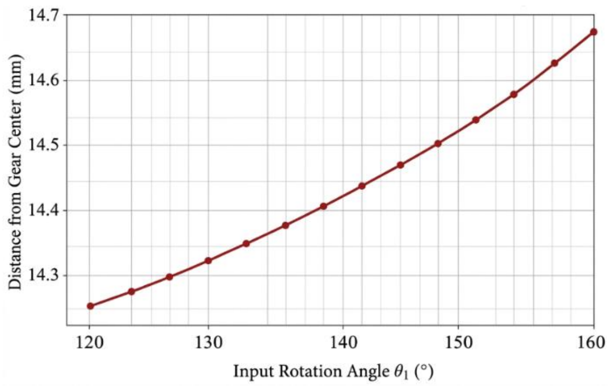

Figure 13. Variation of the contact point as a function of the gear rotation angle

When all the results are evaluated together, it is observed that the contact path in logarithmic spiral spur gears exhibits a curved character and ensures continuity of contact along the tooth surface. This is considered an advantage that could theoretically provide a more balanced load distribution and more stable power transmission.

4. DISCUSSION

In this section, the findings regarding a logarithmic spiral spur gear pair designed using a parametric design algorithm and manufactured via additive manufacturing are evaluated in comparison with classical gear systems featuring a circular centroid geometry. The contact characteristics of the logarithmic spiral spur gear pair have been examined from both geometric and kinematic perspectives. Furthermore, the simplified force analysis conducted revealed the variation of contact forces along the tooth surface, providing meaningful insights into the system's mechanical behavior.

4.1. The Role of Additive Manufacturing and Production Advantages

The manufacture of non-circular gears using traditional production methods (hobbing, planing, or milling) is a costly and time-consuming process due to their complex geometric structures. Since the tooth profile and contact conditions in these gears constantly change, both the cutting tool geometry and the tool-workpiece kinematics must be precisely controlled during the manufacturing process [12]. Due to their reliance on fixed cutting tool geometry, these methods are limited in the production of gears with variable pitch curves [14]. In particular, the fact that each tooth has a different geometry leads to complex synchronization problems and high precision requirements in these methods [13]. For these reasons, the additive manufacturing method used in this study has significantly reduced these challenges. Bodzás and Nemes [18] demonstrated the geometric accuracy of gears with very small deviations by using additive manufacturing technology and 3D scanning methods for gear design, production, and accuracy analysis. In this study, the functional performance of the non-circular gear produced via additive manufacturing serves as evidence of the gear's geometric accuracy. Furthermore, the ability to control the internal solidity ratio through the additive manufacturing method enables the lightweighting of gear geometry and the reduction of inertial effects in dynamic systems. However, the additive manufacturing of gears is only applicable when the required accuracy values can be achieved within the appropriate ISO tolerance range, and when permitted temperature and strength requirements are of secondary importance. Particularly in the production of variable-radius profiles such as logarithmic spirals, the continuously changing cutting geometry and the need for precise adjustments increase the likelihood of errors. Nevertheless, the examination of the variation in contact forces along the gear tooth surface in this study provided meaningful insights into the system's mechanical behavior and demonstrated that it is an effective alternative method for the prototyping of complex gear geometries such as the logarithmic spiral. However, for applications requiring higher strength and precision, metal-based additive manufacturing technologies must be evaluated [17].

4.2. Contact Path Characteristics and Kinematic Behavior

By sequentially connecting the obtained contact points, the contact path of the gear pair was constructed, and the geometric characteristics of this path were analyzed. This approach enables an assessment of the contact continuity and kinematic behavior of logarithmic spiral spur gears.

As a result of the analyses conducted, it was determined that the contact behavior in logarithmic spiral spur gears differs from that of classical involute gears due to the variable-radius structure of the tooth profile. This results in a longer contact duration and a wider contact area in logarithmic spiral gears, in contrast to the limited contact length observed in involute gears. This finding is consistent with the results of a study by Liu et al. [27] on the contact analysis of non-circular gears. In that study, it was observed that the maximum value of the load distribution occurs at the location of the gear's intermediate pitch curve, while the maximum value of tooth displacement occurs at the tooth tip. The effects of this difference observed in logarithmic spiral spur gear sets on mechanical performance indicate significant variations. This finding is consistent with the conclusion reached in the study by Mo et al. [25], which demonstrated that the logarithmic spiral gear drive mechanism is both feasible and efficient. In this study, the authors noted that logarithmic spiral gears represent a gear design solution that provides lower transmission error compared to involute gears and offers good stability and efficiency. This situation may contribute to a more balanced distribution of contact stresses and more homogeneous wear. Xiao et al. [28], in their work on generating contact curves for non-circular gears, demonstrated that they could reliably generate a predefined rotation angle and angular velocity. This finding is consistent with the reliable generation of the contact path curve obtained in this study. This may contribute to a reduction in sudden load changes. It is also anticipated that this could contribute to reducing vibration and noise levels. These assessments were made by considering the obtained contact characteristics and the non-circular gear behavior reported in the literature.

This demonstrates that logarithmic spiral straight-toothed gears can be considered an effective alternative solution, particularly in mechanical systems requiring variable speed ratios.

4.3. Limitations

This study has some limitations. First, the experimental investigations conducted within the scope of this study are based on a visual assessment of contact behavior. No precision measuring equipment was used for the positional measurements of the contact points. Therefore, the experimental findings obtained are of a qualitative nature. Second, since future studies plan to conduct a detailed analysis of stress distributions in logarithmic spiral spur gears under various loading conditions using the finite element method, the determination of contact stresses and deformation distributions via finite element analysis (FEA) was excluded from the scope of this study. This situation limits the results obtained primarily to geometric and kinematic evaluations. Therefore, the findings must be supported by quantitative stress analyses.

5. CONCLUSION

In this study, the parametric design, additive manufacturing, and contact characteristics of gear pairs with logarithmic spiral centroids were examined using a holistic approach. When the findings are evaluated collectively, it is evident that the logarithmic spiral spur gear geometry offers significant advantages in terms of both manufacturability and contact characteristics. It has been confirmed that these complex geometries can be physically manufactured using the additive manufacturing method and function in accordance with CAD analyses.

As a result of the contact analyses conducted, it was determined that the contact path in the logarithmic spiral spur gear pair has a curvilinear rather than a linear character. It was observed that the contact points progress gradually and continuously along the tooth surface, and it was assessed that this allows the contact to spread over a wider area. In this context, the results indicate that logarithmic spiral spur gears can offer an alternative solution in certain mechanical applications due to their variable contact characteristics. The findings demonstrate that logarithmic spiral spur gears are feasible both in terms of design and manufacturing and offer distinct kinematic properties compared to classical gear systems. Future studies plan to experimentally investigate the mechanical performance of these gears under different loading conditions and evaluate their behavior with various material types.

DECLARATION OF ETHICAL STANDARDS

The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

AUTHORS' CONTRIBUTIONS

Mehmet YAZAR: Contribution to the overall study is 100%.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

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