Research Article GU J Sci 31(4): 1166-1177 (2018)

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

Mixed Fuzzy Soft Topological Spaces

Ayten GEZICI[1,*], Cemil YILDIZ[1]

[1]*Department of Mathematics, Faculty of Science, Gazi University, 06500 Teknikokullar Ankara, Turkey*

Article Info

Received: 09/03/2017
Accepted: 14/03/2018

Keywords

Fuzzy soft set
Fuzzy soft topology
First countable space
Second countable space

Abstract

In this paper, we introduce a new type of mixed fuzzy soft topological space over a fuzzy soft set on initial universe set. We define countability on mixed fuzzy soft topological spaces.

1. INTRODUCTION

In 1965, Zadeh [1] introduced the notion of fuzzy sets and fuzzy sets operations. Subsequently, Chang [2] defined the notion of fuzzy topology. Pao Ming et al. [3] defined neighbourhood structure of a fuzzy point.

In 1999, Molodtsov [4] introduced soft set theory for modeling vagueness and uncertainties. He applied soft set theory to several directions, such as game theory, Riemann integration, Perron integration, smoothness of functions. Maji, Biswas and Roy [5] defined and studied several basic notions of soft set theory.

In recent times, researchers have contributed a lot towards fuzzification of soft set theory. In 2001, Maji et al. [6] introduced the concept of fuzzy soft set. Tanay et al. [7] introduced the definition of fuzzy soft topology over a subset of initial universe set while Roy and Samanta [8] gave the definition of fuzzy soft topology over the initial universe set. Varol and Aygün [9,10], Neog et al. [11] and Hussain [12] studied the topological structures of fuzzy soft theory. Simsekler and Yuksel [13,14] introduced fuzzy soft topology over a fuzzy soft set on initial universe set. Tripathy and Ray [15] introduced and studied the concept of mixed fuzzy topological spaces and countability. Tripathy and Ray [16] introduced mixed fuzzy ideal topological spaces. Borah and Hazarika [17] gave the definition mixed fuzzy soft topological space over the initial universe set.

In this paper we introduce mixed fuzzy soft topological space. It is define over a fuzzy soft set instead of initial universe set. In order to define the mixed fuzzy soft topological space over a fuzzy soft set we give definition the complement according to this fuzzy soft set. We introduce the notions of fuzzy soft neighbourhood, Q-fuzzy soft neighbourhood over a fuzzy soft set. Also we define countability on mixed fuzzy soft topological spaces.

*Corresponding author, e-mail: ayten.gezici@gazi.edu.tr

2. PRELIMINARIES

Throughout this paper X denotes initial universe set, E denotes the set of all possible parameters which are attributes, characteristic or properties of the objects in X and the set of all subsets of X will be denoted by $P(X)$.

Definition 1. [1] A fuzzy set A in X is a set of ordered pairs:

$A = \{ (x, \mu_A(x): \ x \in X\},$

where $\mu_A : X \to [0,1] = I$ is called the membership function and $\mu_A(x)$ is grade of membership of x in A. The family of all fuzzy sets in X denoted by I^X.

Definition 2. [1] Let A, B be two fuzzy sets of I^X.

1. A is contained in B if and only if $\mu_A(x) \leq \mu_B(x)$, for every $x \in X$.
2. The union of A and B is a fuzzy set C, denoted by $A \vee B = C$, whose membership function $\mu_C(x) = \max\{ \mu_A(x), \ \mu_B(x)\}$ for every $x \in X$.
3. The intersection of A and B is a fuzzy set C, denoted by $A \wedge B = C$, whose membership function $\mu_C(x) = \min\{ \mu_A(x), \ \mu_B(x)\}$ for every $x \in X$.
4. The complement of A is a fuzzy set, denoted by A^c, whose membership function $\mu_{A^c}(x) = 1 - \mu_A(x)$, for every $x \in X$.

Definition 3. [2] A fuzzy set A is called a null fuzzy set, if $\mu_A(x) = 0, \forall x \in X$ and denoted by $\overline{0}$.

Definition 4. [2] A fuzzy set A is called a absolute fuzzy set, if $\mu_A(x) = 1, \forall x \in X$ and denoted by $\overline{1}$.

Definition 5. [4]

1. Let X be the initial universe set, E be the set of parameters and $A \subset E$. A pair (F, A) or F_A is called a soft set over X, where F is mapping given by $F: A \to P(X)$.
2. Let X be the initial universe set, E be the set of parameters. A pair (F, E) or F_E is called a soft set over X, where F is mapping given by $F: E \to P(X)$.

In other words, the soft set is a parameterized of subsets of the set X. For $e \in E$, $F(e)$ may be considered as the set of e-elements of the soft set (F, E) or as the set of e-approximate elements of the soft set of (F, E).

Definition 6. [8]

1. Let X be the initial universe set, E be the set of parameters and $A \subset E$. A pair (f, A) or f_A is called a fuzzy soft set over X , where f f is mapping given by $f: A \to I^X$.
2. Let X be the initial universe set and E be the set of parameters. A pair (f, E) or f_E is called a fuzzy soft set over X, where f is mapping given by $f: E \to I^X$.

The following definition is the extended of fuzzy soft set f_A .

Definition 7. [8] Let X be an initial universe set, E be a parameters set and $A \subset E$. Then the mapping $f_A: E \to I^X$, defined by $f_A(e) = \mu_{f_A}(e)$, is called fuzzy soft set over X, where $\mu_{f_A}(e) = \overline{0}$, if $e \in E \setminus A$ and $\mu_{f_A}(e) \neq \overline{0}$, if $e \in A$.

The set of all fuzzy soft set over X is denoted by $FS(X,E)$.

Definition 8. [8] The complement of a fuzzy soft set f_A on X which is denoted by f_A^c and $f_A^c: E \to I^X$ is defined by $\mu_{f_A^c}^e = \bar{1} - \mu_{f_A}^e$ if $e \in A$ and $\mu_{f_A^c}^e = \bar{1}$ if $e \in E \setminus A$, where $\bar{1}(x) = 1$ for each $x \in X$.

Definition 9. [8] The fuzzy soft set $f_\emptyset \in FS(X,E)$ is called null fuzzy soft set and it is denoted by Φ. Here $f_\emptyset(e) = \bar{0}$, for every $e \in E$, where $\bar{0}(x) = 0$ for every $x \in X$.

Definition 10. [8] The fuzzy soft set $f_E \in FS(X,E)$ is called absolute fuzzy soft set and it is denoted by $\breve{E}$. Here $f_E(e) = \bar{1}$, for every $e \in E$.

Definition 11. [8] Let f_A and g_B be two fuzzy soft sets on X. f_A is defined to be fuzzy soft subset of g_B, if $\mu_{f_A}^e \leq \mu_{g_B}^e$ for all $e \in E$ and is denoted by $f_A \sqsubseteq g_B$.

Definition 12. [8] Let f_A and g_B be two fuzzy soft sets on X. The union of these two fuzzy soft set is a fuzzy soft set h_C, defined by $h_C(e) = \mu_{h_C}^e = \mu_{f_A}^e \vee \mu_{g_B}^e$ for all $e \in E$, where $C = A \cup B$ and is denoted by $h_C = f_A \sqcup g_B$.

Definition 13. [8] Let f_A and g_B be two fuzzy soft sets on X. The intersection of these two fuzzy soft set is a fuzzy soft set h_C, defined by $h_C(e) = \mu_{h_C}^e = \mu_{f_A}^e \wedge \mu_{g_B}^e$ for all $e \in E$, where $C = A \cap B$ and is denoted by $h_C = f_A \sqcap g_B$.

Definition 14. [3] A fuzzy point x_λ in X a special fuzzy set with membership defined by:

$$x_\lambda(y) = \begin{cases} 0, & x \neq y \\ \lambda & x = y \end{cases}$$

where $0 < \lambda \leq 1$. x_λ is said to have support x, value λ.

Definition 15. [13] Let $x \in X$, $A \subset E$. A soft set $x_A: A \to P(X)$ is called soft point defined by $x_A(e) = \{x\}$, for every $e \in A$.

Definition 16. [9] Let $f_A \in FS(X,E)$ and $\lambda: E \to I$ be a mapping defined by $\lambda(e) \neq 0$, $e \in A$ and $\lambda(e) = 0, e \in E \setminus A$.

The fuzzy soft set f_A is called a fuzzy soft point defined by $f_A(e) = x_{\lambda(e)}$, $\forall\, e \in E$, where $f_A(e) = x_{\lambda(e)}$ for $e \in A$ is a fuzzy soft point and $\lambda(e) = 0$, for $e \in E \setminus A$, $f_A(e)$ is a null fuzzy set. Or equivalently,

$$f_A(e)(y) = x_{\lambda(e)}(y) = \begin{cases} \lambda(e), & x = y, \\ 0, & x \neq y, \end{cases}$$

fuzzy soft point denoted by x_A^λ.

Example 1. Let X={x,y}, E=$\{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\} \subset E$

$$x_A^\lambda = \{x_A^\lambda(e_1) = \{(x, 0.1), (y, 0)\}, x_A^\lambda(e_2) = \{(x, 0.2), (y, 0)\}, x_A^\lambda(e_3) = \{(x, 0), (y, 0)\}\}$$

That x_A^λ fuzzy soft set is a fuzzy soft point in X.

Definition 17. Let g_A be a fuzzy soft subset of f_A and $\lambda: E \to I$ be a mapping defined by $\lambda(e) \neq 0$, $e \in A$ and $\lambda(e) = 0, e \in E \setminus A$.

The fuzzy soft set g_A is called a fuzzy soft point in f_A defined by $g_A(e) = x_{\lambda(e)}$, $\forall e \in E$, where $g_A(e) = x_{\lambda(e)}$ for $e \in A$ is a fuzzy soft point and $\lambda(e) = 0$, for $e \in E \setminus A$, $g_A(e)$ is a null fuzzy set. Or equivalently,

$$g_A(e)(y) = x_{\lambda(e)}(y) = \begin{cases} \lambda(e), & x = y, \\ 0, & x \neq y, \end{cases}$$

fuzzy soft point in f_A denoted by x_A^{λ}.

Definition 18. [14] Let f_A be a fuzzy soft set on X and τ_f be the collection of fuzzy soft subsets of f_A , then τ_f is said to be a fuzzy soft topology if the following conditions hold:

1. Φ, $f_A \in \tau_f$

2. If $g_A, h_A \in \tau_f$ then $g_A \sqcap h_A \in \tau_f$

3. If $(f_{iA}) \in \tau_f$ then $\sqcup_i f_{iA} \in \tau_f$

Then (f_A, τ_f) is called a fuzzy soft topological spaces over f_A.

Definition 19. Let (f_A, τ_f) fuzzy soft topological spaces and $B_f \subset \tau_f$. B_f is said to be base for τ_f, if every members of τ_f is a union of members of B_f.

Definition 20. Let (f_A, τ_f) fuzzy soft topological spaces. (f_A, τ_f) is called indiscrete if it contains only Φ and f_A while the discrete fuzzy soft topology consists of all fuzzy soft subset f_A. That is $\tau_f = P(f_A)$.

Definition 21. Let x_A^{λ} be a fuzzy soft point in f_A and g_A be a fuzzy soft subset of f_A. If $x_{\lambda(e)} \leq g_A(e)$ for every $e \in A$ ($\lambda(e) \leq g_A(e)(x)$, for $x \in X$), then x_A^{λ} belongs to g_A and this denoted by $x_A^{\lambda} \tilde{\in} g_A$.

Definition 22. Let x_A^{λ} be a fuzzy soft point in f_A and g_A be a fuzzy soft subset of f_A and (f_A, τ_f) be a fuzzy soft topological space. g_A is called fuzzy soft neighbourhood x_A^{λ}, if there exists a fuzzy soft open h_A such that $x_A^{\lambda} \tilde{\in} h_A \sqsubseteq g_A$.

The family of neighbourhood of x_A^{λ} is denoted by $N(x_A^{\lambda})$.

Definition 23. Let g_A , h_A be two fuzzy soft subsets of f_A . g_A is said to be quasi-coincident with h_A denoted by $g_A \tilde{q} h_A$ if $g_A(e) q h_A(e)$, $\forall e \in A$, where $g_A(e) q h_A(e)$, $\forall e \in A \Leftrightarrow g_A(e)(x) + h_A(e)(x) > 1$, $x \in X$.

Definition 24. . Let x_A^{λ} be a fuzzy soft point in f_A and g_A be a fuzzy soft subset of f_A . x_A^{λ} is said to be quasi-coincident with g_A denoted by $x_A^{\lambda} \tilde{q} g_A$, if $x_{\lambda}(e) q g_A(e)$, $\forall e \in A$, where $x_{\lambda}(e) q g_A(e)$, $\forall e \in A \Leftrightarrow \lambda(e) + g_A(e)(x) > 1$, $x \in X$.

Definition 25. Let x_A^{λ} be a fuzzy soft point in f_A , g_A be a fuzzy soft subset of f_A and (f_A, τ_f) be a fuzzy soft topological space. g_A is called Q-fuzzy soft neighbourhood x_A^{λ}, if there exists a fuzzy soft open h_A such that $x_A^{\lambda} \tilde{q} h_A \sqsubseteq g_A$.

The family of Q-fuzzy soft neighbourhood of x_A^{λ} is denoted by $N_Q(x_A^{\lambda})$.

3. MIXED FUZZY SOFT TOPOLOGICAL SPACE ON f_A

Simsekler and Yuksel [14] defined fuzzy soft topology over f_A. In this section we give the following definitions and mixed fuzzy soft topology over f_A.

Definition 26. The complement with respect to f_A of a fuzzy soft set g_A is a fuzzy soft subset of f_A which is denoted by g_A^c and $g_A^c : E \to I^X$ is defined by $\mu^e_{g_A^c} = \mu^e_{f_A} - \mu^e_{g_A}, e \in E$.

Example 2. Let $X = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\}$ and

$f_A = \{f(e_1) = \{a_{0.7}, b_{0.8}, c_{0.5}\}, f(e_2) = \{a_{0.4}, b_{0.9}, c_{0.3}\}, f(e_3) = \{a_0, b_0, c_0\}\}$,
$g_A = \{g(e_1) = \{a_{0.6}, b_{0.5}, c_{0.3}\}, g(e_2) = \{a_{0.2}, b_{0.5}, c_{0.1}\}, g(e_3) = \{a_0, b_0, c_0\}\}$

The complement of g_A is defined by $\mu^e_{g_A^c}(x) = \mu^e_{f_A}(x) - \mu^e_{g_A}(x), e \in E, x \in X$.

g_A^c(x)=$\{g^c(e_1) = \{a_{0.1}, b_{0.3}, c_{0.2}\}, g^c(e_2) = \{a_{0.2}, b_{0.4}, c_{0.2}\}, g^c(e_3) = \{a_0, b_0, c_0\}\}$.

Theorem 1. (f_A, τ_1) and (f_A, τ_2) be two fuzzy soft topological spaces over f_A. Consider the collection of fuzzy soft sets $\tau_1(\tau_2) = \{g_A \sqsubseteq f_A$: For any fuzzy soft set h_A be a fuzzy soft subset of f_A with $g_A \tilde{q} h_A$, there exists τ_2- open set h_{A_1} such that $h_{A_1} \tilde{q} h_A$ and τ_1-closure $\overline{h_{A_1}} \sqsubseteq g_A\}$. Then this family of fuzzy soft sets will form a topology on f_A.

Proof: $t_1)$ Since Φ is not quasi-coincident with any fuzzy soft set g_A and therefore, there does not arise any questions of violation of the condition of being member of $\tau_1(\tau_2)$. Therefore $\Phi \in \tau_1(\tau_2)$.

Any fuzzy soft set g_A be a fuzzy soft subset of f_A, such that $g_A \tilde{q} f_A$ and there exists τ_2- open set f_A with $g_A \tilde{q} f_A$ and τ_1- closure $\overline{f_A} = f_A \sqsubseteq f_A$. Therefore $f_A \in \tau_1(\tau_2)$.

$t_2)$ Let $g_A, h_A \in \tau_1(\tau_2)$. We show that $g_A \sqcap h_A \in \tau_1(\tau_2)$.

Let k_A be a fuzzy soft subset of f_A, such that $k_A \tilde{q}(g_A \sqcap h_A)$.

$\Rightarrow k_A(e) q (g_A(e) \wedge h_A(e))$, $\forall e \in A$

$\Rightarrow k_A(e)(x) + (g_A(e) \wedge h_A(e))(x) > 1$, $x \in X, \forall e \in A$

$\Rightarrow k_A(e)(x) + g_A(e)(x) > 1$ and $k_A(e)(x) + h_A(e)(x) > 1$, $x \in X, \forall e \in A$.

$\Rightarrow k_A(e) q g_A(e)$ and $k_A(e) q h_A(e)$, $\forall e \in A$.

$\Rightarrow$ $k_A \tilde{q} g_A$ and $k_A \tilde{q} h_A$.

Since $g_A, h_A \in \tau_1(\tau_2)$,for $k_A \tilde{q} g_A$ there exists τ_2 open set k_{1A} such that $k_{1A} \tilde{q} k_A$ and τ_1 closure $\overline{k_{1A}} \sqsubseteq g_A$ and for $k_A \tilde{q} h_A$ there exists τ_2 open set k_{2A} such that $k_{2A} \tilde{q} k_A$ and τ_1 closure $\overline{k_{2A}} \sqsubseteq h_A$. Now k_{1A}, k_{2A} are τ_2 open set implies $k_{1A} \sqcap k_{2A} \in \tau_1(\tau_2)$.

We have $\overline{k_{1A} \sqcap k_{2A}} \sqsubseteq \overline{k_{1A}} \sqcap \overline{k_{2A}} \sqsubseteq g_A \sqcap h_A$.

$k_{1A} \tilde{q} k_A \Rightarrow k_{1A}(e)(x) + k_A(e)(x) > 1$, $x \in X, \forall e \in A$.

$k_{2A}\tilde{q}k_A \Longrightarrow k_{2A}(e)(x) + k_A(e)(x) > 1\,,\ \ x \in X, \forall\, e\ \in A.$

$\Longrightarrow k_A(e)(x) + (k_{1A}(e) \wedge k_{2A}(e))(x) > 1,\ \ x \in X, \forall\, e \in A\,.$

$\Longrightarrow k_A(e)q(k_{1A}(e) \wedge k_{2A}(e)),\ \ \forall\, e \in A$

$\Longrightarrow k_A\tilde{q}(k_{1A} \sqcap k_{2A}).$

For $k_A\tilde{q}(g_A \sqcap h_A)$, there exists τ_2 open set $k_{1A} \sqcap k_{2A}$ such that $k_A\tilde{q}(k_{1A} \sqcap k_{2A})$ and τ_1 closure $\overline{k_{1A} \sqcap k_{2A}} \sqsubseteq g_A \sqcap h_A$.

Therefore $g_A \sqcap h_A\ \in \tau_1(\tau_2)$.

$t_3)$ Let $g_{iA} \in \tau_1(\tau_2),\ \forall\ i \in \Delta$, then $\sqcup_{i\in\Delta}\, g_{iA} \in \tau_1(\tau_2)$.

Let h_A be a fuzzy soft subset of f_A such that $h_A\tilde{q} \sqcup_{i\in\Delta} g_{iA}$.

$h_A\tilde{q} \sqcup_{i\in\Delta} g_{iA} \Longrightarrow h_A(e)q\vee_{i\in\Delta}g_{iA}(e), \forall\, e \in A.$

$\Longrightarrow h_A(e)(x) + \vee g_{iA}(e)(x) > 1, x\ \in X,\ \ \forall\, e \in A$

$\Longrightarrow h_A(e)(x) + g_{i_0A}(e)(x) > 1,\ \ i_0 \in \Delta, x\ \in X,\ \ \forall\, e \in A.$

$\Longrightarrow h_A(e)q\, g_{i_0A}(e),\ \ i_0 \in \Delta,\ \ \forall\, e \in A \Longrightarrow h_A\tilde{q}g_{i_0A},\ i_0 \in \Delta.$

$g_{i_0A} \in \tau_1(\tau_2)$ and $h_A\tilde{q}g_{i_0A}$, $i_0 \in \Delta$ there exists τ_2 open h_{iA} such that $h_{iA}\tilde{q}h_A$ and τ_1 closure

$\overline{h_{\iota A}} \sqsubseteq g_{i_0}$.

$h_{iA}\tilde{q}h_A \Longrightarrow\ \ h_A(e)(x) + \vee g_{iA}(e)(x) > 1, x\ \in X,\ \ \forall\, e \in A.$

$\Longrightarrow \vee_{i\in\Delta}h_{iA}(e)(x) + h_A(e)(x) > 1\,,, x\ \in X,\ \ \forall\, e \in A.$

$\Longrightarrow \vee_{i\in\Delta}h_{iA}(e)q\, h_A(e)\,,\ \ \forall\ e \in A.$

$\Longrightarrow \sqcup_{i\in\Delta}\, h_{iA}\tilde{q}h_A\,.$

For h_{iA} are τ_2 open set implies $\sqcup_{i\in\Delta} h_{iA} \in \tau_2$ and τ_1 closure, $\overline{\sqcup_{\iota\in\Delta}\, h_{\iota A}} = \sqcup_{i\in\Delta}\, \overline{h_{\iota A}} \sqsubseteq \sqcup_{i\in\Delta}\, g_{iA}$.

For $h_A\tilde{q} \sqcup_{i\in\Delta} g_{iA}$, there exists τ_2 open $\sqcup_{i\in\Delta} h_{iA}$ such that $h_A\tilde{q} \sqcup_{i\in\Delta} h_{iA}$ and τ_1 closure $\overline{\sqcup_{\iota\in\Delta}\, h_{\iota A}} \sqsubseteq \sqcup_{i\in\Delta}\, g_{iA}$. Hence $\sqcup_{i\in\Delta}\, g_{iA} \in \tau_1(\tau_2)$.

Therefore this collection $\tau_1(\tau_2)$ is a fuzzy soft topology on f_A.

Definition 27. $\tau_1(\tau_2)$ defined in Theorem 3.1. is called a mixed fuzzy soft topolgy on f_A and $(f_A, \tau_1(\tau_2))$ is a mixed fuzzy soft topological space.

Definition 28. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space and g_A be a fuzzy soft subset of f_A. g_A is called a fuzzy soft closed set in $(f_A, \tau_1(\tau_2))$ iff its complement g_A^c is a fuzzy soft open set in $(f_A, \tau_1(\tau_2))$.

We give an example to $\tau_1(\tau_2)$ defined in Definition 3.2.

Example 3. Let $X = \{a, b, c, d\}$, $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_3\}$ and fuzzy soft set $f_A = \{f(e_1) = \{a_{0.4}, b_{0.6}, c_{0.5}, d_{0.6}\}, f(e_2) = \{a_{0.3}, b_{0.7}, c_{0.8}, d_{0.5}\}, f(e_3) = \{a_{0,7}, b_{0,4}, c_{0,6}, d_{0,4}\},, f(e_4) = \{a_0, b_0, c_0, d_0\}\}$

$f_{1A} = \{f_1(e_1) = \{a_{0.2}, b_{0.4}, c_{0.3}, d_{0.5}\}, f_1(e_2) = \{a_{0.2}, b_{0.5}, c_{0.5}, d_{0.3}\}, f_1(e_3) = \{a_{0.4}, b_{0.3}, c_{0.5}, d_{0.3}\}, f_1(e_4) = \{a_0, b_0, c_0, d_0\}\}$

$f_{2A} = \{f_2(e_1) = \{a_{0.2}, b_{0.2}, c_{0.2}, d_{0.1}\}, f_2(e_2) = \{a_{0.1}, b_{0.3}, c_{0.2}, d_{0.3}\}, f_2(e_3) = \{a_{0.3}, b_{0.1}, c_{0.1}, d_{0.1}\}, f_2(e_4) = \{a_0, b_0, c_0, d_0\}\}$

The collection $\tau_1 = \{\Phi, f_A, f_{1A}, f_{2A}\}$ and $\tau_2 = \{\Phi, f_A, f_{2A}\}$ are two fuzzy soft topologies on f_A.

We obtain the family fuzzy soft closed sets by the complement of the element of τ_1

$\tau_1^c = \{\Phi^c, f_A^c, f_{1A}^c, f_{2A}^c\}$, where

$\Phi^c = \{\Phi^c(e_1) = \{a_{0.4}, b_{0.6}, c_{0.5}, d_{0.6}\}, \Phi^c(e_2) = \{a_{0.3}, b_{0.7}, c_{0.8}, d_{0.5}\}, \Phi^c(e_3) = \{a_{0.7}, b_{0.4}, c_{0.6}, d_{0.4}\}, \Phi^c(e_4) = \{a_0, b_0, c_0, d_0\}\} = f_A$

$f_A^c = \{f_A^c(e_1) = \{a_0, b_0, c_0, d_0\}, f_A^c(e_2) = \{a_0, b_0, c_0, d_0\}, f_A^c(e_3) = \{a_0, b_0, c_0, d_0\}, f_A^c(e_4) = \{a_0, b_0, c_0, d_0\}\} = \Phi$

$f_{1A}^c = \{f_1^c(e_1) = \{a_{0.2}, b_{0.2}, c_{0.2}, d_{0.1}\}, f_1^c(e_2) = \{a_{0.1}, b_{0.2}, c_{0.3}, d_{0.2}\}, f_1^c(e_3) = \{a_{0.3}, b_{0.1}, c_{0.1}, d_{0.1}\}, f_1^c(e_4) = \{a_0, b_0, c_0, d_0\}\}$,

$f_{2A}^c = \{f_2^c(e_1) = \{a_{0.2}, b_{0.4}, c_{0.3}, d_{0.5}\}, f_2^c(e_2) = \{a_{0.2}, b_{0.4}, c_{0.6}, d_{0.2}\}, f_2^c(e_3) = \{a_{0.4}, b_{0.3}, c_{0.5}, d_{0.3}\}, f_2^c(e_4) = \{a_0, b_0, c_0, d_0\}\}$.

Now we construct the mixed fuzzy soft topological space on f_A from there two fuzzy soft topolgies τ_1 and τ_2.

We show that $\Phi, f_A \in \tau_1(\tau_2)$.

Since Φ is not quasi-coincident with any fuzzy soft set g_A fuzzy soft subset of f_A and therefore, there does not arise any questions of violation of the condition of being member of $\tau_1(\tau_2)$. Then $\Phi \in \tau_1(\tau_2)$.

$f_A \sqsubseteq f_A$ and for any fuzzy soft set g_A fuzzy soft subset of f_A with $f_A \tilde{q}\, g_A$, there exists τ_2 open set f_A such that $f_A \tilde{q}\, g_A$ and τ_1 closure $\overline{f_A} = f_A \sqsubseteq f_A$. Then $f_A \in \tau_1(\tau_2)$.

Let us consider a fuzzy soft set g_A fuzzy soft subset of f_A such that $g_A \tilde{q}\, f_{2A}$.

Now the only τ_2 open sets are f_A and f_{2A} such that $g_A \tilde{q}\, f_{2A}$ and $g_A \tilde{q}\, f_A$.

Again, τ_1 closure of,

$\overline{f_{2A}} = \sqcap \{k_A : k_A \text{ is } \tau_1 \text{ closed and } f_{2A} \sqsubseteq k_A\} = f_A \sqcap f_{1A}^c = f_{2A} \sqsubseteq f_{2A}$

Hence, $f_{2A} \in \tau_1(\tau_2)$ and so $\tau_1(\tau_2) = \{\Phi, f_A, f_{2A}\}$ is a mixed fuzzy soft topology on f_A.

Result 1.

1. Let $(f_A\ \tau)$ be a indiscrete fuzzy soft topological space and $\tau(\tau)$ construct from fuzzy soft topology τ. Then $\tau(\tau)$ is a indiscrete mixed fuzzy soft topolgy.

2.Let (f_A, τ) be a discrete fuzzy soft topological space and $\tau(\tau)$ construct from fuzzy soft topology τ. Then $\tau(\tau)$ is a discrete mixed fuzzy soft topolgy.

3. Let (f_A, τ_1) be a discrete and (f_A, τ_2) indiscrete fuzzy soft topological spaces and $\tau_1(\tau_2)$ construct from these two fuzzy soft topologies τ_1 and τ_2. Then $\tau_1(\tau_2)$ is a indiscrete mixed fuzzy soft topolgy.

4. Let (f_A, τ_1) be a indiscrete and (f_A, τ_2) discrete fuzzy soft topological spaces and $\tau_1(\tau_2)$ construct from these two fuzzy soft topologies τ_1 and τ_2. Then $\tau_1(\tau_2)$ is a indiscrete mixed fuzzy soft topolgy.

Proof 1. We show that $\tau(\tau)$ is a indiscrete mixed fuzzy soft topolgy. Since Φ is not quasi-coincident with any fuzzy soft set g_A and therefore, there does not arise any questions of violation of the condition of being member of $\tau(\tau)$.

Therefore $\Phi \in \tau(\tau)$.

Any fuzzy soft set g_A be a fuzzy soft subset of f_A, such that $g_A \tilde{q} f_A$ and there exists τ-open set f_A with $g_A \tilde{q} f_A$ and τ-closure $\overline{f_A} = f_A \sqsubseteq f_A$. Therefore $f_A \in \tau(\tau)$.

Let $g_A \sqsubseteq f_A$, k_A be a fuzzy soft of f_A, with $g_A \tilde{q} k_A$ and h_A be a τ open fuzzy soft set such that $h_A \tilde{q} k_A$. τ closure $\overline{h_A} = f_A$, because τ is a indiscrete topology. $\overline{h_A} = f_A \not\sqsubseteq g_A$. So $g_A \notin \tau(\tau)$. Therefore $\tau(\tau)$ mixed fuzzy soft topology only contains Φ, f_A. $\tau(\tau) = \{\Phi, f_A\}$ is a indiscrete mixed fuzzy soft topolgy.

2.We show that $\tau(\tau)$ is a discrete mixed fuzzy soft topolgy.

Since Φ is not quasi-coincident with any fuzzy soft set g_A and therefore, there does not arise any questions of violation of the condition of being member of $\tau(\tau)$. Therefore $\Phi \in \tau(\tau)$.

Any fuzzy soft set g_A be a fuzzy soft subset of f_A, such that $g_A \tilde{q} f_A$ and there exists τ –open set f_A with $g_A \tilde{q} f_A$ and τ-closure $\overline{f_A} = f_A \sqsubseteq f_A$. Therefore $f_A \in \tau(\tau)$.

Let $g_A \sqsubseteq f_A$, k_A be a fuzzy soft subset of f_A, with $g_A \tilde{q} f_A$. There exists τ open g_A such that $g_A \tilde{q} f_A$. τ closure $\overline{g_A} = g_A$, because τ is a discrete topology. $\overline{g_A} = g_A \sqsubseteq g_A$. So $g_A \in \tau(\tau)$. Therefore $\tau(\tau)$ mixed fuzzy soft topology contains all fuzzy soft subsets of f_A. $\tau(\tau) = P(f_A)$ is a discrete mixed fuzzy soft topolgy.

3.We show that $\tau_1(\tau_2)$ is a indiscrete mixed fuzzy soft topolgy.

Since Φ is not quasi-coincident with any fuzzy soft set g_A and therefore, there does not arise any question of violation of the condition of being member of $\tau_1(\tau_2)$. Therefore $\Phi \in \tau_1(\tau_2)$.

Any fuzzy soft set g_A be a fuzzy soft subset of f_A, such that $g_A \tilde{q} f_A$ and there exists τ_2 –open set f_A with $g_A \tilde{q} f_A$ and τ_1-closure $\overline{f_A} = f_A \sqsubseteq f_A$. Therefore $f_A \in \tau_1(\tau_2)$.

Let g_A be any subset of f_A, k_A be a fuzzy soft subset of f_A, with $g_A \tilde{q} f_A$. There exists τ_2 open f_A such that $f_A \tilde{q} k_A$. τ_1 closure $\overline{f_A} = f_A \not\sqsubseteq g_A$. So $g_A \notin \tau_1(\tau_2)$. Therefore $\tau_1(\tau_2)$ mixed fuzzy soft topology only contains Φ, f_A. $\tau_1(\tau_2) = \{\Phi, f_A\}$ is a indiscrete mixed fuzzy soft topolgy.

4.We show that $\tau_1(\tau_2)$ is a indiscrete mixed fuzzy soft topolgy.

Since Φ is not quasi-coincident with any fuzzy soft set g_A and therefore, there does not arise any questions of violation of the condition of being member of $\tau_1(\tau_2)$. Therefore $\Phi \in \tau_1(\tau_2)$.

Any fuzzy soft set g_A be a fuzzy soft subset of f_A, such that $g_A \tilde{q} f_A$ and there exists τ_2-open set f_A with $g_A \tilde{q} f_A$ and τ_1-closure $\overline{f_A} = f_A \sqsubseteq f_A$. Therefore $f_A \in \tau_1(\tau_2)$.

Let g_A be any subset of f_A, k_A be a fuzzy soft subset of f_A, with $g_A \tilde{q}\, f_A$. There exists τ_2 open h_A such that $h_A \tilde{q}\, k_A$. τ_1 closure $\overline{h_A}$ $=f_A$. Because τ_1 indiscrete topology. $\overline{h_A}$ $=f_A \not\sqsubseteq g_A$. So $g_A \notin \tau_1(\tau_2)$. Therefore $\tau_1(\tau_2)$ mixed fuzzy soft topoloyg only contains Φ, f_A. $\tau_1(\tau_2) = \{\Phi, f_A\}$ is a indiscerete mixed fuzzy soft topolgy.

4. COUNTABILITY ON MIXED FUZZY SOFT TOPOLOGICAL SPACE ON f_A

In this section, the definitions of neighbourhood, Q neighbourhood, first countability, Q first countability, second countability will be given with respect to the topology on fuzzy soft set f_A instead of the topology on X. Furthermore, the above mentioned definitions and related theorems will be given in mixed fuzzy soft topological space on fuzzy soft set f_A.

Definition 29. Let (f_A, τ_f) fuzzy soft topological space and $N(x_A^\lambda)$ be a family of neighbourhood of a fuzzy soft point x_A^λ in f_A. A subfamily $B(x_A^\lambda)$ of $N(x_A^\lambda)$ is said to be fuzzy soft neighbourhood base of x_A^λ, if for every $g_A \in N(x_A^\lambda)$ there exists $h_A \in B(x_A^\lambda)$ such that $h_A \sqsubseteq g_A$.

Definition 30. A fuzzy soft topological space (f_A, τ_f) is said to be first countable space if and only if every fuzzy soft point in f_A has a countable fuzzy soft neighbourhood base.

The following definition is an alternative to the Definition 4.2.

Definition 31. Let (f_A, τ_f) be a fuzzy soft topological space. Then (f_A, τ_f) is said to be fist countable space, if for each fuzzy soft point $x_A^\lambda (0 < \lambda(e) \leq 1)$ there exists a countable class of fuzzy soft open sets $B(x_A^\lambda)$ such that $x_A^\lambda \in g_A$, for all $g_A \in B(x_A^\lambda)$ and $x_A^\lambda \in h_A$ for some fuzzy soft open set h_A then there exists $k_A \in B(x_A^\lambda)$ such that $k_A \sqsubseteq h_A$.

Definition 32. Let $N_Q(x_A^\lambda)$ be a family of Q fuzzy soft neigbourhood of fuzzy soft point x_A^λ in f_A. A subfamily $B_Q(x_A^\lambda)$ of $N_Q(x_A^\lambda)$ is said to be a Q -fuzzy soft neigbourhood base of x_A^λ if for every $g_A \in N_Q(x_A^\lambda)$ there exists $h_A \in B_Q(x_A^\lambda)$ such that $h_A \sqsubseteq g_A$.

Briefly we will say Q -fuzzy soft neighbourhood base Q -neighbourhood base.

Definition 33. A fuzzy soft topological space (f_A, τ_f) is said to be Q -first countable space if and only if every fuzzy soft point in f_A has countable Q -neighbourhood base.

Definition 34. A fuzzy soft topologial space (f_A, τ_f) is said to be second countable space if there exists a countable base for τ_f.

Definition 35. Let x_A^λ be a fuzzy soft point in f_A, g_A be a fuzzy soft subset of f_A and $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topolgical space. g_A is a called fuzzy soft neighbourhood of x_A^λ, if there exists a fuzzy soft open h_A such that $x_A^\lambda \tilde{\in} h_A \sqsubseteq g_A$.

The family of neighbourhood of x_A^λ is denoted by $N(x_A^\lambda)$.

Definition 36. Let x_A^λ be a fuzzy soft point in f_A, g_A be a fuzzy soft subset of f_A and $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topolgical space. g_A is a called Q -fuzzy soft neighbourhood of x_A^λ, if there exists a fuzzy soft open h_A such that $x_A^\lambda \tilde{q} h_A \sqsubseteq g_A$.

The family of Q -fuzzy soft neighbourhood of x_A^λ is denoted by $N_Q(x_A^\lambda)$.

Definition 37. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space. Let $N(x_A^\lambda)$ be a family fuzzy soft neighbourhood of a fuzzy soft point x_A^λ in f_A. A subfamily $B(x_A^\lambda)$ of $N(x_A^\lambda)$ is said to be a fuzzy soft neighbourhood base of x_A^λ if for every $g_A \in N(x_A^\lambda)$ there exists $h_A \in B(x_A^\lambda)$ such that $h_A \sqsubseteq g_A$.

Definition 38. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space. Then $(f_A, \tau_1(\tau_2))$ is said to be first countable space if every fuzzy soft point in f_A has a countable fuzzy soft neighbourhood base.

Definition 39. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space. Then $(f_A, \tau_1(\tau_2))$ is said to be Q -first countable space if every fuzzy soft point in f_A has a countable Q -neighbourhood base.

Definition 40. A mixed fuzzy soft topological space $(f_A, \tau_1(\tau_2))$ is said to be second countable space if there exists a countable base for $\tau_1(\tau_2)$.

Theorem 2. Let $(f_A, \tau_1(\tau_2))$ be a first countable space. Then it is a Q - first countable space.

Proof: Let x_A^λ be any fuzzy soft point in f_A. Consider a sequence $\{\lambda_n(e)\}n\epsilon N$ in $(1-\lambda(e), 1]$ converging to $1-\lambda(e)$ and let $x_A^{\lambda n} \in f_A$. Since $(f_A, \tau_1(\tau_2))$ is a first countable space for each $n \in N$, there exists a countable open neighbourhood base $B_n(x_A^{\lambda n})\}, n \in N$ of $x_A^{\lambda n}$. We have for each member g_A of $\{B_n(x_A^{\lambda n})\}$, $g_A(e)(x) \geq \lambda_n(e) > 1 - \lambda(e)$

$\Longrightarrow \lambda(e) + g_A(e)(x) > 1$

$\Longrightarrow x_A^\lambda \tilde{q} g_A$

Hence g_A is a Q -neighbourhood of x_A^λ. This the collection $\{B_n(x_A^{\lambda n})\}$ is a family of open Q -neighbourhoods of x_A^λ and hence this family is a countable family of Q -neighbourhood of x_A^λ.

Let h_A be an arbitrary Q -neighbourhood of x_A^λ. Hence $h_A(e)(x) > 1 - \lambda(e)$. Since $\lambda_n(e) > 1 - \lambda(e)$ so there exists $m \in N$ such that $h_A(e)(x) \geq \lambda_m(e) > 1 - \lambda(e) \Longrightarrow x_A^{\lambda m} \tilde{\in} h_A$ and an open neighbourhood of $x_A^{\lambda m}$.

This there exists a member $g_A \in \{B_n(x_A^{\lambda n})\}$ such that $g_A \sqsubseteq h_A$ and $g_A(e)(x) > \lambda_m(e) > 1 - \lambda(e)$ and so g_A is a Q -neighbourhood base of x_A^λ. Hence $(f_A, \tau_1(\tau_2))$ is Q -first countable space.

Theorem 3. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space and x_A^λ is a fuzzy soft point f_A. A subfamily β of $\tau_1(\tau_2)$ is said to base for $\tau_1(\tau_2)$ if and only if a collection $B(x_A^\lambda)$ such that,

$B(x_A^\lambda) = \{k_A \in \beta : x_A^\lambda \tilde{\in} k_A\}$ is a neighbourhood base of x_A^λ.

Proof: Let β be a base for $\tau_1(\tau_2)$. We Show that $B(x_A^\lambda) = \{k_A \in \beta : x_A^\lambda \tilde{\in} k_A\}$ is a neighbourhood base of x_A^λ.

x_A^λ is an arbitrary fuzzy soft point in f_A and h_A be a fuzzy soft neighbourhood of $x_A^\lambda, h_A \in N(x_A^\lambda)$. $h_A \in N(x_A^\lambda) \Longrightarrow$ there exists g_A fuzzy soft open such that $x_A^\lambda \tilde{\in} g_A \sqsubseteq h_A$. Since β is a base for $\tau_1(\tau_2)$, $g_A \in \tau_1(\tau_2)$ is expressed as a union of members of β. Therefore, there exists $k_A \in \beta$ such that $x_A^\lambda \tilde{\in} k_A \sqsubseteq g_A$. Hence k_A becomes a member of neighbourhood base of x_A^λ. $k_A \in B(x_A^\lambda)$ and $B(x_A^\lambda) = \{k_A \in \beta : x_A^\lambda \tilde{\in} k_A\}$ is a neighbourhood base of x_A^λ.

In contrast, let $B(x_A^\lambda)$ be a neighbourhood base of x_A^λ. We Show that β is a base for $\tau_1(\tau_2)$. Let any $g_A \in \tau_1(\tau_2)$ and $x_A^\lambda \tilde{\in} k_A \Longrightarrow g_A \in N(x_A^\lambda)$. From the definition neighbourhood base of x_A^λ there exists

$h_A \in B(x_A^\lambda)$ such that $x_A^\lambda \tilde{\in} h_A \sqsubseteq g_A$ and $h_A \in \beta$. $\forall x_A^\lambda \tilde{\in} g_A$ and $x_A^\lambda \tilde{\in} h_A \sqsubseteq g_A \Longrightarrow g_A = \sqcup_{x_A^\lambda \tilde{\in} g_A} h_A, h_A \in \beta$. g_A is an arbitrary element of $\tau_1(\tau_2)$ and g_A is a union of members of β. Therefore β is a base for $\tau(\tau_2)$.

Proposition 1. Let $(f_A, \tau_1(\tau_2))$ be a mixed fuzzy soft topological space. If $(f_A, \tau_1(\tau_2))$ is second countable space, then it is also first countable space.

Proof: Let $(f_A, \tau_1(\tau_2))$ be a second countable space. There exists a countable base β for $\tau_1(\tau_2)$. B(x_A^λ) neighbourhood base of x_A^λ is a subset of β from. Theorem 4.2. Since β is a countable space, B(x_A^λ) is a countable space. Therefore B(x_A^λ) is a countable neighbourhood base of x_A^λ.

Thus, there are a countable neighbourhood base every fuzzy soft point in f_A and $(f_A, \tau_1(\tau_2))$ is a first countable space.

Result 2. If $(f_A, \tau_1(\tau_2))$ is second countable space, then it also Q -first countable space.

Proof: Let $(f_A, \tau_1(\tau_2))$ be a second countable space. From Proposition 4.1. $(f_A, \tau_1(\tau_2))$ is a first countable space and from Theorem 4.1. $(f_A, \tau_1(\tau_2))$ is a Q -first countable space.

Proposition 2. Let τ_1 and τ_2 be two fuzzy soft topologies for f_A and if the mixed fuzzy soft topology $\tau_1(\tau_2)$ is Q -first countable, then τ_2 is also Q -first countable.

Proof: Let x_A^λ be an arbitrary fuzzy soft point in f_A. Since $\tau_1(\tau_2)$ is a Q-first countable space, therefore there exists a countable Q -neighbourhood base for every fuzzy soft point x_A^λ. Let $g_A \in B_Q(x_A^\lambda)$, where $B_Q(x_A^\lambda)$ is the countable collection of $\tau_1(\tau_2)$ Q-neighbourhood base at x_A^λ. Then g_A is $\tau_1(\tau_2)$ Q-neighbourhood of x_A^λ. There exists $h_A \in \tau_1(\tau_2)$ such that $h_A \sqsubseteq g_A$ and $x_A^\lambda \tilde{q} h_A$. We know that $\tau_1(\tau_2) \sqsubseteq \tau_2$. Therefore $h_A \in \tau_1(\tau_2) \Longrightarrow h_A \in \tau_2$ and $h_A \sqsubseteq g_A, x_A^\lambda h_A$, so g_A also τ_2 Q-neighbourhood of x_A^λ. Thus every member $g_A \in B_Q(x_A^\lambda)$ is τ_2 Q -neighbourhood of x_A^λ. $B_Q(x_A^\lambda)$ is also τ_2 Q -neighbourhood base at x_A^λ. Hence τ_2 is Q -first countable space.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] Zadeh, L.A. “Fuzzy sets”, Information and Control, 338-353, (1965).

[2] C.L. Chang, “Fuzzy topological spaces”, J. Math. Anal. Appl.24, 182-190, (1968).

[3] Pao-Ming, P., and Ying-Ming, L., “Fuzzy topology. I. Neighborhood structure of a fuzzy point and Moore-Smith convergence”, J.Math.Anal.Appl., 76: 571599 (1980).

[4] Molodtsov, D., “Soft set theory - First result”, Computers Math. Appl.,37(4-5): 19-31, (1999).

[5] Maji, P.K, Biswas, R and Roy A.R.“Soft set theory”, Computers Math. Appl.,45(4-5): 555-562, (2003).

[6] Maji, P.K.,Biswas, R. and Roy A.R. “Fuzzy soft sets”, J. Fuzzy Math., 589-602, (2001).

[7] Tanay B., Kandemir M.B., “Topological structure of fuzzy soft sets”, Computers and Mathematics with Appl., 61(10): 2952-2957, (2011).

[8] Roy, S., Samanta, T.K., “A note on fuzzy soft topological spaces”, Annals of Fuzzy Mathematics and Informatics”, 3(2): 305-311, (2012).

[9] Varol, B.P., “Fuzzy soft topological spaces”, Ph.D Thesis, The Graduate School of Natural and Applied Science of Kocaeli University, (2012)

[10] Varol, B.P.,Aygun H., “Fuzzy soft topology”, Hacettepe Journal of Mathematics and Statistics”, 41(3): 404-419, (2012).

[11] Neog, T.J., Sut, D.K., Hazarika, G.C., “Fuzzy soft topological spaces”, Lates Trend Math., 2(1), 54-67, (2012).

[12] Hussain, S., “On some generalized structures in fuzzy soft topological spaces”, Information Sciences Letters, 3: 107-115, (2015).

[13] Dizman, T.H., “Soft ditopological-fuzzy soft topological spaces and the applications in medicine”, Ph.D Thesis, The Graduate School of Natural and Applied Science of Selcuk University, (2014).

[14] Simsekler, T., Yuksel S., “Fuzzy soft topological spaces”, Annals of Fuzzy Math. and Informatics”, 5(1): 87-96, (2013).

[15] Tripathy, B.C., Ray G.C., “On mixed fuzzy topological spaces and countability”, Soft Comput., 16(10): 1691-1695, (2012).

[16] Tripathy, B.C. Ray G.C., “Mixed fuzzy ideal topological spaces”, Appl. Math. and Comput,,220: 602-607, (2013).

[17] Borah, M.J. and Hazarika B., “Some results of mixed fuzzy soft topology and applications in Chemistry”, Annals of Fuzzy Mathematics and Informatics, 12(1): 129-138, (2016).