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Concircular Curvature Tensor on Generalized Kenmotsu Manifolds

Genelleştirilmiş Kenmotsu Manifoldları Üzerinde Concircular Eğrilik Tensörü

İnan ÜNAL*[1,a], Ramazan SARI[2,b], Aysel TURGUT VANLI[3,c]
[1]*Munzur Üniversitesi, Mühendislik Fakültesi, Bilgisayar Mühendisliği Bölümü, Tunceli*
[2]*Amasya Üniversitesi, Gümüşhacıköy Hasan Duman Meslek Yüksekokulu, Amasya*
[3]*Gazi Üniversitesi Fen Fakültesi Matematik Bölümü, Ankara*

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Abstract
The aim of the present paper is to study on concircular curvature tensor on generalized Kenmotsu manifolds. Concircular flat and φ-concircular flat generalized Kenmotsu manifolds are examined. Also some results are given about φ-semi symmetric and φ -concircular semi symmetric generalized Kenmotsu manifolds.

Keywords: Concircular curvature tensor, Generalized Kenmotsu manifolds, φ-semi symmetric, φ-concircular semi symmetric

Öz
Bu çalışmanın amacı genelleştirilmiş Kenmotsu manifoldları üzerinde concircular eğrilik tensörünün çalışılmasıdır. Concircular düz ve φ-concircular düz genelleştirilmiş Kenmotsu manifoldları incelenmiştir. Ayrıca φ-semi simetrik ve φ-concircular semi simetrik genelleştirilmiş Kenmotsu manifoldları üzerine bazı sonuçlar verilmiştir.

Anahtar kelimeler: *Conircular eğrilik tensörü, Genelleştirilmiş Kenmotsu manifoldları, φ-semi simetrik, φ-concircular semi simetrik*

*[a] İnan ÜNAL; inanunal@munzur.edu.tr; Tel: (0428) 2131794 ; *orcid.org/0000-0003-1318-9685*
[b] *orcid.org/0000-0002-4618-8243* [c] orcid.org/0000-0001-9793-7366

1. Introduction

Contact geometry is an important branch of geometry. One can dived into two part this notion: real and complex. The complex contact geometry studied since 1959 and there several works in literature (Blair, 2010; Turgut Vanlı and Unal, 2017). Real contact geometry is studying widely by scientists and has lots of applications to other areas of mathematics and theoretical physics (Kholodenko, 2013). There are several classes of contact manifolds. One of them Kenmotsu manifolds which is called for honor of Katsuei Kenmotsu who was introduced the notion. Kenmotsu proposed to study the properties of warped product of the complex space with real line. This product is one of the three classes in classification was given by S. Tanno (Tanno, 1969). So this problem appears naturally at that time. Kenmotsu obtained some results and gave characterization for this third classes of Tanno's classification. The importance of Kenmotsu manifolds comes from some different properties of these spaces. On of them is that a n almost contact manifold which is satis- fied being Sasakian condition is not Sasakian. Also the contact distribution of any Kenmotsu manifolds is always integrable. A Kenmotsu manifold is not compact and has negative sec- tional curvature, for details see (Pitiş, 2007).

A manifold with the almost contact Riemann structure (φ,ξ,η,g) is an almost Kenmotsu manifold if the following conditions are satisfied

$$d\eta = 0,\ d\Omega = 2\eta\wedge\Omega$$

where Ω is second fundamental form of almost contact structure and defined by $\Omega(X,Y)=g(\varphi X,Y)$ for X, Y ∈ Γ(T M). In 2006, M. Falcitelli and A. M. Pastore introduced Kenmotsu $f.pk$ -manifold (Falciletti and Pastore, 2006). An $f.pk$ -manifold M is (2n+ s)-dimensional manifold with an f –structure with parallelizable kernel. A normal -manifold $(M,\varphi_i,\xi_i,\eta_i,g)$ is called Kenmotsu if 1-forms η^i are closed and $d\Omega = 2\eta^1\wedge\Omega$.

Two of presented authors introduced generalized Kenmotsu manifolds obtained some results on curvature properties (Turgut Vanlı and Sari, 2016). Also same authors studied on invariant submanifolds of generalized Kenmotsu manifold (Turgut Vanlı and Sari, 2015a,b). In 2017 Srikantha and Venkatesha examined this kind of submanifolds by certain conditions (Srikantha and Venkatesha, 2017). In this article we studied on concircular curvature tensor on generalized Kenmotsu manifolds. We gave some results on flatness conditions and symmetric conditions.

2. Preliminaries

Let M be a $(2n+s)-$dimensional differentiable manifold. If, for $(1,1)$ type tensor field φ, $s-$differentiable vector fields $\xi_1,\dots,\xi_s$ (called structure vector fields), $s-$differentiable 1-forms $\eta^1,\dots,\eta^s$, a Riemannian metric g and vector fields $X,T\in\Gamma(TM)$ on M we have

$$\varphi^2 = -I+\sum_{i=1}^{s}\eta^i\otimes\xi_i,\ \ \eta^i(\xi_j)=\delta_{ij},\varphi\xi_i=0,\ \ \eta^i\circ\varphi=0 \qquad (1)$$

$$g(\varphi X,\varphi T)=g(X,T)-\sum_{i=1}^{s}\eta^i(X)\eta^i(T) \qquad (2)$$

then we recall M is f-manifold (Goldberg and Yano,1971). In addition we get

$\eta^i(X)=g(X,\xi_i),\ g(X,\varphi T)=-g(\varphi X,T).$

The fundamental second form of the contact structure is defined by $\Phi(X,T)=g(X,\varphi T)$.

If we denote $\mathcal{L}=\{X:\eta^\alpha(X)=0, 1\le\alpha\le s\}$ and $\mathcal{M}=sp\{\xi_1,\xi_2,...,\xi_s\}$ then we can write $TM=\mathcal{L}\oplus\mathcal{M}$ Let U be a coordinate neighborhood and E_1 be a unit vector field on U orthogonal to the structure vector fields. M has local an orthonormal basis $\{E_1,\dots,E_n,\varphi E_1,\dots,\varphi E_n,\xi_1,\dots,\xi_s\}$.

A metric f-manifold is normal if $[\phi,\phi]+2\sum_{i=1}^{s}d\eta^i\otimes\xi_i=0$, where $[\varphi,\varphi]$ is the Nijenhuis tensor of φ. A 2-form Φ on M such that $\eta^1\wedge...\wedge\eta^s\wedge\Phi^n\neq 0$ then M is called an almost s-contact metric manifold. A normal almost s-contact metric manifold is called an s-contact metric manifold.

Definition 2.1 Let $\left(M^{(2n+s)},\varphi,\xi^i,\eta^i\right), s\ge 1,$ be an almost $s-$contact metric manifol. If for all $1\le i\le s,$ are closed and $d\Phi = 2\sum_{i=1}^{s}\eta^i\wedge\Phi$ then M is called generalized almost Kenmotsu manifol.

If, in addition, M is normal, then it is called a generalized Kenmotsu manifold.
Let R be Riemannian curvature tensor of M which is defined as:

$$R(X,Y)W = \nabla_X \nabla_Y W - \nabla_Y \nabla_X W - \nabla_{[X,Y]} W \quad (3)$$

for X,Y,W vector fields on M. The Riemann curvature of a generalized Kenmotsu manifold have following equations (Turgut Vanli and Sari, 2016)

$$R(X,T)\xi_i = \sum_{j=1}^{s} \{\eta^j(T)\varphi^2 X - \eta^j(X)\varphi^2 T\} \quad (4)$$

$$R(X,\xi_i)T = \sum_{j=1}^{s} \{\eta^j(T)\varphi^2 X - g(X,\varphi^2 T)\xi_j\} \quad (5)$$

$$R(X,\xi_j)\xi_i = \varphi^2 X, \qquad R(\xi_k,\xi_j)\xi_i = 0 \quad (6)$$

$$\eta^i(R(X,T)W) = \sum_{j=1}^{s} \{\eta^j(T)g(X,W) - \eta^j(X)g(T,W)\}. \quad (7)$$

Also for $X,Y,T \in \Gamma(TM)$ we get

$$R(X,Y)\varphi T - \varphi R(X,Y)T = g(Y,T)\varphi X - g(X,T)\varphi Y - g(Y,\varphi T)X + g(X,\varphi T)Y, \quad (8)$$

$$R(\varphi X,\varphi Y)T = R(X,Y)T + g(Y,T)X - g(X,T)Y + g(Y,\varphi T)\varphi X - g(X,\varphi T)\varphi Y, \quad (9)$$

$$R(\varphi X,Y)T = \varphi R(X,Y)T + g(Y,T)\varphi X - g(X,T)\varphi Y - g(\varphi X,\varphi T)\varphi Y - g(\varphi Y,\varphi T)\varphi X. \quad (10)$$

For the Ricci curvature S of M we have

$$S(X,\xi_i) = -2n\sum_{j=1}^{s} \eta^j(X), \quad S(\xi_k,\xi_i) = -2n. \quad (11)$$

The notion of curvature is one of important tools for understanding Riemannian geometry of manifolds. Some geometric features of structure on manifolds could be restricted from curvature tensors. One of the important curvature tensor is concircular curvature tensor which is invariant under concircular transformations (Yano and Kon, 1984). If concircular curvature tensor vanishes then we recall the manifold concircularly. Concircular curvature tensor on a $(2n+s)-$dimensional generalized Kenmotsu manifold is given by

$$Z(X,Y)T = R(X,Y)T - \frac{r}{(2n+s)(2n+s-1)}\left[g(Y,T)X - g(X,T)Y\right]. \quad (12)$$

where r is the scalar curvature of M. Yano proved that a concircular flat Riemann manifold is space of constant curvature (Yano, 1940). Also an Einstein space is invariant under concircular transformation. We can express that this tensor measure the difference the manifold from being space forms. In order to brevity we take $A = \frac{r}{(2n+s)(2n+s-1)}$.

From the definition of concircular curvature tensor on a generalized Kenmotsu manifold M we have

$$Z(\xi_k,\xi_j)\xi_i = 0 \quad (13)$$

$$Z(X,\xi_j)\xi_i = -X + \sum_{\alpha=1}^{s} \eta^\alpha(X)\xi_\alpha - A\eta^j(X)\xi_i \quad (14)$$

$$Z(X,\xi_i)Y = \sum_{j=1}^{s} \left(g(X,Y)\xi_j - \eta^j(Y)X\right) + A(g(X,Y)\xi_i - \eta^i(Y)X) \quad (15)$$

$$Z(X,Y)\xi_i = \sum_{j=1}^{s} (\eta^j(X) - \eta^j(Y)X) + A(\eta^i(X)Y - \eta^j(Y)X) \quad (16)$$

for all $X,Y \in \Gamma(TM)$ and $1 \le i, j, \alpha \le s$.
In the contact geometry $\eta-$ Einstein metric has important position. This metric is the generalization of Einstein metric. Let λ, μ some functions and X,T be vector fields on M if the Ricci tensor of a generalized Kenmotsu manifold satisfies

$$S(X,T) = \lambda g(X,T) + \mu \sum_{i=1}^{s} \eta^i(X)\eta^i(T)$$

then M is called $\eta-$Einstein generalized Kenmotsu manifold. Since $TM = \mathcal{L} \oplus sp\{\xi_1, \xi_2, ..., \xi_s\}$ we can write

$$X = X_0 + \sum_{i=1}^{s} \eta^i(X)\xi_i \text{ and } T = T_0 + \sum_{i=1}^{s} \eta^i(T)\xi_i$$

where $X_0, T_0 \in \mathcal{L}$. Then we get

$$S(X,T) = S(X_0,T_0) - 2n\sum_{i=1}^{s} \eta^i(X)\eta^i(T). \quad (17)$$

There we get that if $S(X_0,T_0) = \alpha g(X_0,T_0)$ then M is $\eta-$Einstein.
By similar consideration for sectional curvature of a generalized Kenmotsu manifold we have

$$\begin{aligned} k(X,T) &= k(X_0,T_0) + \sum_{\alpha,\beta=1}^{s} \eta^\alpha(T)\eta^\beta(T)k(T_0,\xi_\alpha) + \sum_{\alpha,\beta=1}^{s} \eta^\alpha(X)\eta^\beta(X)k(X_0,\xi_\alpha) \\ &+ \sum_{\alpha,\beta=1}^{s} [\eta^\alpha(X)\eta^\beta(T)]^2 k(\xi_\alpha,\xi_\beta). \end{aligned} \quad (18)$$

3. Flatness Of Generalized Kenmotsu Manifold Via Concircular Curvature Tensor

As we know any concircular flat Riemann manifold is space of constant curvature. There is also a similar results for generalized Kenmotsu manifold. By an easy and direct computation;

Theorem 3.1 Any concircularly flat generalized Kenmotsu manifold is Einstein.

Definition 3.2 Let M be a generalized Kenmotsu manifold. For $X,Y,W \in \Gamma(TM)$ if

$$\phi^2(Z(\varphi X, \varphi Y)\varphi W) = 0 \quad (19)$$

then M is called $\varphi-$concircularly flat.

Theorem 3.3 $\varphi-$concircularly flat generalized Kenmotsu manifold is $\eta-$Einstein.

Proof Let M be a $\varphi-$concircularly flat generalized Kenmotsu manifold. From the definition of concircular curvature tensor and by using (10) for $X,Y,U \in \Gamma(TM)$ we get

$$\begin{aligned} &Z(\varphi X, \varphi Y,)\varphi U = \varphi R(X,Y)U + g(Y,U)\phi X - g(X,U)\varphi Y \\ &+(1+A)g(\varphi X, \varphi U)\phi Y - (1+A)g(\varphi Y, \varphi U)\varphi X. \end{aligned}$$

Since M is $\varphi-$concircularly flat by applying φ^2 to this equation we have

$$\begin{aligned} &0 = -\varphi R(X,Y)U - g(Y,W)\phi X + g(X,U)\varphi Y - (1+A)g(\varphi X, \varphi U)\varphi Y \\ &+(1+A)g(\varphi Y, \varphi U)\varphi X. \end{aligned}$$

Let apply φ again to last equation and from (1),(2) and (7) we obtain

$$\begin{aligned} &R(X,Y)U = \sum_{\alpha=1}^{s}\sum_{j=1}^{s}\{\eta^j(X)g(Y,U) - \eta^j(Y)g(X,U)\}\xi_\alpha \\ &-\{g(Y,U) - (1+A)g(\varphi Y, \varphi U)\}\phi^2 X + \{g(X,U) - (1+A)g(\varphi X, \varphi U)\}\phi^2 Y. \end{aligned} \quad (20)$$

Let choose $Y = U = E_i$, $X = X_0, T = T_0$ for $X_0, T_0 \in \mathcal{L}$ and taking sum over i to $2n+s$ for the basis $\{E_1, \ldots, E_n, \varphi E_1, \ldots, \varphi E_n, \xi_1, \ldots, \xi_s\}$ from (20) we get

$$\sum_{i=1}^{s} g(R(X_0,E_i),E_i,T_0) = -\sum_{i=1}^{s}\{\{g(E_i,E_i)-(1+A)g(\phi E_i,\phi E_i)\}g(\phi^2 X_0,T_0)\}$$
$$+\sum_{i=1}^{s}\{\{g(X_0,E_i)-(1+A)g(\phi X_0,\phi,E_i)\}g(\phi^2 E_i,T_0)\}$$

and thus we have

$$\sum_{i=1}^{s} g(R(X_0,E_i),E_i,T_0) = -(2n+s)g(\phi^2 X_0,T_0)-(1+A)2ng(\phi^2 X_0,T_0)+g(\phi^2 X_0,T_0)$$
$$-(1+Ag(\phi^2 X_0,\phi^2 T_0)).$$

Therefore we get

$$S(X_0,T_0) = (4n+s+(2n-1)A)g(X_0,T_0)$$

and from (17) we have

$$S(X,T) = (4n+s-(2n-1)A)g(X,T)+(-6n-s+(2n-1)A)\sum_{j,\alpha=1}^{s}\eta^{\alpha}(X)\eta^{\alpha}(T)$$

for arbitrary vector fields X,T on M. So this shows that M is $\eta-$Einstein.

Definition 3.4 Let M be a generalized Kenmotsu manifold. For $X,Y,W \in \Gamma(TM)$ if

$$g\left(Z(\phi X,Y)T,\phi W\right) = 0 \quad (21)$$

then M is called pseudo-concircularly flat.

Theorem 3.5 Let M be pseudo-concircularly flat generalized Kenmotsu manifold then the Riemannian curvature of M has the following form:

$$R(T,W)X = -g(W,\phi X)T - g(T,\phi X)W - g(W,X)\phi T + g(T,X)\phi W$$
$$+A\big(g(\phi X,W)T - g(\phi X,T)\phi W\big) + \sum_{\alpha=1}^{s}\{\eta^{\alpha}(R(T,W)X) + g(W,\phi X)\eta^{\alpha}(T) \quad (22)$$
$$+g(T,\phi X)\eta^{\alpha}(W)\}\xi_{\alpha}$$

where $T,W,X \in \Gamma\left(TM\right)$.

Proof Let M be pseudo-concircularly flat generalzed Kenmotsu manifold. For $X,Y,T,W \in \Gamma(TM)$ from (12), (8) we get

$$\phi R(T,W)X = -g(W,\phi X)\phi T + g(T,\phi X)\phi W - g(W,X)T - g(T,X)W$$
$$+A(g(\phi X,W)T - g(\phi X,T)W. \quad (23)$$

By applying φ to the (23) and from (1), (7) we obtain (22).

From this theorem we have following corollary;

Corollary 3.6 Let M be a pseudo-concircularly flat generalized Kenmotsu manifold for $X_0,T_0 \in \Gamma(\mathcal{L})$ and $\xi_\alpha \in \Gamma(\mathcal{M})$ the sectional curvature of a pseudo-concircularly flat generalized Kenmotsu manifold is given $k(X_0,T_0) = k(X_0,\xi_\alpha) = k(\xi_\alpha,\xi_\beta) = 0.$

Thus from (18) also we get;

Corollary 3.7 The sectional curvature of a pseudo-concircularly flat generalized Kenmotsu manifold vanishes.

Corollary 3.8 The $\varphi-$sectional curvature of a pseudo-concircularly flat generalized Kenmotsu manifold M is equal to 1.

4. Some Symmetry Properties of a Generalized Kenmotsu Manifold

The notion of symmetry has the important position to study the Riemannain geometry of manifolds. Locally symmetry of a Riemann manifold could state by curvature tensor.

For a S tensor $R(X,Y).S$ is defined by

$$R(X,Y).S = \nabla_X\nabla_Y S - \nabla_Y\nabla_X S - \nabla_{[X,Y]}S.$$

Also if $R.\mathcal{T}=0$ then the manifold is called $T-$ semi symmetric. For a Riemannian manifold if $R.R=0$ then the manifold is called locally symmetric. Locally symmetric generalized Kenmotsu manifold studied by Turgut Vanli and Sari (Turhut Vanlı and Sarı, 2016). They proved that $\varphi-$sectional curvature of any semi symmetric $(2n+s)$-dimensional generalzed Kenmotsu manifold $\left(M,\varphi,\xi_i,\eta^i,g\right)$ is equal to $-s$. Same authors also studied on Ricci-semi symmetric generalized Kenmotsu manifolds. In 2005 Blair, Kim and Tripathi studied on concircular curvature tensor of a contact metric manifold (Blair vd., 2005). They classify the special class of contact $N(k)$ manifolds by concircularly symmetric (which means $\nabla\mathcal{Z}=0$) condition. The concircular geometry has interesting results in the contact geometry. In this section we study on generalized Kenmotsu manifold via symmetry condition with these tensor.

Let M be a generalized Kenmotsu manifold. If $R(X,Y).\varphi=0$ then M is called $\varphi-$semi symmetric. For $X,Y,U\in\Gamma(TM)$ $R(X,Y).\varphi$ is given by

$$(R(X,Y).\phi)U=R(X,Y)\phi U-\phi R(X,Y)U.$$

Similarly if $\mathcal{Z}(X,Y).\varphi=0$ then the manifold is called $\varphi-$concircular semi symmetric. For $X,Y,W\in\Gamma(TM)$ $\mathcal{Z}(X,Y).\varphi$ is given by

$$(Z(X,Y).\varphi)W=Z(X,Y)\varphi W-\varphi Z(X,Y)W.$$

Also for arbitrary vector fields X,Y,Z,T,U and W on M we have

$$(R(X,Y).Z)(T,U)W=R(X,Y)Z(T,U)W-Z(R(X,Y)T,U)W-Z(T,R(X,Y)U)W-Z(T,U)R(X,Y)W.$$

A genarlized Kenmotsu manifold with $R.\mathcal{Z}=0$ is called concircular semi symmetric.

In this section we give some results on symmetry properties with related to concircular curvature tensor of generalized Kenmotsu manifold.

Theorem 4.1 A genarlized Kenmotsu manifold is $\varphi-$concircular semi symmetric if and only if the scalar curvature $r\neq-(2n+s)(2n+s-1)$ and the manifold is $\varphi-$semi symmetric.

Proof Let M be a $\varphi-$semi symmetric generalzed Kenmotsu manifold. From the definition and by using (8), (9) we get

$$0=(1+A)(g(Y,W)\varphi X-g(X,U)\varphi Y-g(Y,\varphi U)X+g(X,\varphi U)Y).$$

If $1+A\neq 0$ i.e $r\neq-(2n+s)(2n+s-1)$ we get

$$(g(Y,U)\varphi X-g(X,U)\varphi Y-g(Y,\varphi U)X+g(X,\varphi U)Y=0.$$

This means M is $\varphi-$semi symmetric. Conversely let M be a $\varphi-$semi symmetric generalzed Kenmotsu manifold and $r\neq-(2n+s)(2n+s-1)$ then we get

$$(Z(X,Y)\varphi)U=-A(g(Y,\varphi U)X-g(X,\varphi U)Y-g(Y,U)\varphi X+g(X,U)\varphi Y).$$

So the manifold M is $\varphi-$concircular semi symmetric.

Theorem 4.2 The Riemannian curvature tensor R of the a concircularly semi symmetric generalized Kenmotsu manifold is

$$R(T_0,U_0)Y_0=(s-2A)g(Y_0,T_0)U+(1+A)g(Y_0,U)T_0 \tag{24}$$

where Y_0,U_0,T_0 are vector fields orthogonal to ξ_i on M.

Proof Let choose $X=W=\xi_i$ and Y_0,U_0,T_0 orthogonal to ξ_i then we get

$$R(\xi_i,Y_0)Z(T_0,U_0)\xi_i-Z(R(\xi_i,Y_0)T_0,U)\xi_i-Z(T_0,R(\xi_i,Y_0)U)\xi_i-Z(T_0,U_0)R(\xi_i,Y_0)\xi_i=0.$$

From equalities (4) ,(5), (6), (14), (15) and (16) and by some computations we get (24).

For vector field V_0 is orthogonal to ξ_i from (24) we get

$$g(R(T_0,U_0)Y_0,V_0)=(s-2A)g(Y_0,T_0)g(U_0,V_0)+(1+A)g(Y_0,U_0)g(T_0,V_0)$$

and by choosing $U_0 = V_0 = E_i$, taking sum from i to $2n$ we obtain
$S(T_0,Y_0) = (2n+s-1-(2n-1)A)g(T_0,Y_0).$

Then from (18) for arbitrary vector fields $T,Y \in \Gamma(TM)$ we get

$$S(T,Y) = (2n+s-1-(2n-1)A)g(T,Y) - (-4n-s-1+(2n-1)A)\sum_{\alpha=1}^{s}\eta^{\alpha}(Y)\eta^{\alpha}(Y).$$

Thus we have proved following corollary.

Corollary 4.3 A concircularly semi symmetric generalized Kenmotsu manifold is $\eta-$Einstein.

Corollary 4.4 The sectional curvature of a generalized Kenmotsu manifold for unit ant mutually orthogonal vector field $T,W \in \Gamma(TM)$ is given by

$$k(T,W) = (1+A)\left(1+\sum_{\alpha,\beta=1}^{s}\left(\eta^{\alpha}(T)\eta^{\beta}(T)+\eta^{\alpha}(W)\eta^{\beta}(W)+\left(\eta^{\alpha}(T)\eta^{\beta}(W)\right)^{2}\right)\right).$$

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