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A STUDY ON MOTION PLANNING OF FOUR-BAR MECHANISMS IN THE PRESENCE OF SINGULARITIES

(DÖRT-ÇUBUK MEKANİZMALARININ TEKİLLİKLERİN VARLIĞINDA HAREKET PLANLAMASI ÜZERİNE BİR ÇALIŞMA)

Mustafa ÖZDEMİR[1]

ABSTRACT

Type II singularities are a phenomenon encountered in closed kinematic chains. Characteristically, they lead to diverging actuator forces and loss of motion control. As a general solution to this problem, it is suggested in the literature that the dynamic model of the mechanism is made consistent at the singular configuration so that the singularity can be passed through smoothly. In line with this principle, the present paper analytically explores the guidelines for motion planning of four-bar mechanisms in the presence of type II singularities. With this purpose in mind, four theorems and two corollaries are developed and proved. Considering that four-bar mechanisms are widely used in various industrial applications, the theoretical outcomes of the paper are believed to be important also from a practical point of view.

Keywords: Four-bar mechanism, Motion planning, Singularity

ÖZ

Tip II tekillikler kapalı kinematik zincirlerde karşılaşılan bir olgudur. Karakteristik olarak, eyleyici kuvvetlerinin ıraksamasına ve hareket kontrolünün kaybına neden olurlar. Bu probleme genel bir çözüm olarak, literatürde tekilliğin sorunsuz bir şekilde geçilebilmesini teminen, mekanizmanın dinamik modelinin tekil konumda tutarlı hale getirilmesi önerilmiştir. Bu prensip çerçevesinde, bu makale tip II tekilliklerin varlığında dört-çubuk mekanizmalarının hareket planlamasına dair temel ilkeleri analitik olarak ortaya koymuştur. Bu amaçla dört teorem ve iki sonuç teoremi geliştirilmiş ve ispatlanmıştır. Dört-çubuk mekanizmalarının muhtelif endüstriyel uygulamalarda yaygın olarak kullanıldığı göz önüne alınarak, makalenin teorik sonuçlarının pratik açıdan da önem taşıdığı düşünülmektedir.

Anahtar Kelimeler: *Dört-çubuk mekanizması, Hareket planlaması, Tekillik*

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1. INTRODUCTION

A great number of studies on robotics have been devoted to closed-loop (or parallel) manipulators. Despite their numerous advantages over serial robotic arms, parallel manipulators have workspace limitations, mainly stemming from type II singularities [1-3]. Motion control is lost and the mechanism may be damaged due to the divergence of the actuator forces around these positions [1, 4, 5]. Therefore, it is crucial to develop effective methods for enabling a closed-loop mechanism to smoothly pass through this kind of singular configurations.

The inherent solution is to use redundancy [6]. Recent variations of this kind of attempts can be found in [7, 8]. However, non-redundancy offers improvements in terms of cost and complexity. One of the solutions in this regard is the planning of the motion in order to ensure the consistency of the dynamic model even at the singular position [9-13]. This motion planning technique has been also adapted to rigid-link flexible-joint and flexible parallel robots [14, 15]. Two alternative methods have been recently proposed by the present author. These alternatives are the method of singularity robust balancing and the singularity-consistent payload placement method [16, 17].

Although the general conditions for the dynamic model of parallel robots to be consistent at a singular configuration are well identified in the literature, deriving a complete set of guidelines for the motion planning of a particular mechanism passing through such configurations still requires a thorough analysis of the problem. While the existing studies have focused primarily on high degree-of-freedom mechanisms, one-degree-of-freedom mechanisms are also widely used in numerous industrial systems [18]. Use of flywheels or a double four-bar mechanism has been suggested particularly for dealing with singularities of four-bar mechanisms [19]. Hence, this paper is aimed at analytically deriving the guidelines for the motion planning of four-bar mechanisms in the presence of type II singularities. The paper is organized as follows: First, Section 2 provides some background on the dynamic model and type II singularities of the mechanism. Then, in Section 3, the motion planning guidelines in the presence of type II singularities are derived in the form of four theorems and two corollaries. Following this, in Section 4, a demonstrative example is presented. Finally, Section 5 concludes the paper by summarizing the highlights of the study.

2. PRELIMINARIES

Referring to Figure 1, the closed-loop constraint equations of a planar RRRR four-bar mechanism can be written as follows:

$$a_2 \cos\theta_2 + a_3 \cos\theta_3 = a_1 + a_4 \cos\theta_4 \tag{1}$$

$$a_2 \sin\theta_2 + a_3 \sin\theta_3 = a_4 \sin\theta_4 \tag{2}$$

where $a_1 = AD$, $a_2 = AB$, $a_3 = BC$ and $a_4 = CD$. From Eqs. (1) and (2), one can obtain the velocity-level constraints in matrix form as

$$\mathbf{\Phi}\dot{\mathbf{q}} = \mathbf{0} \tag{3}$$

where

$$\mathbf{q} = \begin{bmatrix} \theta_2 & \theta_3 & \theta_4 \end{bmatrix}^T \quad (4)$$

and

$$\mathbf{\Phi} = \mathbf{\Phi}(\mathbf{q}) = \begin{bmatrix} -a_2 \sin\theta_2 & -a_3 \sin\theta_3 & a_4 \sin\theta_4 \\ a_2 \cos\theta_2 & a_3 \cos\theta_3 & -a_4 \cos\theta_4 \end{bmatrix} \quad (5)$$

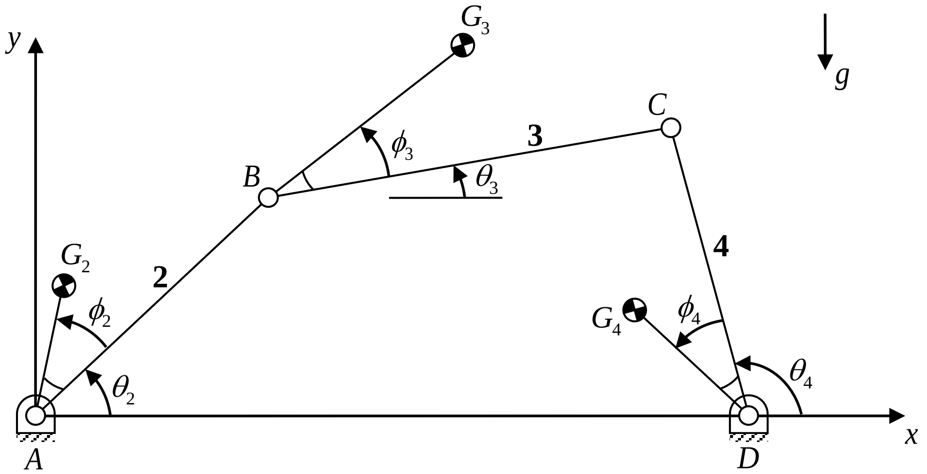

Figure 1. A four-bar mechanism

Assuming that θ_2 is the input variable and choosing $\mathbf{q}$ as the generalized coordinates, the dynamic equations of the mechanism can be written using the Lagrangian method as follows.

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{N} + \mathbf{G} = \mathbf{T} + \mathbf{\Phi}^T \boldsymbol{\lambda} \quad (6)$$

where

$$\mathbf{M} = \mathbf{M}(\mathbf{q}) = \begin{bmatrix} M_{11} & M_{12} & 0 \\ M_{21} & M_{22} & 0 \\ 0 & 0 & M_{33} \end{bmatrix} \quad (7)$$

$$\mathbf{N} = \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} N_1 & N_2 & 0 \end{bmatrix}^T \quad (8)$$

$$\mathbf{G} = \mathbf{G}(\mathbf{q}) = \begin{bmatrix} G_1 & G_2 & G_3 \end{bmatrix}^T \quad (9)$$

$$\mathbf{T} = \begin{bmatrix} T & 0 & 0 \end{bmatrix}^T \quad (10)$$

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}^T \quad (11)$$

and

$$M_{11} = m_2 c_2^{\,2} + I_2 + m_3 a_2^{\,2} \tag{12}$$

$$M_{12} = M_{21} = m_3 a_2 c_3 \cos\left(\theta_2 - \theta_3 - \phi_3\right) \tag{13}$$

$$M_{22} = m_3 c_3^{\,2} + I_3 \tag{14}$$

$$M_{33} = m_4 c_4^{\,2} + I_4 \tag{15}$$

$$N_1 = m_3 a_2 c_3 \dot{\theta}_3^{\,2} \sin\left(\theta_2 - \theta_3 - \phi_3\right) \tag{16}$$

$$N_2 = -m_3 a_2 c_3 \dot{\theta}_2^{\,2} \sin\left(\theta_2 - \theta_3 - \phi_3\right) \tag{17}$$

$$G_1 = \left[m_2 c_2 \cos\left(\theta_2 + \phi_2\right) + m_3 a_2 \cos\theta_2 \right] g \tag{18}$$

$$G_2 = m_3 g c_3 \cos\left(\theta_3 + \phi_3\right) \tag{19}$$

$$G_3 = m_4 g c_4 \cos\left(\theta_4 + \phi_4\right) \tag{20}$$

Here, m_i and I_i denote the mass and the centroidal moment of inertia of the i^{th} link, respectively; g is the gravitational acceleration, T the motor torque associated with θ_2; the distances to the center of gravities are labeled as $c_2 = AG_2$, $c_3 = BG_3$, $c_4 = DG_4$; and λ_1 and λ_2 are the Lagrange multipliers.

For a prescribed motion of the mechanism, Equation 6 is a system of three equations in three unknown variables, namely, T, λ_1 and λ_2. However, these three variables appear together only in the first row of the matrix Equation 6 whereas T does not show up in its second and third rows. Therefore, provided the second and third rows of the matrix Equation 6 can be solved for the Lagrange multipliers, one can obtain the necessary motor torque from its first row. However, this becomes impossible and a type II singularity occurs when

$$\Delta = \left|\mathbf{H}\right| = \begin{vmatrix} -a_3 \sin\theta_3 & a_3 \cos\theta_3 \\ a_4 \sin\theta_4 & -a_4 \cos\theta_4 \end{vmatrix} = a_3 a_4 \sin\left(\theta_3 - \theta_4\right) = 0 \tag{21}$$

or, equivalently, when

$$\theta_3 - \theta_4 = 0,\ \pm\pi \tag{22}$$

It should be obvious at this point that $\mathbf{H}$ is the coefficient matrix of the Lagrange multipliers in the second and third rows of the matrix Equation 6, i.e. it is nothing but a partition obtained by deleting the first row of $\mathbf{\Phi}^T$.

3. THEOREMS ON MOTION PLANNING OF FOUR-BAR MECHANISMS IN THE PRESENCE OF TYPE II SINGULARITIES

In the rest of the paper, the superscript * over a variable x denotes the value of that variable at a type II singular configuration, i.e. $x^* = x\left(t = t_{sc}\right)$ where t is the time and t_{sc} the time at which the singularity is encountered.

Definition 1. *Let $\sigma = 1$ if $\theta_3^* - \theta_4^* = 0$, and $\sigma = -1$ if $\theta_3^* - \theta_4^* = \pm\pi$. Then define the constants δ_1, δ_2 and δ_3 as follows:*

$$\delta_1 = \sigma \frac{a_3}{a_4}$$

$$\delta_2 = \frac{-\sigma a_3 + a_4 \delta_1^2}{a_2 \sin\left(\theta_2^* - \theta_4^*\right)}$$

$$\delta_3 = \frac{a_2}{a_4} \delta_2 \cos\left(\theta_2^* - \theta_4^*\right)$$

Definition 2. *In accordance with Definition 1, define the constants α, β and γ as follows:*

$$\alpha = \frac{a_4}{a_3} \delta_2 M_{21}^* + \sigma \delta_3 M_{33}$$

$$\beta = \frac{a_4}{a_3} M_{22} + \sigma \delta_1 M_{33}$$

$$\gamma = \frac{a_4}{a_3} G_2^* + \sigma G_3^*$$

It should be noted before proceeding further that the joint displacements at the singular configuration can be determined by solving Equation 1, Equation 2 and Equation 22 simultaneously. Also notice from Equation 14 and Equation 15 that M_{22} and M_{33} are constant and positive.

Theorem 1. *Consider a four-bar mechanism, as shown in Figure 1. Let θ_2 and θ_3 be its input and output variables, respectively. Then, in accordance with Definitions 1 and 2, the condition that should be satisfied by the motion in order to render the dynamic equations consistent at a type II singularity can be expressed by the following equation:*

$$\alpha\left(\dot{\theta}_3^*\right)^2 + \beta \ddot{\theta}_3^* + \gamma = 0$$

Proof. The proof starts by noting that, at a type II singularity, the rows of the $\mathbf{H}$ matrix are linearly dependent [10] through the following relation:

$$\frac{a_4}{a_3}H_{1j}+\sigma H_{2j}=0\,,\qquad j=1,2 \tag{23}$$

For a physically realizable system, the governing dynamic equations should be consistent at the singular position [9-13], i.e.

$$\frac{a_4}{a_3}\left[M_{21}^{*}\ddot{\theta}_2^{*}+M_{22}\ddot{\theta}_3^{*}+N_2^{*}+G_2^{*}\right]+\sigma\left(M_{33}\ddot{\theta}_4^{*}+G_3^{*}\right)=0 \tag{24}$$

For a prescribed motion of the mechanism, the joint velocities and accelerations are calculated as

$$\dot{\theta}_2=-\frac{a_3\sin\left(\theta_3-\theta_4\right)}{a_2\sin\left(\theta_2-\theta_4\right)}\dot{\theta}_3 \tag{25}$$

$$\dot{\theta}_4=\frac{a_3\sin\left(\theta_2-\theta_3\right)}{a_4\sin\left(\theta_2-\theta_4\right)}\dot{\theta}_3 \tag{26}$$

$$\ddot{\theta}_2=\frac{-a_2\dot{\theta}_2^{\,2}\cos\left(\theta_2-\theta_4\right)-a_3\dot{\theta}_3^{\,2}\cos\left(\theta_3-\theta_4\right)+a_4\dot{\theta}_4^{\,2}-a_3\ddot{\theta}_3\sin\left(\theta_3-\theta_4\right)}{a_2\sin\left(\theta_2-\theta_4\right)} \tag{27}$$

$$\ddot{\theta}_4=\frac{-a_2\dot{\theta}_2^{\,2}-a_3\dot{\theta}_3^{\,2}\cos\left(\theta_2-\theta_3\right)+a_4\dot{\theta}_4^{\,2}\cos\left(\theta_2-\theta_4\right)+a_3\ddot{\theta}_3\sin\left(\theta_2-\theta_3\right)}{a_4\sin\left(\theta_2-\theta_4\right)} \tag{28}$$

Now consider the singular configuration. First, by substituting Equation 22 into Equation 25, $\dot{\theta}_2^{*}$ becomes simply zero, i.e.

$$\dot{\theta}_2^{*}=0 \tag{29}$$

In a similar manner, substitution of Equation 22 into Equation 26 gives

$$\dot{\theta}_4^{*}=\sigma\frac{a_3}{a_4}\dot{\theta}_3^{*} \tag{30}$$

Next, by substituting Equation 22 and Equation 29 into Equation 27, $\ddot{\theta}_2^{*}$ can be expressed as

$$\ddot{\theta}_2^{*}=\frac{-\sigma a_3\left(\dot{\theta}_3^{*}\right)^2+a_4\left(\dot{\theta}_4^{*}\right)^2}{a_2\sin\left(\theta_2^{*}-\theta_4^{*}\right)} \tag{31}$$

Similarly, by the use of Equation 22 and Equation 29, $\ddot{\theta}_4^*$ can be obtained as follows.

$$\ddot{\theta}_4^* = \frac{\left[-\sigma a_3 \left(\dot{\theta}_3^*\right)^2 + a_4 \left(\dot{\theta}_4^*\right)^2\right]\cos\left(\theta_2^* - \theta_4^*\right)}{a_4 \sin\left(\theta_2^* - \theta_4^*\right)} + \sigma \frac{a_3}{a_4} \ddot{\theta}_3^* \tag{32}$$

The rest of the proof directly follows from the substitution of Equations 13-15, Equation 17, Equation 19, Equation 20 and Equations 29-32 into Equation 24, and rearrangement of the resulting expression according to Definitions 1 and 2. ***Q.E.D.***

Remark 1. The condition given in Theorem 1, in general, describes a parabola in the $\dot{\theta}_3^*\,\ddot{\theta}_3^*$-plane.

A corollary to Definition 2 can be stated as follows:

Corollary 1. *β is always positive, being given by*

$$\beta = \frac{a_4}{a_3} M_{22} + \frac{a_3}{a_4} M_{33}$$

Proof. Substituting the given expression for δ_1 into β expression in Definition 2, the proof follows from the facts that $\sigma^2 = 1$ and that M_{22} and M_{33} are positive constants. ***Q.E.D.***

Remark 2. If the mechanism works in the horizontal plane, then the $\mathbf{G}$ vector will be a zero vector and, consequently, γ will be simply zero.

Remark 3. In order to prevent the mechanism from being instantaneously at rest at the singular position, $\dot{\theta}_3^*$ should be selected to be non-zero.

Theorem 2. *Consider a four-bar mechanism, as shown in Figure 1. Let θ_2 and θ_3 be its input and output variables, respectively. Then, in accordance with Definitions 1 and 2, the lower or upper bound for output angular accelerations that are realizable at a type II singularity is as follows:*

(i) *For $\alpha < 0$, $\ddot{\theta}_3^* > -\dfrac{\gamma}{\beta}$.*

(ii) *For $\alpha > 0$, $\ddot{\theta}_3^* < -\dfrac{\gamma}{\beta}$.*

Proof. The condition of Theorem 1 can be rearranged in the following form:

$$\ddot{\theta}_3^* = -\frac{\alpha}{\beta}\left(\dot{\theta}_3^*\right)^2 - \frac{\gamma}{\beta} \tag{33}$$

Then, setting $d\ddot{\theta}_3^* / d\dot{\theta}_3^*$ to zero gives the critical point as follows:

$$\frac{d\ddot{\theta}_3^*}{d\dot{\theta}_3^*} = -2\frac{\alpha}{\beta}\dot{\theta}_3^* = 0 \tag{34}$$

or, given that $\alpha \neq 0$,

$$\dot{\theta}_3^* = 0 \tag{35}$$

The value of $\ddot{\theta}_3^*$ at this critical point can be obtained by simply setting $\dot{\theta}_3^*$ to zero in Equation 33.

$$\ddot{\theta}_3^* = -\gamma/\beta \qquad \text{at } \dot{\theta}_3^* = 0 \tag{36}$$

Noting that $\frac{d}{d\dot{\theta}_3^*}\left(\frac{d\ddot{\theta}_3^*}{d\dot{\theta}_3^*}\right) = -2\frac{\alpha}{\beta}$ and recalling Remark 1, it can be concluded that the obtained value of $-\gamma/\beta$ is the unique global minimum (maximum) for $\ddot{\theta}_3^*$ if $\alpha < 0$ (>0) by applying the second derivative test. The rest of the proof follows from Remark 3. ***Q.E.D.***

A useful corollary to Theorem 2 is as follows:

Corollary 2. *Under the definitions and assumptions of Theorem 2, in order to obtain a realizable motion passing through a type II singularity:*

- **(i)** *For $\alpha < 0$ and $\gamma \leq 0$, $\ddot{\theta}_3^*$ cannot be assigned a non-positive value.*
- **(ii)** *For $\alpha > 0$ and $\gamma \geq 0$, $\ddot{\theta}_3^*$ cannot be assigned a non-negative value.*

Proof. Keeping in mind Corollary 1 and Remark 3, for $\alpha < 0$, the $-\frac{\alpha}{\beta}\left(\dot{\theta}_3^*\right)^2$ term in Equation 33 is always positive. Furthermore, under the condition that $\gamma \leq 0$, one can conclude that the $-\gamma/\beta$ term in Equation 33 is always non-negative. Hence, $\ddot{\theta}_3^*$ is always positive in such a case. Similarly, when $\alpha > 0$, the $-\frac{\alpha}{\beta}\left(\dot{\theta}_3^*\right)^2$ term is always negative, and given that $\gamma \geq 0$, the $-\gamma/\beta$ term is always non-positive. Therefore, $\ddot{\theta}_3^*$ is always negative in such a case. ***Q.E.D.***

Theorem 3. *Consider a four-bar mechanism, as shown in Figure 1. Let θ_2 and θ_3 be its input and output variables, respectively. If the output angular acceleration is desired to be zero at a type II singularity, then, in accordance with Definitions 1 and 2, $\frac{\gamma}{\alpha} < 0$ must hold for it to be realizable. Moreover, if that is the case, then $\dot{\theta}_3^*$ should be equal to $\sqrt{-\gamma/\alpha}$ or $-\sqrt{-\gamma/\alpha}$.*

Proof. When $\ddot{\theta}_3^* = 0$, the condition of Theorem 1 reduces to

$$\alpha\left(\dot{\theta}_3^*\right)^2+\gamma=0 \tag{37}$$

or, solving for $\dot{\theta}_3^*$ gives

$$\dot{\theta}_3^*=\pm\sqrt{-\gamma/\alpha} \tag{38}$$

Therefore, $-\dfrac{\gamma}{\alpha}>0$ must hold for $\dot{\theta}_3^*$ to be real and non-zero (recall Remark 3). ***Q.E.D.***

Remark 4. Theorem 3 is useful for, but not limited to, the tasks that are prescribed with a constant angular velocity of the output link.

Theorem 4. *Consider a four-bar mechanism, as shown in Figure 1. Let θ_2 and θ_3 be its input and output variables, respectively. In accordance with Definitions 1 and 2, if $\alpha=0$, $\ddot{\theta}_3^*$ should be equal to $-\gamma/\beta$ for a realizable motion passing through a type II singularity.*

Proof. The proof directly follows from the substitution of $\alpha=0$ into the condition of Theorem 1, and solution of the resulting expression for $\ddot{\theta}_3^*$. ***Q.E.D.***

4. NUMERICAL EXAMPLE

This section presents a numerical example to show the application and effectiveness of the proposed guidelines. The parameters of the mechanism are chosen as given in Table 1. The mechanism is assumed to work in the horizontal plane (i.e. $\gamma=0$). The task is described as follows:

- The mechanism will start from $\theta_3=345°$. The remaining joint variables at this initial configuration will be $\theta_2=83.5°$ and $\theta_4=173.8°$.
- The coupler link will rotate an angle of $10°$ in the counterclockwise direction. The joint variables at this final position will be $\theta_2=81.5°$, $\theta_3=355°$ and $\theta_4=158.7°$.
- The motion will be completed in $t_{end}=1$ s.
- The initial and final velocities and accelerations will be zero.

While executing the prescribed task, a type II singularity arises when $\theta_3=348.5°$. At this singular configuration, $\theta_2=84.3°$ and $\theta_4=\theta_3-\pi$ (so $\sigma=-1$), and Figure 2 shows the corresponding consistent motion design parabola whose equation is as follows:

$$3.2664\left(\dot{\theta}_3^*\right)^2+10\ddot{\theta}_3^*=0 \tag{39}$$

By setting $t_{sc}=0.4$ s, and by selecting $\dot{\theta}_3^*=0.2$ rad/s and $\ddot{\theta}_3^*=-0.0131$ rad/s^2 in order to satisfy Equation 39, a consistent motion fulfilling the aforementioned task specifications can be planned for the coupler link as below:

$$\theta_3(t) = 6.0214 + 6.6860t^3 - 31.5065t^4 + 65.0950t^5 - 68.2196t^6 + 35.1461t^7 - 7.0266t^8 \quad (40)$$

Table 1. Numerical values assumed for the parameters of the mechanism

Parameter	Numerical value
a_1	5 m
a_2	1 m
a_3	3 m
a_4	2 m
m_2	1 kg
m_3	3 kg
m_4	2 kg
I_i	$\frac{m_i a_i^2}{12}$ for $i = 2,3,4$
c_i	$\frac{1}{2} a_i$ for $i = 2,3,4$
ϕ_i	0 for $i = 2,3,4$

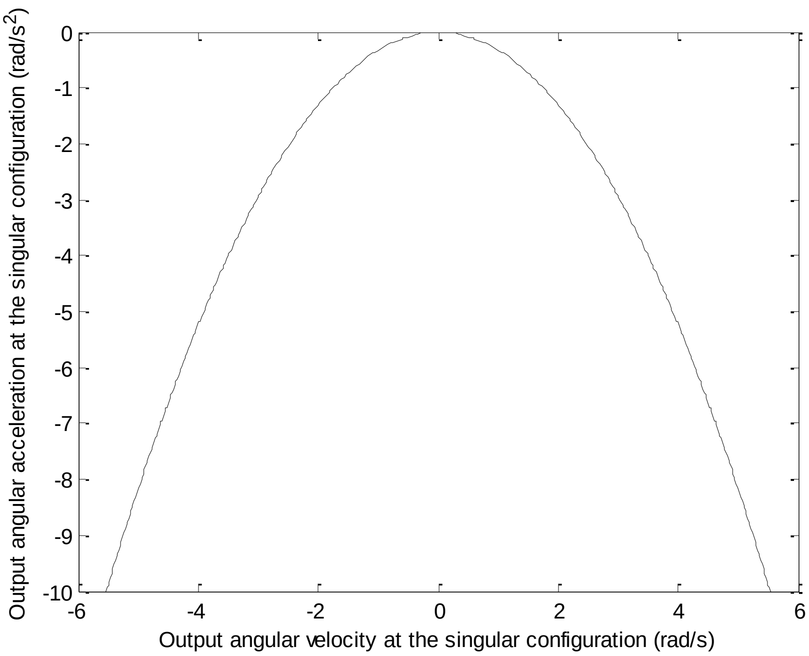

Figure 2. The consistent motion design parabola for the encountered type II singularity

As can be understood from Figure 3, since the motion is planned according to Equation 39, the mechanism can pass smoothly through the singularity.

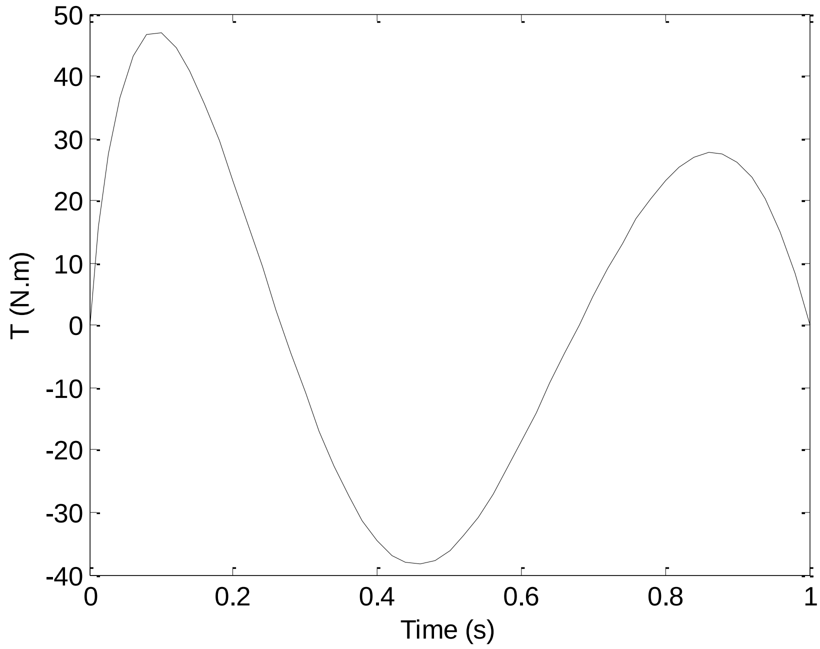

Figure 3. The necessary motor torque for executing the planned motion

5. CONCLUSIONS

This paper studies in depth the motion planning of the classical four-bar mechanism in the presence of type II singularities by proposing four theorems and two corollaries. There should be no doubt on that four-bar mechanisms are used in an enormous range of applications in industry and machinery. The highlights of the study can be summarized as follows:

- Theorem 1 gives, in its simplest form, the relation between the consistent angular velocities and accelerations of the output link at the singular configuration.
- It is shown analytically that the consistent angular velocities and accelerations of the output link at the singularity, in general, describe a parabola in the $\dot{\theta}_3^* \ \ddot{\theta}_3^*$ -plane.
- Theorem 2 gives the lower or upper bound for output angular accelerations that are realizable at the singular configuration.
- Corollary 2 states the realizable sign of the output angular acceleration at the singular position.
- Theorem 3 is able to cover tasks that are prescribed with a constant angular velocity of the output link.

Last but not least, the analysis presented here is believed to bring new insights into the synthesis of four-bar mechanisms.

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Upon graduating from Ankara Science High School in 2001, he received the B.S., M.S. and Ph.D. degrees, all in Mechanical Engineering, from Middle East Technical University, in 2005, 2008 and 2013, respectively. Between 2005 and 2015 he worked as a Research Assistant and as an Instructor in the Department of Mechanical Engineering at Middle East Technical University. In October 2014, he completed his military service as a Reserve Officer with the rank of Artillery Second Lieutenant. Since 2015, he has been an Assistant Professor of Mechanical Engineering Department in Faculty of Engineering at Marmara University His research interests include parallel robots and vehicle safety.

2001 yılında Ankara Fen Lisesi'nden mezuniyetini takiben, Lisans, Yüksek Lisans ve Doktora derecelerini sırasıyla 2005, 2008 ve 2013 yıllarında Orta Doğu Teknik Üniversitesi Makine Mühendisliği Bölümünden almıştır. 2005-2015 yılları arasında Orta Doğu Teknik Üniversitesi Mühendislik Fakültesi Makine Mühendisliği Bölümünde Araştırma Görevlisi ve Öğretim Görevlisi olarak çalışmıştır. Ekim 2014'te Yedek Subay olarak askerlik hizmetini tamamlayarak Topçu Teğmen rütbesi ile terhis olmuştur. 2015 yılından bu yana Marmara Üniversitesi Mühendislik Fakültesi Makine Mühendisliği Bölümü Konstrüksiyon Anabilim Dalında Yardımcı Doçent Doktor unvanı ile görev yapmaktadır. Araştırma alanları arasında paralel robotlar ve taşıt güvenliği yer almaktadır.