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Investigation of the Nuclear Structure of Unstable Isotopes ^{3}Li, ^{4}Li and ^{5}Li

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Abstract: Lithium (Z = 3) is one of the lightest nuclei and exhibits a wide range of nuclear structure phenomena that are highly sensitive to neutron number. While the stable isotopes ^{6}Li and ^{7}Li are well established in nuclear technology, the unstable isotopes ^{3}Li, ^{4}Li, and ^{5}Li provide an important testing ground for theoretical models due to their weak binding and short lifetimes. In this work, the nuclear ground-state properties of these unstable lithium isotopes are investigated within the Hartree–Fock framework using Skyrme-type effective interactions. Root-mean-square (rms) radii for proton, neutron, and mass distributions are calculated, and the total energies and binding energies per nucleon are analyzed. In addition, mass radii are estimated using the semi-empirical liquid-drop model for comparison. A systematic study of various Skyrme parameterizations is performed, and the GS4 and SKx forces are identified as providing the closest agreement with available experimental data for binding energies and radii. The calculated results are compared with experimental values and liquid-drop model predictions, highlighting both the capabilities and the limitations of mean-field approaches in describing very light, unstable nuclei. The present study contributes to a better understanding of the nuclear structure of exotic lithium isotopes and provides useful benchmarks for future theoretical and experimental investigations.

^{3}Li, ^{4}Li ve ^{5}Li Kararsız İzotoplarının Nükleer Yapısının İncelenmesi

Anahtar Kelimeler
Lityum,
^{3,4,5}Li,
Hartree-Fock,
Skyrme,
Rms yarıçapı,
Bağlanma enerjisi,
Nükleer yapı

Öz: Lityum (Z = 3), en hafif çekirdeklerden biridir ve nötron sayısına oldukça duyarlı olan geniş bir nükleer yapı olgusu yelpazesini sergiler. Kararlı izotoplar olan ^{6}Li ve ^{7}Li nükleer teknolojide iyi bilinirken, kararsız izotoplar ^{3}Li, ^{4}Li ve ^{5}Li zayıf bağlanmaları ve kısa ömürleri nedeniyle teorik modellerin test edilmesi için önemli bir alan sağlar. Bu çalışmada, bu kararsız lityum izotoplarının nükleer temel durum özellikleri, Skyrme tipi etkili etkileşimler kullanılarak Hartree–Fock çerçevesinde incelenmiştir. Proton, nötron ve kütle dağılımlarının kök ortalama kare (rms) yarıçapları hesaplanmış; toplam enerjileri ve nükleon başına bağlanma enerjileri analiz edilmiştir. Ayrıca karşılaştırma amacıyla yarı-deneysel sıvı-damla modeli kullanılarak kütle yarıçapları tahmin edilmiştir. Çeşitli Skyrme parametrelerinin sistematik çalışması yapılmış ve GS4 ile SKx kuvvetlerinin, mevcut deneysel bağlanma enerjileri ve yarıçap verileriyle en yakın uyumu sağladığı belirlenmiştir. Hesaplanan sonuçlar, deneysel değerler ve sıvı-damla modeli tahminleri ile karşılaştırılmış; ortalama alan yaklaşımlarının çok hafif, kararsız çekirdeklerin tanımlanmasındaki yetenekleri ve sınırlamaları ortaya konmuştur. Bu çalışma, egzotik lityum izotoplarının nükleer yapısının daha iyi anlaşılmasına katkı sağlamış ve gelecekteki teorik ve deneysel araştırmalar için faydalı referanslar sunmaktadır.

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1. Introduction

Lithium, with atomic number 3, is a light alkali metal notable for its nuclear properties and important applications in nuclear technology. The stable isotopes ^{6}Li and ^{7}Li constitute essential components in nuclear reactor technologies. Specifically, ^{6}Li is utilized extensively for tritium generation through neutron capture based on the following reaction:

$$^{6}\text{Li}+n \rightarrow {}^{3}\text{H}+{}^{4}\text{He} + 4.78\ \text{MeV} \qquad (1)$$

while ^{7}Li serves as a neutron moderator and coolant, especially in advanced reactor designs. Beyond these, unstable isotopes such as ^{3}Li, ^{4}Li, and ^{5}Li provide valuable insights into nuclear structure and reaction dynamics, exhibiting phenomena like neutron halos that challenge traditional models [3,4].

^{3}Li is a reference case for nuclear instability. The asymmetry term in the semi-empirical mass formula, together with the Coulomb repulsion between protons, clearly explains why a neutron-free nucleus is energetically unstable and cannot exist as a bound system. ^{3}Li is an extremely short-lived nucleus with an experimental binding energy (BE) of approximately 6.81 MeV, corresponding to a binding energy per nucleon (BE/A) of about 2.27 MeV/nucleon. Its ground state spin-parity is assigned as $3/2^{-}$ and it has a measured half-life on the order of 10^{-22} seconds, indicating an unbound or resonant nature [7,8]. For ^{4}Li, experimental data indicate BE close to 4.6 MeV and binding energy per nucleon roughly 1.15 MeV/nucleon. Its ground state exhibits a spin-parity of 2^{+}, with a half-life similarly very short, on the order of 10^{-22} seconds, reflecting its position beyond the drip line and rapid decay. ^{4}Li acts as a compound nucleus in neutron- and proton-induced reactions and also used Breit–Wigner resonance calculations, neutron scattering and absorption models. In the previous study, root mean square (rms) nuclear radii and binding energies of ^{7}Li nuclei were investigated using the Skyrme-Hartree-Fock (SHF) method, and the theoretical results were compared to experimental data from nuclear data libraries. In the study, it has been seen that SHF method has quite successful to explain the ground state features of Li nuclei for chosen Skyreme parameters [17,18].

Although ^{4}Li does not appear explicitly in nuclear data libraries, its resonance properties influence is important since evaluated cross sections of ^{6}Li and ^{7}Li and background reaction channels [17,18]. On the other hand, ^{4}Li is also used in Lithium-based neutron sources, fusion blanket material modeling, and Lithium coolants in advanced reactor concept. ^{5}Li plays a key role as an intermediate resonance in fusion reactions and it is also used in calculations such as energy deposition, reaction probabilities, particle emission angular distributions calculations. The ^{5}Li isotope, which is slightly more stable than these other light exotic nuclei, has an experimental binding energy of about 26.3 MeV and a binding energy per nucleon near 5.26 MeV. It is characterized by a $3/2^{-}$ ground-state spin-parity and has an extremely short half-life of around 3.7×10^{-22} seconds. These experimental values provide critical benchmarks for theoretical models aiming to describe such loosely bound and resonant nuclear systems.

The conventional compactness of ^{6}Li and ^{7}Li is well-documented, yet as the neutron number increases, particularly in ^{11}Li. The onset of halo phenomena induces a substantial expansion of the nuclear radius and yields a more diffuse density profile. Precise experimental determinations of root mean square (rms) values for charge, neutron, and proton radii, as well as mass distributions, have been comprehensively catalogued in recent data evaluations [1,2], providing an essential reference point for testing theoretical approaches. Conversely, the unstable isotopes ^{3}Li, ^{4}Li, and ^{5}Li, despite their short lifetimes and absence in nature, provide valuable insights into nuclear structure and reaction mechanisms [3,4].

The Hartree-Fock (HF) method, developed as a self-consistent mean-field approach, has become a cornerstone in nuclear structure calculations by providing a microscopic description of nucleon interactions within the nucleus. The development of Skyrme-type effective interactions by Skyrme in the 1950s represented a major breakthrough in nuclear structure studies, enabling the efficient and successful application of the Hartree–Fock (HF) approach to finite nuclei [9].

Over the decades, the Skyrme Hartree-Fock (SHF) approach has been extensively refined and successfully applied to describe binding energies, radii, deformation, and other ground-state properties of both stable and exotic nuclei [5,10]. Given its computational tractability combined with reliable predictive capabilities, the SHF approach represents a viable method for investigating light unstable isotopes whose experimental data remain scarce or challenging to acquire.

In this study, the experimental ground-state properties of the isotopes ^{3}Li, ^{4}Li, and ^{5}Li are systematically compared and analyzed alongside theoretical values obtained from self-consistent Hartree-Fock calculations with Skyrme-type effective interactions. This comparative approach serves to evaluate the accuracy of the theoretical model and to provide deeper insights into the nuclear structure of these unstable isotopes. Reliable theoretical modeling of ground-state properties—such as binding energies and root mean square (rms) radii—is central to advancing our understanding of the nuclear behavior of lithium isotopes. The structural characteristics of various lithium isotopes are investigated within this

theoretical framework, aiming to refine nuclear data that are essential for both fundamental nuclear physics research and practical applications. The theoretical results are benchmarked against the latest experimental measurements and curated nuclear data, enabling a direct assessment of the model's predictive reliability [1,2]. Particular attention is devoted to some isotopes, where extended mass distributions present significant challenges to conventional mean-field approaches and underscore areas requiring model enhancement. This analysis not only contributes to the benchmarking of nuclear interaction models, but also deepens our understanding of structural evolution in light, exotic nuclei.

2. Material and Methods

The total energy of the nucleus can be written within the Hartree–Fock framework as follows:

$$E = E_{\text{Skyrme}} + E_{\text{Coulomb}} + E_{\text{pair}} - E_{\text{cm}} \tag{2}$$

where E is the total energy of nucleus, E_{Skyrme} is the energy of Skyrme interaction energy, $E_{Coulomb}$ is the Coulomb interaction energy, E_{pair} is the two-nucleon interaction pairing energy and E_{cm} is the energy for correction for the spurious center-of-mass motion of the mean field [11]. Skyrme force is given as:

$$V_{Skyrme} = t_0(1+x_0P_\sigma)\delta(\vec{r}) + \frac{1}{2}t_1(1+x_1P_\sigma)\{\delta(\vec{r})\vec{k}^2 + \vec{k}'^2\delta(\vec{r}))\}$$
$$+ t_2(1+x_2P_\sigma)\vec{k}'\cdot\delta(\vec{r})\vec{k} + \frac{1}{6}t_3(1+x_3P_\sigma)\rho^\alpha(\vec{R})\delta(\vec{r}) + it_4\vec{k}'\cdot\delta(\vec{r})(\vec{\sigma_i}+\vec{\sigma_j})\times\vec{k} \tag{3}$$

where $\vec{k}$ is the relative momentum, $\delta(\overrightarrow{r})$ is the delta function, P_σ is the space exchange operator , $\vec{\sigma}$ is the vector of Pauli spin matrices $t_0, t_1, t_2, t_3, t_4, x_0, x_1, x_2, x_3, \alpha$ are Skyrme force parameters. The values of these Skyrme force parameters are provided in detail in Refs. [12,13].

The neutron, proton or charge densities are given by

$$\rho_q(r) = \sum_{\beta\in q} w_\beta^2 \left|\psi_\beta(r)\right|^2 \quad , \quad q = n, p, c \tag{4}$$

where q denotes neutron, proton, or charge densities, ψ_β is the single-particle wave function and w_β is the occupation probability of state β respectively. The rms radii of neutron, proton, charge and mass distrubutions can be evaluated from using Eq. (4) with the following expression:

$$\langle r_q^2\rangle^{1/2} = \left[\frac{\int r^4\rho_q(r)dr}{\int r^2\rho_q(r)dr}\right]^{1/2} \tag{5}$$

Semi–empirical binding-energy formula is given by:

$$SBE(A,Z) = a_vA - a_sA^{\frac{2}{3}} - a_c\frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_{sym}\frac{(A-2Z)^2}{A} \pm \frac{\delta}{\sqrt{A}} \tag{6}$$

is given and where the coefficients $a_v = 15.5$ MeV, $a_s = 16.8$MeV, $a_c = 0.72$MeV, $a_{\text{sym}} = 23$MeV, and $\delta = 34$MeV were adopted from Ref. [14].

3. Results and Discussions

In this study, the nuclear structural properties of lithium isotopes ^{3}Li, ^{4}Li, and ^{5}Li have been investigated using the Skyrme-Hartree-Fock (SHF) mean-field approach with GS4 and SKx Skyrme parameterizations. The Skyrme force parameters used in this study have been given in Table 1.

Table 1. Skyrme Force Parameters

	SKx	***GS4***
t_0 *(MeV.fm³)*	-1445.3	-1242
t_1 *(MeV.fm⁵)*	246.9	760
t_2 *(MeV.fm⁵)*	-131.8	-146.2
t_3 *(MeV.fm³)*	12103.9	19362
t_4 *(MeV.fm⁵)*	-	-2157
x_0	0.34	0.026
x_1	0.58	0
x_2	0.127	0
x_3	0.03	1
α	0.5	1

The harmonic oscillator potential wave functions were initially calculated with the HAFOMN code [15], and pairing correlations were treated approximately via the constant-force BCS formalism. The calculations employed a self-consistent Hartree-Fock method under the assumption of spherical symmetry for the nuclei. The Skyrme interaction, characterized by central, zero-range forces with radial dependence only, provides an effective framework for describing nucleon-nucleon interactions within the mean-field approximation. This study aimed to evaluate the predictive power of different Skyrme parameter sets in reproducing experimental observables, specifically

the total energies and mass root-mean-square (rms) radii of light lithium isotopes.

It should be noted that the Liquid Drop Model is primarily intended for medium and heavy-mass nuclei. For very light systems (A < 10), shell effects, clustering phenomena, and quantum correlations become dominant, reducing the predictive capability of the macroscopic model. Therefore, the LDM results presented in this work are included only for qualitative comparison.

Table 2. Total Energy Calculations Results (in MeV)

Skyrme Force	^{3}Li	^{4}Li	^{5}Li
SKM	1.267	14.030	29.500
SKM*	4.982	13.432	28.454
SKx	4.549	13.625	30.590
T	5.108	12.952	23.465
T3	4.820	13.020	27.684
Z	4.050	11.066	28.889
Z-SIGMA	3.946	12.198	29.383
Z-SIGMA*	4.180	12.297	29.067
E	4.135	9.988	26.546
E-SIGMA	3.982	11.098	27.154
R-SIGMA	5.322	15.632	31.830
G-SIGMA	5.393	15.740	31.832
SGI	4.913	13.339	29.518
SGII	5.014	15.448	32.058
SLY4	4.379	12.305	27.751
SLY5	4.427	12.356	27.775
SLY6	4.390	12.971	29.191
SLY7	4.374	13.178	29.703
S1	1.072	10.652	29.511
S2	0.592	8.298	20.003
S3	4.365	10.666	27.260
S4	4.673	9.057	26.934
S6	4.145	11.265	27.513
SKa	4.891	11.665	27.249
SKb	5.044	12.492	27.485
GS1	4.628	6.248	16.700
GS2	4.606	6.216	20.443
GS3	4.577	18.575	48.108
GS4	4.720	6.262	15.965
GS5	4.559	6.087	18.780
GS6	4.527	16.989	38.557
SBE	63.230	14.800	18.350
Experiment[16]	6.810	4.600	26.330

Previous investigations have shown that the GS4 and SKx parameter sets provide theoretical predictions of the ^{7}Li nuclear radius and binding energy that are in accuracy with the corresponding experimental data [17]. Therefore, these parameterizations were adopted in the present study for the calculation of nuclear radii and binding energies. In particular, the GS4 interaction provides reasonable agreement with experimental total binding energies, whereas the SKx interaction reproduces the experimental nuclear radii more accurately. The use of both parameter sets also enables a comparative assessment of their predictive performance for both stable and unstable lithium isotopes. Regarding the total energy, the calculated values for the ^{3}Li, ^{4}Li, and ^{5}Li isotopes obtained with the GS4 and SKx parameter sets are summarized in Table 2 together with the available experimental data [16]. As expected from their successful performance in describing the properties of ^{7}Li, both parameterizations reproduce the overall binding-energy trends. However, noticeable quantitative deviations remain, particularly for the unstable and weakly bound isotopes, reflecting the limitations of the mean-field approximation in very light nuclear systems. For ^{3}Li, the GS4 and SKx forces yield binding energies of 4.549 MeV and 4.720 MeV, respectively, which underestimate the experimental value of 6.81 MeV by approximately 2 MeV. The discrepancy becomes more pronounced for ^{4}Li, where the calculated energies (13.625 MeV for SKx and 6.262 MeV for GS4) diverge significantly from the experimental binding energy of 4.60 MeV. Similarly, for ^{5}Li, despite an experimental binding energy of 26.330 MeV, the GS4 and SKx calculations produce 30.590 MeV and 15.965 MeV, respectively.

Table 3. Skyrme Mass Radii (in fm, r_0 =1.2 fm)

Skyrme Force	^{3}Li (×10^{7})	^{4}Li	^{5}Li
SKM	23.213	10.4	4.002
SKM*	1.800	11.4	1.939
SKx	1.651	4.40	2.138
T	1.841	8.00	1.552
T3	1.778	9.00	0.506
Z	1.447	12.8	2.519
Z-SIGMA	1.430	7.60	0.559
Z-SIGMA*	1.518	9.30	1.047
E	1.490	14.7	2.330
E-SIGMA	1.435	6.10	0.474
R-SIGMA	1.920	7.00	0.787
G-SIGMA	1.936	7.00	0.740
SGI	1.833	8.60	1.481
SGII	1.813	7.90	0.635
SLY4	1.626	10.9	2.202

SLY5	1.642	11.0	2.172
SLY6	1.632	10.4	1.876
SLY7	1.627	10.2	1.798
S1	17.510	49.6	4.115
S2	17.883	-	-
S3	1.592	5.10	1.902
S4	1.749	9.30	2.166
S6	1.563	12.6	2.956
SKa	1.808	10.5	2.157
SKb	1.874	9.40	2.156
GS1	1.732	-	3.904
GS2	1.711	-	2.567
GS3	1.685	5.60	1.466
GS4	1.774	-	3.950
GS5	1.687	-	2.170
GS6	1.656	9.20	2.323
$r = r_0 A^{1/3}$	1.730	1.90	2.052

Table note: The radius r is given as the nuclear radius in the liquid-drop model and – indicates that no converged physical solution was obtained.

Regarding the mass radius, the mass rms radii computed with GS4 and SKx parameter sets are compared with liquid-drop model predictions in Table 3. Since the unstable ^{3}Li nucleus has no neutrons, mass rms radius calculations have yielded meaningless. These numerical results does not carry physical significance; rather, it reflects the inability of the mean-field Hartree–Fock framework to produce a stable bound-state solution for a neutron-deficient and highly unbound system such as ^{3}Li. For ^{4}Li, the SKx calculation significantly overestimates the radius while S2, GS1, GS2, GS4 and GS5 yields an unphysically large value indicating a breakdown of the model in describing unstable, diffuse systems. The ^{5}Li isotope presents a somewhat intermediate case: SKx predicts a radius of 2.138 fm, consistent with the liquid-drop model value of 2.052 fm, while GS4 overestimates it at 3.950 fm, suggesting a more diffuse nuclear mass distribution.

In isotopes with extremely low neutron numbers, such as ^{3}Li and ^{4}Li, the mean-field assumption inherent in the Skyrme Hartree–Fock framework may become less reliable. This limitation is especially evident in neutron density calculations, where strong quantum fluctuations and many-body correlation effects associated with the small neutron population can significantly affect the accuracy of the predicted density distributions.

These findings are consistent with the understanding that classical mean-field models, although effective for stable nuclei, fail to capture the complex spatial correlations and extended mass distributions characteristic of halo or weakly bound states. The anomalously large radius predicted by S2, GS1, GS2, GS4 and GS5 for ^{4}Li indicates computational instabilities and the need for models incorporating beyond-mean-field effects. These results underscore the limitations of mean-field approximations for nuclei near the drip line or with resonance and halo characteristics. These discrepancies primarily reflect the intrinsic limitations of the mean-field approximation in very light nuclei rather than deficiencies of the Skyrme interaction itself. Consequently, while mean-field methods capture the gross features of stable nuclei, their predictive power diminishes markedly in unstable systems.

Although the Skyrme–Hartree–Fock framework has proven highly successful for medium- and heavy-mass nuclei, its applicability becomes increasingly limited for extremely light systems such as ^{3}Li and ^{4}Li. In these nuclei, continuum coupling, resonance behavior, and clustering effects become increasingly important, leading naturally to larger deviations between theoretical predictions and experimental observations.

The overall analysis demonstrates that, while GS4 and SKx parameterizations yield reasonable agreement with experimental binding energies and radii for unstable lithium isotopes, significant discrepancies exist for nuclei exhibiting halo or resonance features. These deviations highlight the necessity of including three-body forces, nucleon-nucleon correlations, and the effects of quantum fluctuations effects not fully accounted for in standard SHF approaches. The mean-field framework, by construction, treats nucleons as independent particles moving in an average potential, which limits its applicability to systems where many-body correlations dominate. For unstable and short-lived isotopes like ^{4}Li and ^{5}Li, advanced theoretical methods such as ab-initio calculations or configuration interaction approaches are essential to achieve accurate descriptions. The present results are in line with previous studies emphasizing that reliable predictions for exotic nuclei require extensions beyond the mean-field paradigm, including explicit treatment of three-nucleon forces and pairing correlations with improved microscopic input.

Figures 1–4 present the proton, neutron, and mass density distributions calculated with the GS4 and SKx parameter sets for the ^{4}Li and ^{5}Li nuclei.

The binding energies, mass radii, and density distributions calculated using the GS4 and SKx parameter sets provide valuable benchmarks and can serve as initial inputs for more complex reaction models, such as pre-compound exciton calculations, thereby contributing to a deeper understanding of the nuclear structure and reactions of light nuclei. Figure1 displays the proton, neutron, and mass density distributions of the ^{4}Li nucleus calculated with SKx

parameter. These density distributions indicate that the ^{4}Li nucleus exhibits a compact structure, with the proton density dominating in the central region. The rapid decrease of the density profiles with increasing radius suggests the absence of a pronounced nuclear halo structure.

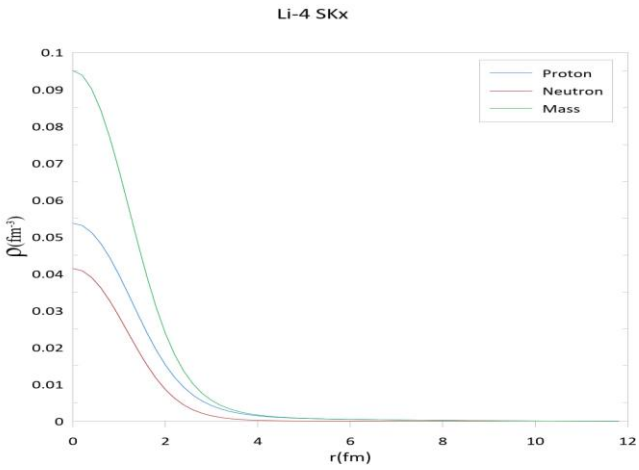

Fig. 1. Proton, neutron, and mass density distributions of the ^{4}Li nucleus calculated using the SKx parameter set.

In Figure2 GS4 results show a compact ^{4}Li nucleus with a dominant proton density distribution. Although the density profiles exhibit a slight radial extension, no clear evidence of a well-developed nuclear halo is observed.

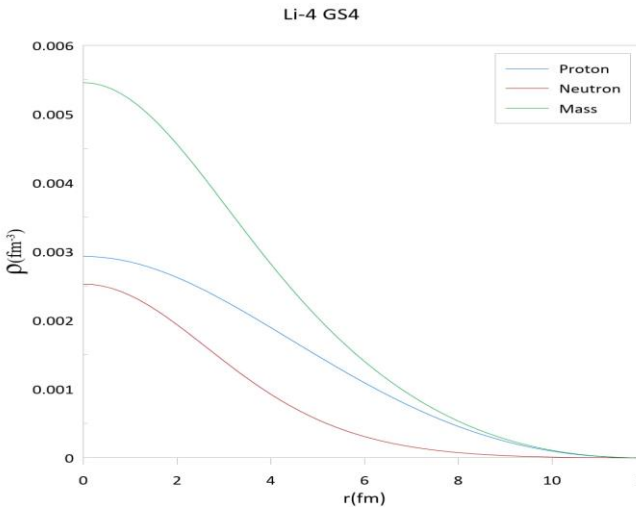

Fig. 2. Proton, neutron, and mass density distributions of the ^{4}Li nucleus calculated using the GS4 parameter set.

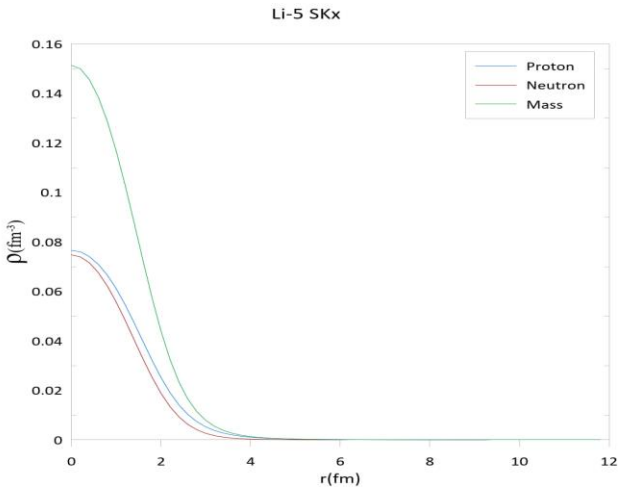

Fig. 3. Proton, neutron, and mass density distributions of the ^{5}Li nucleus calculated using the SKx parameter set.

Figure3 illustrates the proton, neutron, and mass density distributions of the ^{5}Li nucleus calculated with the SKx parameter set. The proton density exhibits a slightly more extended radial distribution than the neutron density, consistent with the proton-rich nature of the nucleus. The overall density profiles indicate a diffuse nuclear surface associated with the weakly bound resonant character of ^{5}Li, rather than a well-developed halo structure. Figure4 shows the proton, neutron, and mass density distributions of the Li^{5} nucleus calculated using the GS4 parameter set. These distrubitions reveal a more diffuse nuclear mass. The differences in density profiles emphasize the role of the effective interaction in shaping the nuclear periphery and surface properties.

Overall, SKx predicts a more compact structure with neutron skin/halo features, while GS4 suggests a more diffuse and extended mass distribution. The GS4 parameter results in a larger nuclear radius, indicating greater diffuseness and lower stability then that predicted by SKx. The density profiles reflect how different effective interactions influence the predicted surface properties and stability of the ^{5}Li nucleus.

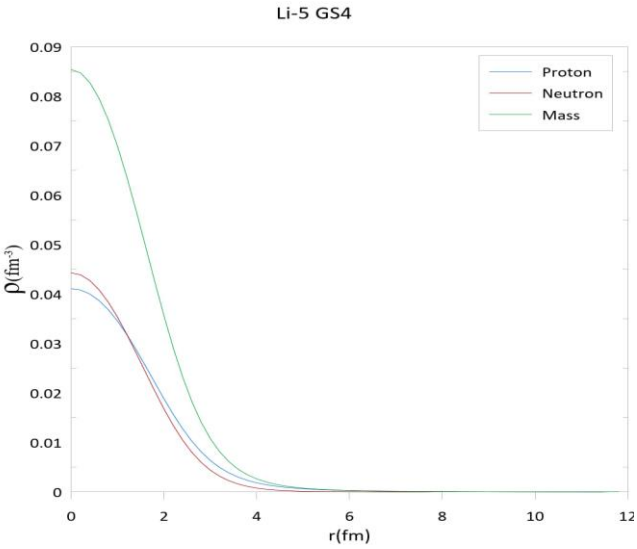

Fig. 4. Proton, neutron, and mass density distributions of the ^{5}Li nucleus calculated using the GS4 parameter set.

4. Conclusion

In this study, the proton, neutron, and mass radii, as well as the total energies of ^{3}Li, ^{4}Li, and ^{5}Li isotopes, were initially determined using Skyrme-Hartree-Fock (SHF) theoretical frameworks. The SHF method has demonstrated significant efficacy in characterizing the ground-state properties of these light lithium nuclei. Our analysis revealed that the selection of equidistant radial grid parameters is critical for the precision of Skyrme force calculations. As a result of the analyses, it was determined that GS4 and SKx parameter sets shows the closest agreement with the experimental total binding energy and mass rms radii values respectively. According to Table 2, the GS4 parameter emerge as the suitable values for ^{4}Li, whereas the SKx parameter appears to be the most compatible for ^{5}Li. Additionally, an examination of Table 3, concerning the rms radii, reveals that the SKx Skyrme parameter is consistent with the data for both ^{4}Li and ^{5}Li.

We observed a significant discrepancy in calculation precision when examining the data on an isotopic basis. For instance, the error margin in mass radii calculations using the SKx parameter set is approximately 131.58% for the ^{4}Li isotope, whereas this rate decreases substantially to 4.24%-for the ^{5}Li isotope. Regarding total energy, the error margin for the ^{4}Li isotope calculated with the GS4 force was found to be approximately 36.13%, while a much lower error margin of 39.37% was recorded for the ^{5}Li isotope.

In conclusion, while Skyrme parameterizations and the Hartree-Fock mean-field approach exhibit strong alignment with experimental data for stable Lithium isotopes, significant discrepancies in both binding energy and nuclear radii arise when applied to very light and unstable nuclei—particularly those characterized by resonance, halo, or diffuse structures. These findings underscore the critical importance of incorporating three-body forces and nucleon correlations into theoretical models for these specific nuclear regions.

From a physical standpoint, although classical mean-field models are capable of reproducing the fundamental dynamics and structural properties of stable nuclei with sufficient precision, they prove inadequate for unstable systems. This limitation clearly necessitates a transition toward *ab-initio* frameworks and advanced correlated theoretical approaches in nuclear physics. Future research, by integrating many-body effects and modern parameterizations, is expected to provide more realistic and robust predictions for unstable and short-lived nuclei. Furthermore, the results established in this study contribute to a deeper understanding of the nuclear structure of lithium isotopes and light elements, while providing a reliable basis for determining initial exciton numbers in the calculation of pre-compound reaction cross sections.

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Declaration of Ethical Code

In this study, we undertake that all the rules required to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with, and that none of the actions stated under the heading "Actions Against Scientific Research and Publication Ethics" are not carried out.

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