# Cycle Duration in Production with Periodicity – Evidence from Turkey

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Received: 03.07.2018 Accepted: 23.09.2018 Published: 30.09.2018

## ABSTRACT

In this paper, the sub-cycles in two economic activity measures are analyzed by using periodogram analyses. Our results from Turkish data suggest that industrial production and capacity utilization rate consist of various cycles including seasonal cycles. These series also have common cycles which correspond about four and seven years. Last, industrial production has also additional cycles compare to capacity utilization ratio cycle.

**Key words:** *Business Cycle, Spectral Analysis, Periodogram*
JEL Classifications: C22, C32, E32

## 1. INTRODUCTION

Business cycle theory developed by Mitchell (1927), Mitchell and Burns (1938) and Burns and Mitchell (1946), who are economists of National Bureau of Economic Research (NBER)[1], examined the dynamics of fluctuations in overall economic activities. They defined business cycle as fluctuations in the overall economic activities.

Burns and Mitchell (1946) calculated the minimum and maximum duration of the US and the UK business cycles. For the United States data, they find a minimum duration between 16 and 22 months and a maximum duration between 100 and 106 months. Also, for the United Kingdom, they find a minimum duration between 16 and 22 months and maximum duration between 135 and 141 months. Following the Burns and Mitchell (1946), several other studies proposed different new methods to search minimum and maximum durations in business cycles. These methods are classified as time domain analyses. Prescott (1986), Singleton (1988), King and Rebelo (1993) and Cogley and Nason (1995) use high-pass filter methods to separate the trend from the cyclical components to define the duration of business cycles. King and Plosser (1994), Stock and Watson (1999), Baxter and King (1999) and Christiano and Fitzgerald (2003) use band-pass filters to identify the durations. On the other hand, there are several different new methods have been developed to identify the minimum and maximum duration of business cycles in developed countries. These time domain methods are categorized into two groups as parametric and non-parametric methods.

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[1] NBER has been publishing and updating the duration of US business cycles data which begins from 1854 to the presents.

Parametric method has four approaches. The first three approaches depend on the linear stationary process but the last approach non-linear stationary process. To determine the duration in business cycles; (i) Beveridge and Nelson (1981), Nelson and Plosser (1982) and Campbell and Mankiw (1987) followed ARMA or ARIMA processes. (ii) Harvey (1985), Watson (1986) and Clark (1987) applied linear unobserved components models. (iii) Granger et al. (1993) search the duration by using Engle and Granger (1987)'s cointegration specification. (iv) Hamilton (1989), Chib (1993), McCulloch and Tsay (1994), Durland and McCurdy (1994) Filardo (1994) and Filardo and Gordon (1998) used Markov Switching Models.

Non-parametric method developed by Bry and Boschan (1971) find the turning points of a data series without a statistical model. Other studies using non-parametric methods are Diebold and Rudebusch (1990), Diebold et al. (1993), Watson (1992), Ohn et al. (2004) and Castro (2010). However, these non-parametric studies cited above cannot detect sub-cycles in the time series. Analyzing business cycles with periodograms, which is a non-parametric method, detects the sub-cycles (or hidden periodicities) in the time series. Thus, periodograms are superior to the other methods in term of finding sub-cycles in the series. A'Hearn and Woitek (2001) use periodograms to detect sub-cycles in North Atlantic economies' industrial production series. Al Zoubi and Maghyereh (2006) examine the sub-cycles in the industrial production series of Jordanian and Israel.

The purpose of this paper is to search the existence of sub-cycles in production. There are various reasons to use the periodograms to search sub-cycles in the series such as

(i) The method is invariant to the model specifications. That is, periodograms can be calculated by trigonometric transformations without depending on any model specifications. The method is also invariant to the mean.

(ii) Since the critical values of the distribution do not depend on the sample size, the method will give better results for small samples.

(iii) In estimating the model parameters, there is no need for estimating any parameters other than the variance of the white noise series.

(iv) The normalized periodogram is asymptotically distributed as $\chi^2$ with 2 degrees of freedom under the alternative hypothesis. Therefore, the analytical power function exists for the test.

(v) The method seems to be more robust if the data has periodic components.

The contribution of this paper is to find the different periodicities in production by using periodograms which allow determining sub-cycles in the series at the same time. The results of the periodogram analysis for Turkey suggest that industrial production index and capacity utilization ratio consist of seasonal periods, some common periods and different periods different from each other. The common periods in these series corresponds about four and seven year cycles.

The remainder of this paper is organized as follows. In Section 2, we introduce the method. Section 3 presents empirical evidence. Finally, in Section 3, we conclude.

## 2. METHOD

Periodograms are often used to reveal hidden periodicities in the data set (Fuller, 1996; Wei, 2006 and Brockwell and Davis, 1987). Periodograms are also used to test for a unit root (Akdi and Dickey, 1998).

We consider given set of a time series data $Y_1, Y_2, \ldots, Y_n$ which may contain periodic components. We consider a trigonometric regression model to capture the periodic components as

$$Y_t = \mu + R\cos(wt+\phi) + e_t \,,\; t=1,2,\ldots,n. \tag{2.1}$$

where $\mu$, $R$, $\phi$ and $w$ represent the expected value, amplitude, phase, and frequency of the series respectively. Also, $e_t$ is the white noise sequence. From the properties of the cosine function, $a = R\cos(\phi)$ and $b = R\sin(\phi)$, the model given in equation (2.1) can be written as

$$Y_t = \mu + a\cos(w_k t) + b\sin(w_k t) + e_t,\; t=1,2,\ldots,n. \tag{2.2}$$

When $w_k = 2\pi k/n$ is selected, the OLS estimators of $\mu$, $a$ and $b$ can be calculated as

$$\hat{\mu} = \bar{Y}_n, \tag{2.3}$$

$$a_k = \frac{2}{n}\sum_{t=1}^{n}(Y_t - \bar{Y}_n)\cos(w_k t) \tag{2.4}$$

and

$$b_k = \frac{2}{n}\sum_{t=1}^{n}(Y_t - \bar{Y}_n)\sin(w_k t) \tag{2.5}$$

here $a_k$ and $b_k$ estimators are known as Fourier coefficients. Because of the properties of trigonometric functions, then we can write

$$\sum_{t=1}^{n}\cos(w_k t) = \sum_{t=1}^{n}\sin(w_k t) = 0 \tag{2.6}$$

and the periodogram ordinates are invariant to the mean. Therefore, Fourier coefficients can also be calculated as

$$\hat{\mu} = \bar{Y}_n, \tag{2.7}$$

$$a_k = \frac{2}{n}\sum_{t=1}^{n} Y_t\cos(w_k t) \tag{2.8}$$

and

$$b_k = \frac{2}{n}\sum_{t=1}^{n} Y_t\sin(w_k t). \tag{2.9}$$

From these OLS estimators based on the model given in equation (2.2), the periodogram ordinate at the frequency $k$ is calculated as

$$I_n(w_k) = \frac{n}{2}\left(a_k^2 + b_k^2\right). \tag{2.10}$$

## 2.1. A Periodogram Based Unit Root Test

Periodograms can also be used to test for a unit root. A periodogram based unit root test is proposed by Akdi and Dickey (1998). Under the null hypothesis that the series has a unit root, Akdi and Dickey (1998) show that for each fixed $k$ the asymptotic distribution of $T_n(w_k)$ is distributed as a mixture of $\chi^2$ given by

$$T_n(w_k) = \frac{2(1-\cos(w_k))}{\hat{\sigma}_n^2} I_n(w_k) \xrightarrow{D} Z_1^2 + 3Z_2^2,\; n\to\infty \tag{2.11}$$

where $T_n(w_k)$ is the value of unit root test statistics based on the periodogram. $Z_1$ and $Z_2$ are independent standard normal random variables and $\hat{\sigma}_n^2$ is any consistent estimator of the error variance. Under the assumption that the series has a unit root, asymptotic distribution is written as

$$T_n(\mathrm{w_k}) \xrightarrow{D} \chi_1^2 + 3\chi_2^2,\; n\to\infty \tag{2.12}$$

If the value of $T_n(w_k)$ is smaller than the critical value $c_\alpha$ ($\alpha$ is significance level), then we reject the null hypothesis of a unit root.

There is also a transition between the autocorrelation function of the series and the spectral density function (Brokwell and Davis, 1987). If $f(w_k)$ is the spectral density function of the stationary time series, the asymptotic distribution of $I_n(w_k)/f(w_k)$ is the $\chi^2$ (the exponential whose expected value is 1) with a degree of freedom of 2. That is, the probability density function of asymptotic distribution of normalized periodograms can be expressed as

$$f(x)=\begin{cases} e^{-x} & , x>0 \\ 0 & , o.w. \end{cases} \tag{2.13}$$

Therefore, periodograms can be taken as an estimator of the spectral density function. Fuller (1996), Brockwell and Davis (1987) and Wei (2006) show $f(w_k)$ is the spectral density function of the stationary time series as;

$$I_n(w_k)/f(w_k) \xrightarrow{D} \chi_2^2,\ n\rightarrow\infty \tag{2.14}$$

## 2.2. Periodicity

The periodograms, which is given in equation (2.2), can be used to search the hidden periodicities in the data for a stationary time series.

Under the null hypothesis that there is no periodic component, $P(V>c_\alpha)=m(1-c_\alpha)^{m-1}$ where $m=(n\text{-}1)/2$, if $n$ is odd and $m=(n/2)\text{-}1$, and if $n$ is even. If $V$ is bigger than the critical values $c_\alpha$ for a given significance level $\alpha$, we reject the null hypothesis $H_0: a=b=0$ that data has no periodic component[2]. Then, we can say that the model contains a periodic component (Wei, 2006). Let $I_n(w_{(1)})=\max\{I_n(w_k)\}$ for $k=1,2,\ldots[n/2]$ where $[n/2]$ is the integer part of $n/2$ and define as

$$V=I_n(w_{(1)})\left[\sum_{k=1}^{[n/2]} I_n(w_k)\right]^{-1}. \tag{2.15}$$

In order to search for another periodic component in the series, let $I_n(w_{(i)})$ be the $i^{\text{th}}$ largest periodogram ordinate and define a test statistic as

$$V_i=\frac{I_n(w_{(i)})}{\sum_{k=1}^{[n/2]} I_n(w_k)\text{-}\sum_{k=1}^{i\text{-}1} I_n(w_{(k)})} \tag{2.16}$$

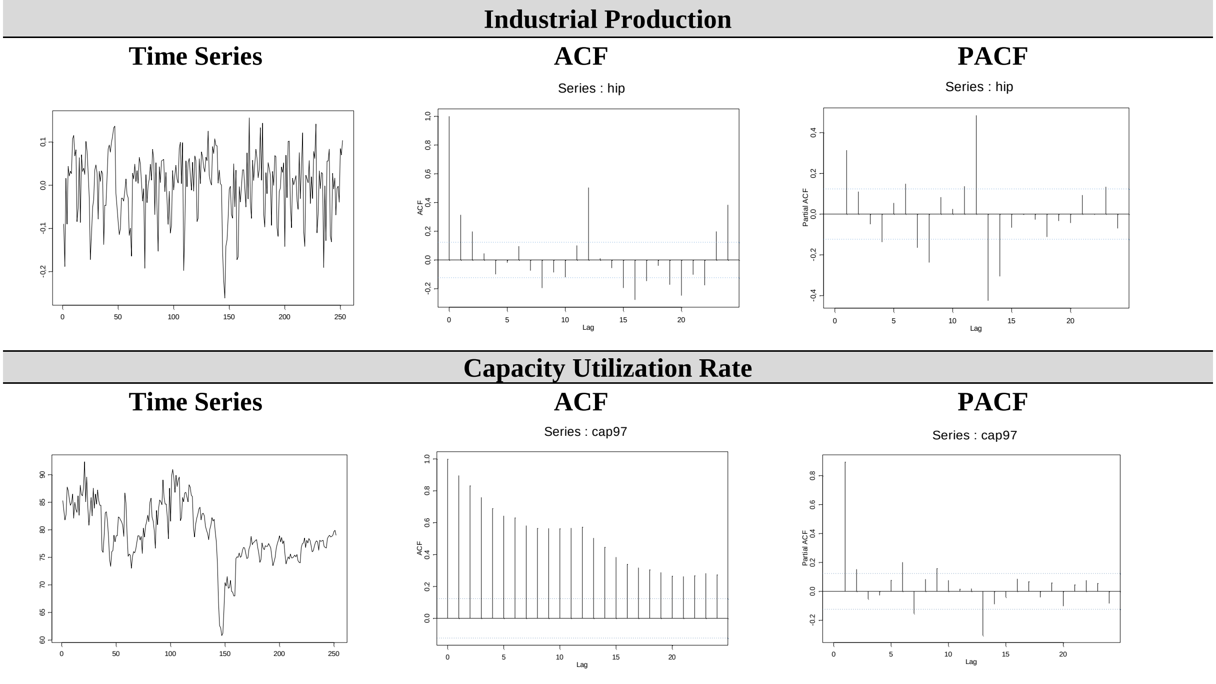
where $V_i$ is the test statistics for determination of a periodic component in series. If $V_i>c_\alpha$, we reject the null hypothesis of no periodic component and conclude that the series has a periodic component at the corresponding frequency (Wei, 2006:295).

## 3. EMPIRICAL EVIDENCE

We employ monthly data from 1997:1 to 2017:12 for industrial production index and capacity utilization ratio. We use industrial production series (IP) as the difference of the logarithmic industrial production index minus Hodrick-Prescott filtered logarithmic industrial production index. Also we use capacity utilization ratio series (CU) at the level. Dataset is gathered from CBRT's Electronic Data Delivery System (EDDS). The time series plot, the identifications plots Autocorrelation Function (ACF), and Partial Autocorrelation Function (PACF) of IP and CU are given in Figure 3.1. Autocorrelation function of IP is rapidly decreasing. Therefore, it suggests that IP does not have a unit root. However, although the autocorrelation function of CU tends not to decrease, it can be said that this decrease rate is rather slow. Table 3.1 also reports the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. The Table 3.1 suggests that IP series is stationary according to both ADF and PP tests. However, the CU series is stationary according to PP test but it has a unit root as to ADF test. Therefore, we applied periodograms based unit root test.

[2] We can test this hypothesis with Fisher's test based on the periodograms. Since the frequency of $w_k$ is unknown, using standard F statistic in not appropriate (Wei, 2006).

**Figure 3.1** Time Series, ACF and PACF Graphs of IP and CU




| Variable | ADF | | PP | |
|---|---|---|---|---|
| | Constant | Constant and Trend | Constant | Constant and Trend |
| IP | -5.1555*** | -5.1338*** | -11.4262*** | -11.4032*** |
| CU | -2.4224 | -2.6968 | -3.5504*** | -4.0409*** |

**Table 3.1** ADF and PP Unit Root Tests.
*Note:* *** is for 1% level of significance.

The critical values of the test statistic given in the equation (2.11) are given in Table 3.2. We calculate $I_n(w_1) = 0.0001$ and $T_n(w_1) = 0.00000208$ for IP, $I_n(w_1) = 2009.82$ and $T_n(w_1) = 0.25807$ for CU. Here, $T_n(w_1) = 0.00000208 < 0.0348 = c_{0.01}$ for the IP. Thus we reject the null of a unit root at the 1% level, and we conclude that the series is stationary. The value of test statistics for the capacity utilization rate is $T_n(w_1) = 0.25807$ which is greater than 5% critical value (0.178). However, the value of the test is less than 8% critical value ($T_n(w_1) = 0.25807 < 0.2908 = c_{0.08}$). That is the CU series is stationary at 8% level. Here, we assume that both series are I(0).

| $\alpha$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_\alpha$ | 0.0348 | 0.0700 | 0.1057 | 0.1418 | 0.1784 | 0.2154 | 0.2528 | 0.2908 | 0.3292 | 0.3681 |

**Table 3.2** The Critical Value of Unit Root Test Statistics Based on the Periodogram ($T_n(w_k)$).
*Note:* The critical values for $\alpha = 0.01, 0.05$ and 0.10 are gathered from Akdi and Dickey (1998), and other values are simulated similarly.

| $\alpha$ | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 | 0.10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $c_\alpha$ | 0.0732 | 0.0680 | 0.0650 | 0.0628 | 0.0611 | 0.0597 | 0.0585 | 0.0575 | 0.0566 | 0.0558 |

**Table 3.3** The Critical Value of Test Statistics for Determination of Periodic Component in series ($V$).
*Note:* The critical values $c_\alpha$ are calculated by the formula $P(V > c_\alpha) = m(1-c_\alpha)^{m-1} \cong \alpha$ given by (Wei, 2006:294).

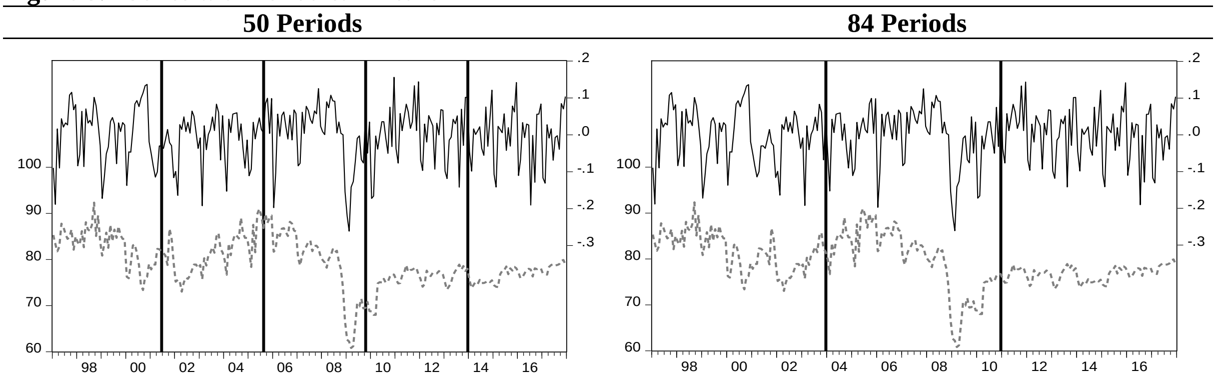
To search the periodicities in IP and CU series, we use the periodograms. The critical values of test statistics $V$ are given in Table 3.3 and corresponding values of $V$ statistics and periods are tabulated in Table 3.4.

Periods of 3, 6 and 12 months correspond to seasonality. Moreover, there are common periods in IP and CU series. These are the 50 and 84 month periods. Also we find other period in IP series. These are 32 and 42 month periods. The main common patterns for these two series are 50 and 84 month periods corresponds about four and seven years. Figure 3.2 plots the IP growth and CU rates with corresponding breaks. The visual inspections further support the 50 and 84 month periods.

| Period | Industrial Production | | Capacity Utilization Rate | |
|---|---|---|---|---|
| | $I_n(w_{(i)})$ | $V_i$ | $I_n(w_{(i)})$ | $V_i$ |
| 3 | 0.1159 | 0.1164 | ----- | ----- |
| 6 | 0.2507 | 0.1713 | ----- | ----- |
| 12 | 0.2180 | 0.1797 | 388.807 | 0.1637 |
| 32 | 0.0555 | 0.0741 | ----- | ----- |
| 42 | 0.0642 | 0.0790 | ----- | ----- |
| 50 | 0.0385 | 0.0763 | 209.144 | 0.1053 |
| 84 | 0.0360 | 0.0774 | 2048.780 | 0.2815 |

**Table 3.4** Periodogram Values and Their Corresponding Values of *V* Statistics.
*Note:* Critical Values are $c_{0.01} = 0.07275$, $c_{0.05} = 0.06073$, $c_{0.10} = 0.05551$ for $V_i$.

**Figure 3.2** Series with Periodical Breaks




*Note:* CU is dashed gray line and is shown in right vertical axis. IP is a black line and is shown in the left vertical axis. Vertical bars indicate periods.

## 4. CONCLUSION

Periodograms allow us to search for a unit root and hidden periodicities in the series. In this paper, we examine the different periodicities in Turkish production data for the period of 1997:1–2017:12 by using a periodogram based analysis. We reveal the existence of sub-cycles in production series.

Our results suggest that industrial production index and capacity utilization ratio have seasonal periods. There are also common cycles of 50 and 84 which correspond about four and seven years. Moreover, industrial production has different sub-cycles from capacity utilization ratio.

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