Research Article

Broadband Wilkinson power divider based on chebyshev impedance transform method

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ABSTRACT

Broadband impedance matching techniques widely used in microwave circuits. In this study, we proposed a 2-way multi-layer micro strip Wilkinson Power Divider (WPD) circuit matched by Chebyshev Impedance matching technique. The design was chosen at 1 GHz center frequency and as four layers. The design was carried out as 3 dimensions on Advanced Design tool (ADS 2009) which is 3D microwave circuits’ simulation tool. For -20 dB return loss reference level, while the reflection bandwidth was 2.5% in the basic quarter wave matched WPD, it could be increased up to 132% in the Chebyshev matching. Additionally, for the reference power transmission of -4.3 dB, the transmission bandwidth raised up to %160. In the range, the proposed design could transfer 75% of the input power to both output ports.

1. Introduction

Maximum power transmission problem is one of most important parameter characterizing basic circuits or systems properties in electronics [1]. In a circuit, the level of power transmission indicates the operating efficiency of the circuit [2-5].

Impedance matching is named as balancing of an electromagnetic (EM) wave propagation between load and transmission line impedance [3, 6]. At the microwave (MW) circuits, the matched situation must exist in the inter-sections of all transmission lines. Otherwise, some problems such as standing wave and unintended physical defects may occur [7-9].

Actually, the impedance matching is a maximum power transmission (MPT) problem [2]. There are various MPT technics in literature. Each of them has contributed many novelties. One of these technics is multi-layer matching. In this technic, it is possible to build broad band circuits. Contrary to lumped elements and single section (quarter wave) matching, multi-layer (section) matching can obtain wider frequency interval. That means the less reflection between source and load [10, 11]. Thus, at microwave frequencies, a better power transmission can be achieved [12].

There are a lot of studies in the literature focusing on eliminating the impedance mismatching between the branches of WPD. There are also studies attempting to obtain broader bandwidth and less reflection over the WPDs by the impedance matching techniques [13]. Those studies have developed power transfer through different mathematical distributions and equations [14, 15]. In addition to these, studies that succeed the high power transmission with the help of Microwave circuit analysis applications are increasing day by day[16, 17].

In this study, it was aimed to eliminate mismatching between the input and output ports of the WPDs. Furthermore, four section Chebyshev polynomials matching technic was used. For 1GHz center frequency, reflection (S11) and transmission (S21 and S31) parameters were evaluated by using Chebyshev impedance matching method. As design tool, the Advanced Design System (ADS 2009) which is 3D EM simulation program was preferred.
2. Power divider technic and theoretical impedance matching approach

2.1 The Power Divider Technic

In microwave circuits, the basic problem of T-junctions is that all ports do not have impedance matching [18]. Wilkinson Power Divider (WPD) was developed to eliminate this mismatching. WPD circuits split the power transmitted through the lossless transmission lines equally into two or more ports without loss [19].

In the microwave circuits, power transmission is stated as logarithmic (dB) units. Over both output ports of WPD, half of the input power is delivered [20]. Ideally, in lossless environment, transferred power in a branch is 50 percent and means -3 dB of power. In fact, real lines have minor loss. If the loss is very little, then the delivered power will be closer to -3 dB levels [21].

In WPDs, input transmission line has Z₀ and each output ports have 2Z₀ characteristic line impedance. Therefore, a mismatching situation is observed between input and output ports. One of the solutions is a general method known as single section (quarter wave) matching. Technically, a single section line with Z = \sqrt{2}Z₀ of characteristic and \( \theta = \beta l = \lambda/4 \) line length is added between T-junction point and output port [21]. General schematic view of WPD is shown in Figure 1 where \( w1=2.9 \) mm (50 Ω), \( w2=1.55 \) mm (70.7 Ω). Since it was cheap and readily available, FR4 whose permittivity was \( \varepsilon_r = 4.3 \) and loss tangent was \( \tan d=0.025 \) was chosen as substrate material. The effective relative permittivity of FR4 is \( \varepsilon_{eff} = 3.26 \). At the center frequency of 1GHz, the quarter wave length was computed as \( \lambda/4 = 41 \text{mm} \).

![Figure 1. General schematic view of WPD](image)

In order for the Wilkinson power divider to be able to transmit power without loss, mode analysis should be performed. In the theoretical even-odd mode calculation, the resistance at the output is determined as \( R=100 \) Ω.

Thus, 50 Ω output impedance and -3 dB of input power will be obtained at ports 2 and 3 [18,20,22].

2.2 Chebyshev Matching Approach

In the literature, there are a lot of methods for modelling multi-layer impedance matching. One of the most used modelling techniques is Chebyshev transform [3,4]. In this technique, while calculating each matching section, Chebyshev polynomials are used. General equations of Chebyshev Polynomials are given in equation 1 [3,10].

\[
T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)
\]  

(1)

As in multi-layer matching, each step of the Chebyshev transform depends on previous one since all steps are calculated sequentially. One disadvantage of this technique is fluctuation on the pass band frequencies. However, comparing with single section and some other multi-layer matching methods, Chebyshev Transform is a design from which more performance can be obtained [3]. In Figure 2, there is a comparison of reflection graphic between single section (quarterwave) and four-section (N=4) Chebyshev transform. It is seen obviously that despite of fluctuations, Chebyshev Transform is better than single section matching.

![Figure 2. Comparison of reflection graphic between single section (quarter wave) and four-section (N=4) Chebyshev transform](image)

Modelling of Chebyshev Polynomials to impedance matching is given in equation 2 [3].

\[
T_n(\sec \theta_m) = \frac{1}{\Gamma_m} \left[ \frac{Z_L-Z_0}{Z_L+Z_0} \right] \approx \frac{1}{2\Gamma_m} \ln \left[ \frac{Z_L}{Z_0} \right]
\]  

where, \( \theta \) is line angle \( \beta l \), \( \theta_m \) is an angle obtained for maximum tolerable reflection coefficient \( \Gamma_m \). As shown in equation 3, generally Nth degree of Chebyshev polynomial( \( T_N(\sec \theta) \) ) is obtained by adapting to reflection coefficient.
\[ \Gamma(\theta) = A \cdot e^{-j\theta} T_N(\sec \theta_m \cos \theta) \]  

(3)

where \( A \) is design factor and calculated for each section as in equation 4 [3].

\[ A = \frac{z_1 - z_0}{z_1 + z_0} \frac{1}{\Gamma_m(\sec \theta_m)} \]  

(4)

In this study, impedance mismatching of each branch of WPDs’ output ports \((2Z_0 = 100 \, \Omega)\) was attempted to eliminate by proposed method. It was decided that the number of section was 4 \((N=4)\) and maximum tolerable reflection coefficient was selected as \( \Gamma_m = 0.01 = -20 \, dB \) because of minimum mismatching loss.

3. Proposed Design

In this study, two-way WPD whose each branch was matched with Chebyshev design was proposed. The dielectric material thickness of FR4 was \( d = 1.5 \, mm \). Lower and upper cover of the substrate was copper where the thickness was \( t = 0.035 \, mm \). The upper micro strip base was the place where the design was built. The substrate dimensions were \( L = 100 \, mm \) and \( W = 80 \, mm \). In Figure 3, perspective (upper) and cross section view (under) of the design are shown.

![Figure 3. Perspective (upper) and cross sectional view (under) of the design](image)

Lengths and inner dimensions of WPD are shown in Figure 4. Here, each length of sections are equal and they are \( \theta = \lambda/4 = 41 \, mm \). Thickness of each layer was computed according to Chebyshev method \((\Gamma_m = 0.01 = -20 \, dB)\). It was also calculated as \( w1 = 2.9 \, mm \) \((50 \, \Omega)\), \( w3 = 0.89 \, mm \) \((90.15 \, \Omega)\), \( w4 = 1.3 \, mm \) \((77 \, \Omega)\), \( w5 = 1.93 \, mm \) \((63.3 \, \Omega)\), \( w6 = 2.55 \, mm \) \((54.08 \, \Omega)\). The second and third ports were terminated at \( wl1 = 2.9 \, mm \) \((50 \, \Omega)\) thickness. Between these two ports, there was a resistor \( R = 100 \, \Omega \) due to theoretically analysis.

To avoid the circuit being too long, the matching lines were bended till terminals. Therefore, it was determined as \( l1 = 12.6 \, mm \), \( l2 = 11 \, mm \) and \( l3 = 3 \, mm \). Thus, each length of line was equalized to \( \lambda/4 \).

![Figure 4. Lenghts and inner dimensions of power divider circuit](image)

4. Results

The reflection in the input port of the designed power divider circuit has improved compared to the single section. In Figure 5, comparison between quarter wave and Chebyshev design of input reflection \((S_{11})\) graphic was seen. As chosen input reflection level was smaller than \( S_{11} < -20 \, dB \); while single section was 0.9-1.3 GHz band width, the Chebyshev design showed 0.350-1.67 GHz performance. The formula of percentage of bandwidth is given in equation 5 [3].

\[ BW\% = \frac{f_u - f_l}{f_0} \]  

(5)

where, \( BW\% \) is percentage of bandwidth, \( f_u \) is the upper band and \( f_l \) is the lower band and \( f_0 \) is the center frequency of 1 GHz.

While the percentage of bandwidth in the quarter wave matching is 25%, it shows 132% BW performance in the Chebyshev Design.

![Figure 5. Comparison between quarter wave and Chebyshev design of input reflection (S_{11}) graphic](image)
In a power divider circuit, one of the most important parameters is power transmission line coefficient ($S_{21}$ and $S_{31}$). In Figure 6, a comparison about logarithmic power transmission characteristic between quarter wave and Chebyshev design is shown. In a symmetrical circuit with respect to the input port, output power of second and third port are identical. If the reference level is accepted as $S_{21} = S_{31} > -4.3 \text{ dB}$, the transmission frequencies will be between 0.1 and 1.7 GHz. Then the percentage of transmission bandwidth would be %160 and it could be said that the method is successful on the matching. The logarithmic and percentage conversion of power transmission is given in equation (6) [3].

$$P \% = \frac{10^{P_{(dB)}}}{10} \times 100$$

Figure 6. Compared logarithmic transmission graphics of quarterwave and Chebyshev design

5. Conclusion

In the study, the impedance was matched through WPD Chebyshev Impedance matching technique. The proposed design was compared with the quarter wave technique in the literature. For -20 dB return loss reference level; while basic WPD (quarter wave matched) bandwidth was 25%, the Chebyshev matched WPD’s bandwidth reached 132%. In addition, the proposed design could transfer 75% of input power to both output ports in the range where the transmission power level is -4.3 dB and the transmission bandwidth (BW) became %160. Regarding the reference studies presented in Table 1, it can be said that this study conforms to the literature and contributes to the recent studies.

As expected, the obtained results indicate that proposed design enables the power division/transmission in a wider frequency range. This study showed that power divider circuit can be designed with broadband Microwave impedance matching techniques. Besides, obtained results conform to the literature about Chebyshev broadband impedance transform studies.

Table 1. Comparison of this study with the reference studies

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Method</th>
<th>Freq (GHz)</th>
<th>$S_{11}$ Ref. Level (dB)</th>
<th>$S_{11}$ BW (%)</th>
<th>$S_{21}$ Ref. Level (dB)</th>
<th>$S_{21}$ BW (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[16]</td>
<td>Non Uniform Transm. Line</td>
<td>1.25</td>
<td>-10</td>
<td>56</td>
<td>-4.25</td>
<td>24</td>
</tr>
<tr>
<td>[13]</td>
<td>Stub Matching</td>
<td>1 - 2</td>
<td>-20</td>
<td>4</td>
<td>-3.5</td>
<td>22</td>
</tr>
<tr>
<td>[15]</td>
<td>Z Polynomials</td>
<td>0.7 - 2</td>
<td>-20</td>
<td>60</td>
<td>66</td>
<td>-3.22 max</td>
</tr>
<tr>
<td>[17]</td>
<td>Gysel Power Divider</td>
<td>1.5</td>
<td>-18</td>
<td>92</td>
<td>1.92 max</td>
<td>-</td>
</tr>
<tr>
<td>[14]</td>
<td>Analytically Compensated</td>
<td>9</td>
<td>-10</td>
<td>160</td>
<td>-4.5</td>
<td>77</td>
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<td>This Study</td>
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<td>1</td>
<td>-20</td>
<td>132</td>
<td>-4.3</td>
<td>160</td>
</tr>
</tbody>
</table>

References
12. Cheng, K.K.M. and C. Law, A novel approach to the design and implementation of dual-band power


