Kastamonu Education Journal, 2026, Vol. 34, No. 2, 207-237

https://doi.org/10.24106/kefdergi.1939228

|Research Article / Araştırma Makalesi|

Enhancing Gifted Students' Mathematical Thinking Skills (An Example of the Waldorf Approach)

Özel Yetenekli Öğrencilerin Matematiksel Düşünme Becerilerinin Geliştirilmesi (Waldorf Yaklaşımı Örneği)

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Keywords

Mathematical thinking skills

Gifted students

Waldorf educational approach

Abstract

This study investigates the potential of activities structured around the Waldorf educational approach's emphasis on play, art, nature, and practical implementation to enhance the mathematical thinking skills of gifted students. It was conducted within a qualitative action research design over a twelve-week period with four gifted fourth-grade students enrolled in a formal educational institution, the Science and Art Center. The process began with initial activities and interviews, followed by the development and implementation of four action plans based on play, art, nature, and practical implementation, designed by the researchers with expert guidance. Data were collected through a mathematical thinking test, semi-structured interviews, and field notes recorded by one of the researchers, and were analyzed through content analysis. The findings revealed significant progress in students' specializing, generalizing, and conjecturing skills. However, compared with other dimensions of mathematical thinking, improvement in justifying and convincing skills remained relatively limited. The study is significant in addressing the limited body of research on the role of the Waldorf approach in fostering the mathematical thinking of gifted students. The findings indicate that art, nature-, and play-based pedagogy can make a meaningful contribution to the development of several dimensions of mathematical thinking, while also revealing the need for more explicit instructional support in the areas of justification and convincing. In this respect, the study offers both theoretical and practical implications for teachers and researchers concerning the use of alternative pedagogical approaches in mathematics education.

Anahtar Sözcükler

Matematiksel düşünme becerileri

Üstün yetenekli öğrenciler

Waldorf eğitim yaklaşımı

Öz

Bu çalışma, Waldorf eğitim yaklaşımının öğrenme süreçlerinde öne çıkardığı oyun, sanat, doğa ve pratik uygulama temelli etkinliklerin üstün yetenekli öğrencilerin matematiksel düşünme becerilerini geliştirme potansiyelini incelemeyi amaçlamaktadır. Araştırma, nitel araştırma yaklaşımı çerçevesinde eylem araştırması deseninde yürütülmüştür. Çalışma grubu, dördüncü sınıf düzeyinde öğrenim gören ve aynı zamanda formel bir eğitim kurumu olan Bilim ve Sanat Merkezinde de eğitim gören dört üstün yetenekli öğrenciden oluşmaktadır. Araştırma süreci on iki hafta sürmüş; başlangıç etkinlikleri ve görüşmelerin ardından, uzman görüşü doğrultusunda araştırmacılar tarafından oyun, sanat, doğa ve pratik uygulama kavramlarına dayalı dört eylem planı geliştirilmiş ve uygulanmıştır. Veriler, matematiksel düşünme testi, yarı yapılandırılmış görüşmeler ve araştırmacılardan birine ait saha notları aracılığıyla toplanmış; elde edilen veriler içerik analiziyle çözümlenmiştir. Bulgular, öğrencilerin özelleştirme, genelleme yapma ve varsayım oluşturma becerilerinde belirgin bir gelişim gösterdiğini ortaya koymuştur. Buna karşılık, gerekçelendirme ve ikna etme becerilerindeki gelişimin diğer matematiksel düşünme becerilerine kıyasla daha sınırlı kaldığı belirlenmiştir. Sonuç olarak çalışma, Waldorf yaklaşımına dayalı sanat, doğa, oyun ve deneyim odaklı pedagojik uygulamaların üstün yetenekli öğrencilerin belirli matematiksel düşünme becerilerini desteklemede işlevsel bir potansiyel taşıdığını göstermektedir. Ayrıca bulgular, gerekçelendirme ve ikna etme becerilerinin geliştirilmesi için daha açık yapılandırılmış öğretimsel desteklere ihtiyaç duyulduğunu ortaya koyarak matematik eğitiminde alternatif pedagojik yaklaşımların uygulanabilirliğine ilişkin kuramsal ve pratik çıkarımlar sunmaktadır.

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Citation/Atıf: Demirci, N., & Ergül, E. (2026). Enhancing gifted students' mathematical thinking skills (an example of the Waldorf Approach). *Kastamonu Education Journal, 34*(2), 207-237. https://doi.org/10.24106/kefdergi.1939228

Received / Başvuru: 04.04.2025
Accepted / Kabul: 10.02.2026

Introduction

Mathematical thinking is defined by researchers as a rich and multidimensional framework that is divided into various stages and types of knowledge. It includes activities such as reasoning, problem solving, generalizing, and abstracting (Goos & Kaya, 2020; Pedersen, 2023; Villa-Ochoa & Suarez-Tellez, 2021). Some researchers emphasize that generalizing, drawing inferences, proving, and convincing are fundamental elements of mathematical thinking (Breen & O'Shea, 2011). Mason et al. (2010), on the other hand, regard mathematical thinking as a structure comprising specializing, generalizing, conjecturing, and justifying. Specializing refers to working with particular examples to reach general conclusions and is characterized by analyzing examples, identifying and demonstrating them, and discovering patterns (Mason et al., 2010; Stacey, 2006). Generalizing involves starting from specific cases to describe broader phenomena and includes classification, prediction, and expressing relationships in mathematical or verbal forms (Gunadi et al., 2023). Conjecturing covers proposing statements, testing them, forming claims, developing representations, and revising assumptions (Villa-Ochoa & Suarez-Tellez, 2021). The stage of justifying and convincing involves explaining the obtained results to oneself or others in order to persuade them of the solution's correctness (Mason et al., 2010). These processes help individuals evaluate problems from different perspectives, discover alternative solutions, and enhance their decision-making and creative problem-solving abilities (Delima et al., 2021; Gunadi et al., 2023). In this study, the mathematical thinking framework of Mason et al. (2010) -widely used in the relevant literature-has been adopted (Amador, 2022; Barnes, 2021; Celik & Ozdemir, 2020; Faizah et al., 2020; Lorenza et al., 2024). Among these components, problem-solving processes deserve particular attention as they possess a sequential and spiral structure (Burton, 1984; Stacey, 2006). Therefore, problem-solving activities are regarded as an effective method for enhancing mathematical thinking skills. Researchers emphasize that gifted students can maximize their problem-solving potential through authentic and open-ended tasks designed to meet their needs for independence, creativity, and challenge (Bonotto & Santo, 2015; Eysink et al., 2015; VanTassel-Baska, 2013).

Mathematical thinking skills are highly important for gifted students to effectively understand and apply mathematical concepts (Assmus & Fritzlar, 2022). Developing these skills plays a critical role in both their academic success and overall life skills (Kroesbergen & Schoevers, 2017). Research indicates that mathematical thinking enhances analytical abilities and creativity, thereby enabling students to grasp abstract concepts more easily (Assmus, 2018; Lev & Leikin, 2017). These skills also facilitate students' comprehension of complex mathematical ideas (Pitta-Pantazi et al., 2011). As gifted students develop their mathematical thinking skills, they also improve their problem-solving strategies, leading to richer learning experiences (Lim et al., 2023). Therefore, supporting mathematical thinking is crucial not only for academic achievement but also for personal development (Delima et al., 2021). Executive functions-such as updating-are acknowledged in the literature as playing a pivotal role in mathematical thinking and problem-solving processes. Through these cognitive mechanisms, essential aspects of personal development, including analytical reasoning, self-discipline, and academic self-efficacy, are effectively nurtured (Van der Ven et al., 2012). Education systems must implement strategies aimed at fostering these skills and provide supportive learning environments for gifted students.

Education systems aiming to foster mathematical thinking in gifted students have adopted a variety of pedagogical strategies, including inquiry-based learning, problem-based tasks, enrichment programs, and differentiated instruction tailored to students' advanced cognitive profiles (Assmus & Fritzlar, 2022; Leikin, 2011). For instance, the Montessori and Reggio Emilia approaches emphasize student agency and hands-on exploration, while more structured models such as the TAG (Talented and Gifted) programs prioritize accelerated content delivery and abstract reasoning challenges. In contrast, the Waldorf education approach adopts a holistic perspective that integrates imagination, aesthetics, and individual pacing-an orientation that is comparatively less explored in relation to mathematical thinking. Examining the Waldorf approach alongside these more commonly studied frameworks offers the potential to generate novel insights into how alternative pedagogies may contribute to the development of high-level mathematical thinking skills in gifted learners.

The Waldorf education approach, which prioritizes creativity, problem-solving skills, and conceptual understanding, offers a promising framework for meeting the needs of gifted students (Steiner, 1996). Emphasizing holistic comprehension and sensory interaction, Waldorf philosophy stands in marked contrast to rigid educational systems (Aljabreen, 2020; Attfield, 2024; Barth & Wiehl, 2024). Known as “education toward freedom,” it encourages individuals to discover their own paths (Carlgren & Klingborg, 1976). Because of this aspect, it is particularly beneficial for gifted students (Goldshmidt, 2017; Huchingson & Huchingson, 1993; Piske & Stoltz, 2021). The curriculum used in Waldorf schools is structured according to different developmental stages (Uhrmacher, 1995) and aims to reveal students’ cognitive potential (Nordlund, 2013). By integrating emotional, social, and cognitive dimensions, this holistic approach provides meaningful learning experiences (Goldshmidt, 2017). The flexibility of Waldorf pedagogy reduces potential barriers faced by gifted students, enabling them to succeed academically and personally (Paschen, 2014; Piske & Stoltz, 2021). This approach enhances gifted students’ cultural awareness while encouraging them to gain practical and artistic skills and develop a profound respect for nature (Goldshmidt, 2017; Huchingson & Huchingson, 1993). Maker (1982) emphasized that specially designed educational programs foster the abilities of gifted students. The intellectual curiosity and creativity nurtured by the Waldorf philosophy have proven effective in achieving these goals (Huchingson, 1990; Huchingson & Huchingson, 1993). Some researchers also suggest that gifted students may thrive in less structured environments (Assmus & Fritzlar, 2022; DeDonno, 2016; Maker, 1982), indicating that the Waldorf educational philosophy holds significant, lasting potential for gifted students as well (Waldorf World List, 2024).

From its inception, Waldorf educational philosophy has been extensively researched in terms of its theoretical framework, fundamental philosophy, and characteristics (Aljabreen, 2020; Attfield, 2024; Barth & Wiehl, 2024; Berner, 2023; Boyd, 2018; Daskolia & Koukouzeli, 2023; Goldshmidt, 2017; Kaya & Gündüz, 2015; Lutzker, 2024; Masters, 2009; Neumann, 2024; Oberski et al., 2007; Paschen, 2014; Rawson, 2024; Taplin, 2024; Uceda, 2015; Veiga, 2015; Wright, 2013). Some comparative studies have also examined the extent to which national curricula align with the Waldorf curriculum (Bak, 2023; O’Connor & Angus, 2014; Shank, 2016). However, research on how Waldorf philosophy contributes to students’ development remains limited (Nicholson, 2000; Van Schie & Vedder, 2023), indicating a noticeable lack of concrete evidence regarding the developmental outcomes of the Waldorf philosophy. In the field of gifted education, studies on Waldorf philosophy also appear to be constrained in scope. Although pilot studies conducted in previous years at public schools hinted at the potential benefits of this approach (Huchingson, 1990; Huchingson & Huchingson, 1991; as cited Huchingson & Huchingson, 1993, p. 402), recent research has not focused on its developmental contributions. As reported by Huchingson (1990) and Huchingson & Huchingson, (1993), activities implemented under a curriculum inspired by Waldorf Principles-applied by teachers providing Waldorf education to fourth- and eighth-grade students-led to superior outcomes in areas such as artistic ability, environmental awareness, public speaking, and creativity among students participating in the Waldorf program, compared to those receiving traditional education. Additionally, Waldorf students displayed higher levels of self-confidence and eagerness to learn than their peers in traditional education. It was also observed that, as the duration of program participation increased, students made notable progress in terms of motivation and creativity. Similarly, Piske and Stoltz (2021) highlighted the significant role Waldorf-based activities play in fostering creativity and social interaction, thereby supporting gifted students’ advancement across various domains of knowledge-a finding that confirms the continuing potential impact of Waldorf philosophy in more recent times.

Building on Goldshmidt’s (2017) view that Waldorf philosophy can yield effective outcomes in mathematics education, this study aims to develop the mathematical thinking skills of four gifted fourth-grade students attending a Science and Art Center (SAC), whose skills in this area have identifiable aspects open to improvement. These centers are specialized education institutions operating under the Ministry of National Education (MoNE) in Türkiye, established to enhance the potential of primary, secondary, and high school students identified as gifted in general mental ability, visual arts, or music through standardized tests and expert evaluations. In these centers, students participate in a tiered educational framework outside of formal school hours, which includes progressive stages such as orientation, support education, individual talent recognition, special talent development, and project production (MoNE, 2025). Moving beyond the conventional curriculum, this pedagogical process is designed with an interdisciplinary approach, aiming to foster students' higher-order cognitive skills-including creative thinking, problem-solving, and scientific research techniques-within a workshop-based and project-oriented structure that differs from traditional school environments (Sak, 2024).

During the intervention process, elements emphasized in Waldorf educational practices-such as play, art, nature, and practical learning-were utilized to address the identified developmental needs of the students. The Waldorf approach holds strong potential for gifted students, as it supports creative and holistic thinking skills through art-, nature-, and play-based learning processes. This approach enables students to make sense of mathematical concepts not only at an abstract level but also in aesthetic and experience-based contexts. Thus, higher-order skills such as specializing, generalizing, conjecturing, and justifying can develop in a multifaceted manner. The limited number of applied studies in the existing literature examining the effects of the Waldorf approach on mathematical thinking skills-particularly in the context of gifted students-increases the originality of this research. The contribution of this study to educators is to provide practical examples and strategies for integrating Waldorf pedagogy into mathematics education, offering an alternative perspective for instructional design. For students, it transforms the learning process into a more meaningful, motivating, and creativity-enhancing experience. In terms of the learning environment, it contributes to the construction of mathematical thinking in a more natural and lasting way by creating an interdisciplinary, interactive, and participatory learning atmosphere. Accordingly, the following research question was addressed: In light of the students' experiences, how do Waldorf-based activities foster gifted students' skills in specializing, generalizing, conjecturing, justifying, and convincing?

Method

This study employed the action research design proposed by Kemmis and McTaggart's (2014) to enhance gifted students' mathematical thinking skills. Rather than testing a hypothesis, the goal of action research is to examine specific educational contexts in depth. For this reason, the action research method is highly suitable for investigating the learning processes of a particular group (Patton, 2014). In this research, changes in gifted students' mathematical thinking skills were described by utilizing activities based on the concepts of play, art, nature, and practical implementation, all of which are emphasized in the Waldorf educational approach. Due to the unique cognitive characteristics of gifted students, a consciously selected small sample group (gifted students) was chosen in order to explore their mathematical thinking processes in detail. This small sample provided an opportunity to closely observe the individual reasoning strategies, challenges, and conceptual development of students who are often overlooked in large-scale studies (Robson, 2024). Therefore, the chosen approach aligns with the strengths of qualitative research, which prioritizes rich, context-focused insights over broad generalizations (Creswell & Poth, 2018). The action plan of the study can be seen in Table 1.

Table 1

The Action Plan of the Study

Planning	Implementation and Observation	Reflection
Literature review	Implementation of the "Square Sequence" problem	Exploration of the effects of the implementation
A reassessment of the research problem in light of the literature	Weeks 1 and 2 First implementation: Mathematics Through Play / Multiplication and Area Calculation Game Time: This cycle consisted of four class sessions, each lasting 40 minutes	A joint reflective discussion of the implementation process between the instructor and the researcher
Collaborative planning of the implementation process by the researcher and the instructor	Observation of the implementation and assignment of the "Hat Average" task	Analysis and interpretation of the research data
	Revision of the subsequent action plan based on observations and student evaluations	Reporting of the results
	Weeks 3, 4, 5, and 6	

Planning	Implementation and Observation	Reflection
	Second implementation: Art and Mathematics / Geometric Designs and Perspective Calculations Time: The second cycle consisted of eight class sessions, each lasting 40 minutes	
	Observation of the implementation and assignment of the “Odd Number Pattern” task	
	Revision of the subsequent action plan based on observations and student evaluations	
	Weeks 7, 8, and 9 Third implementation: Mathematics in Nature and the Human Body / Fibonacci Numbers and Patterns Time: The third cycle consisted of six sessions, each lasting 40 minutes	
	Observation of the implementation and assignment of the “Pizza Ratio” task	
	Revision of the subsequent action plan based on observations and student evaluations	
	Weeks 10, 11, and 12 Fourth implementation: Mathematics in Daily Life / Market Shopping and Picnic Budgeting Time: The fourth cycle consisted of six sessions, each lasting 40 minutes	
	Observation of the implementation and assignment of the “Problem-Design Task”	

Participants

The participants in this study include the researchers and the students. The first researcher has been working as a teacher for 16 years and has been serving at the SAC since 2021, currently working as a classroom teacher and acting as the practitioner of the research. The study was carried out during the 2024-2025 academic year with four fourth-grade students attending a SAC under the MoNE (SACs are specialized, after-school public educational institutions in Türkiye designed to foster the potential of students identified as gifted in general mental ability, visual arts, or music through standardized assessments.). The study was launched when the first researcher, having observed the need for support in courses designed to enhance creative thinking, problem-solving, and deep learning skills among SAC students, consulted the second researcher for collaboration. The second researcher has 12 years of experience teaching in public schools and has continued mathematics education research for the past four years as a faculty member at a state university. The second researcher collaborated with the practitioner in devising action plans, monitoring the implementation process, addressing shortcomings, and analyzing the data.

The first researcher, holds a doctorate in mathematics education. Based on long-term observations in the courses conducted with gifted students at the SAC for the stated objectives, the researcher identified that the students experienced persistent difficulties in defining and extending patterns, performing calculations related to these patterns, engaging in reasoning, and applying these skills in real-world contexts. To address these challenges and precisely diagnose the problem, the teacher administered the “Square Sequence” problem (Stacey, 2006) to 52 gifted students aged 9 to 10 as a regular classroom activity on a typical school day. The results of this problem were then shared with the second researcher.

Figure 1

Square Sequence Problem

Pattern Number Squaers

A series of squares contain certain numbers.

- The 1st square contains the number 1.
- The 2nd square contains the number 3.
- The 3rd square contains the number 6.
- The 4th square contains the number 10.
- The 5th square contains the number 15.

The pattern continues following the established rule.

Based on this information:

a) What number will appear in the 6th square?

b) What pattern or rule can you identify from these numbers? Explain the logic behind the numerical increase.

c) If the pattern continues, what numbers will appear in the 7th and 8th squares?

d) Create three similar problems inspired by this pattern, categorized by difficulty levels (easy, medium, and challenging), and clearly outline each level.

The students' written and oral responses to the "Square Sequence Problem" were meticulously analyzed by the researchers according to the evaluation criteria presented in Appendix 1. Based on the analysis results, the overall performance levels of the students' mathematical thinking skills are presented in Table 2.

Table 2

Initial Performance Levels in Mathematical Thinking Skills (All Students)

*Performance Level	Number of Students (n)
Weak Performance (WP)	4
Moderate Performance (MP)	2
Good Performance (GP)	16
High Performance (HP)	30
Total	**52**

Note. * Please refer to Appendix 1 for the definitions and examples of performance levels.

According to Table 2, students in the entire class exhibit different levels of mathematical thinking skills. Despite the first researcher, who is a teacher in the SAC class, having previously implemented various engaging activities (e.g., algorithmic coding tasks, robotics design projects, and non-routine problem-solving exercises), some students have still not reached the desired level of mathematical thinking skills. In this study, the number of participants was deliberately limited in accordance with the nature and scope of the action research design. Action research prioritizes in-depth, context-specific investigations over generalizability and is based on iterative processes (Creswell & Guetterman, 2019). Since the intervention process involved providing individualized guidance, conducting interdisciplinary activities, and maintaining one-on-one interactions, working with a small group was pedagogically necessary. Furthermore, the pedagogical principles of the Waldorf approach-which centers on art-, nature-, and play-based learning-are better suited to small-group implementations as they emphasize individualized learning experiences. Increasing the number of participants could have compromised the authenticity of the intervention and the integrity of the instructional design.

Therefore, based on a systematic needs analysis, the study was structured with a limited number of participants to allow for intensive work with students whose mathematical thinking skills were identified as having the greatest potential for development. Accordingly, a total of four students—two girls and two boys—were included in the study based on their own consent and the informed consent of their families. To protect their privacy, personal information has been anonymized and pseudonyms have been used to represent each individual ("Ali," "Zeynep," "Mehmet," and "Elif"). Detailed results of the participants' mathematical thinking skills and performance levels are presented in Table 3 using these pseudonyms. This classification was determined through a comprehensive analytical rubric that evaluates each skill-specializing, generalizing, conjecturing, justifying, and convincing-against specific behavioral indicators derived from the preliminary assessment tasks. The theoretical and operational framework for defining these levels (Weak, Medium, or Strong) is detailed in Appendix 1. Readers may refer to this appendix for a thorough breakdown of the scoring parameters and performance descriptors used in the categorization process.

Table 3

Initial Performance Levels in Mathematical Thinking Skills (Participating Students)

Student	Mathematical Thinking Skills	Numbers in Squares Problem	Overall Performance Level	Problem-Posing Skills
Ali	Specializing	Could not detect the rules of numerical change within the squares. (WP)	Weak Performance (WP)	Easy: Unable to define the problem. Moderate: Attempted solving with only a few examples. Difficult: Unable to personalize the problem or progress the solution
	Generalizing	Calculated only a few steps but could not derive a general rule. (WP)		
	Conjecturing	Formed a conjecture such as “If the number in one square doubles, how do the others change?” but could not develop it further. (MP)		
	Justifying and Convincing	Did not present any justification for the solution. (WP)		
Zeynep	Specializing	Took certain numbers as examples but failed to recognize the pattern. (WP)	Weak Performance (WP)	Easy: Had difficulty defining the problem. Moderate: Tried solving with only one example. Difficult: Struggled to identify the steps required to solve the problem.
	Generalizing	Did not attempt to create any general formula. (WP)		
	Conjecturing	Did not form a conjecture. (WP)		
	Justifying and Convincing	Did not offer explanations regarding the solution. (WP)		
Mehmet	Specializing	Partially noticed the pattern but could not fully explain its rules. (MP)	Moderate Performance (MP)	Easy: Tried to solve the problem by giving basic examples. Moderate: Explored some solution approaches but lacked clarity in explanation. Difficult: Unable to analyze the problem in depth.
	Generalizing	Made small-scale generalizations but left them incomplete. (MP)		
	Conjecturing	Formed a conjecture such as “If the numbers in the squares increase by three each time, how would the result change?” but could not prove it. (MP)		
	Justifying and Convincing	Provided no logical explanations for his solution. (MP)		
Elif	Specializing	Did not recognize the pattern in the squares. (WP)	Weak Performance (WP)	Easy: Had difficulty defining the problem. Moderate: Attempted simple examples. Difficult: Could not proceed step by step to solve the problem.
	Generalizing	Unable to formulate a different rule. (WP)		
	Conjecturing	Did not make any conjectures. (WP)		
	Justifying and Convincing	Did not provide any explanation for her solution. (WP)		

According to Table 3, three out of four volunteer students exhibit weak mathematical thinking skills, while one demonstrates a moderate level. However, almost all four students showed weak performance in specializing, generalizing, conjecturing, and justifying and convincing skills. This needs analysis indicates that the participant group faces significant challenges in applying mathematical thinking skills during problem-solving activities. This finding suggests that these students require support and interventions to further develop their mathematical thinking skills.

Data Collection Tools

Mathematical Thinking Test

In this study, the assessment instrument developed by Cai (2003) was used to evaluate students' mathematical thinking processes (see Appendix 2). The instrument consists of four mathematical problems; three of these are open-ended tasks designed to assess mathematical thinking and problem-solving abilities, while the fourth evaluates problem-posing skills. The relevant instrument was presented in a published source and did not require additional permission for research use. Validity of the instrument was supported by aligning the problems with curricular standards, which was further confirmed through expert reviews and pilot testing. All items were translated into Turkish by language experts, and content validity was ensured through the evaluations of three experts in the field of mathematics education. Subsequently, a pilot study was conducted with 150 fourth-grade students across three primary schools. The reliability of the Turkish version was confirmed with a KR-21 value of 0.83, indicating high internal consistency. For the detailed evaluation criteria of the problems, please refer to Appendix 4.

These mathematical tasks were administered as tasks at the end of each cycle. The purpose of assigning these problems after each cycle is to provide qualitative evidence for interpreting the changes in the students' mathematical thinking skills, which result from the Waldorf philosophy-based activities, and to determine the content of the next action plan.

Semi-Structured Interview Form

Semi-structured interviews were conducted to evaluate the changes in students' mathematical thinking skills based on their experiences gained through activities rooted in the Waldorf philosophy. These interviews allowed students to explain their thought processes during the mathematical thinking tasks. Each interview lasted approximately 30 minutes and was audio-recorded. The interview questions were developed in accordance with the research objectives and tailored to the students' cognitive levels (see Appendix 3).

The interview form consists of two main sections: the first section includes an instructional introduction explaining the purpose of the study and confidentiality principles, along with a personal information part for recording the student's age and pseudonym. The second section comprises four open-ended questions focusing on mathematical reasoning.

In terms of language, grammar, and scientific validity, the questions were reviewed by a classroom teacher, a Turkish language teacher, and a mathematics education expert. To test the appropriateness and clarity of the questions, a pilot study was conducted with 13 fourth-grade students from three different schools. The number of questions in the form remained constant at four before and after the pilot application; however, based on the feedback received from the pilot study, the wording of particularly the second and third questions was simplified and revised to ensure better comprehension for fourth-grade students.

Field Notes

Throughout the research process, the practitioner kept field notes to monitor and reflect on the development of the students' mathematical thinking skills. The changes observed during the Waldorf philosophy-based activities were documented as weekly written notes following each activity.

Development and Implementation of Action Plans

In order to create action plans in line with the needs analysis results for the students participating in this study, the Ministry of National Education's mathematics curricula (MoNE, 2018; 2024) and the SAC implementation plan outcomes were examined. Additionally, the literature on methods effective for developing gifted students' mathematical thinking skills was reviewed. Mathematical thinking can be fostered through various approaches such as Science, Technology, Engineering, Mathematics (STEM) education, problem posing, computational thinking, and the use of digital tools (Dick et al., 2022; Gunadi et al., 2023; Lambert et al., 2018; Tran et al., 2017; Wu & Yang, 2022). However, for a special group like gifted students, activities based on the Waldorf philosophy have been found to provide conducive learning environments.

Considering the characteristics of the target group, four activities were designed specifically around play, nature, art, and practical life implementations—elements emphasized by the Waldorf philosophy. Prior to the main implementation, a pilot study was conducted with 13 fourth-grade students from three different schools to test the feasibility and clarity of the research instruments. These students shared similar demographic characteristics with the target group but were not included in the final data collection. This stage focused on assessing the comprehensibility of the tasks and the duration of the interviews.

Following the pilot study, the activities presented in Table 4 were submitted for professional evaluation to two associate professors of mathematics education-one from within the country and one from abroad-who possess advanced knowledge of the Waldorf educational philosophy. Based on their feedback and the insights gained from the pilot study, minor revisions were made to ensure the content validity of the activities. Specifically, these revisions involved simplifying the instructions in some of the open-ended tasks to better suit the cognitive level of fourth-grade students and refining the selection of natural materials to enhance their tactile qualities. Furthermore, the experts suggested strengthening the connection between mathematical concepts and practical life implementations-such as gardening or craft-based measurements-to ensure the activities more authentically reflected the Waldorf pedagogical framework. Through this synthesis of expert evaluations and pilot findings, the activities were finalized to achieve their most effective form. The activities prepared by the researchers were then carried out within the specified timeframes.

Table 4

Implementations and the Planning Process

Implementation	Date
Implementation of the "Square Sequence" problem	March 18–22, 2024
Completion of the Multiplication and Area Calculation Game and the "Hat Average" task	March 25–April 5, 2024
Completion of Geometric Designs and Perspective Calculations, and the "Odd Number Pattern" task	April 8–May 3, 2024
Completion of Fibonacci Numbers and Patterns, and the "Pizza Ratio" task	May 6–24, 2024
Completion of Market Shopping and Picnic Budgeting, and the "Problem-Posing" task	May 27–June 14, 2024

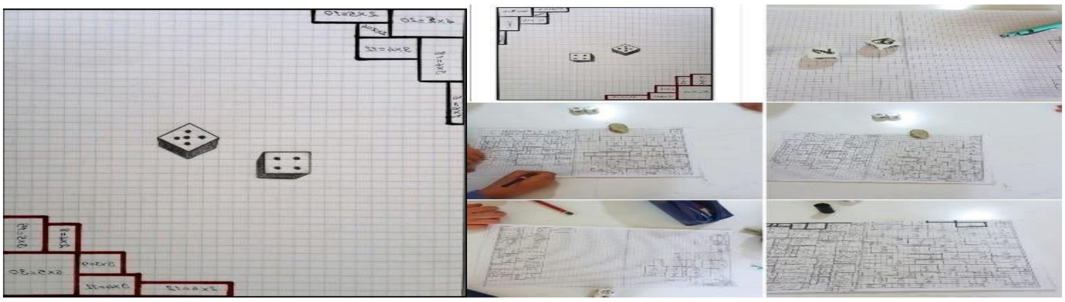
Other details regarding the activities implemented over a 12 week period as part of the action plans, as well as sample images of the implementations, can be presented as follows:

Implementation 1: Mathematics Through Play / Multiplication and Area Calculation Game

The first implementation involves the concept of play, which is one of the fundamental elements of the free and natural learning environments encouraged in the Waldorf philosophy. In this pedagogical approach, play is regarded as an integral part of learning, promoting creativity, critical thinking, and collaboration (Kodsi, 2022; Shank, 2016). The primary objective of this process—during which gifted students calculate the areas of geometric shapes and visualize these areas by drawing patterns on graph paper—is to encourage strategic thinking so that they can utilize their total area in the most efficient manner. Students were expected to aim to exceed the total area of their competitors in an environment where competition and collaboration are balanced.

Figure 2

Multiplication and Area Calculation Game



Following these activities and the completion of the first cycle, the “Hat Average” problem from Cai’s (2003) mathematical thinking test was assigned to the students as a task. The findings obtained from the evaluations of the results and students’ progress guided the planning of the next cycle.

Implementation 2: Art and Mathematics / Geometric Designs and Perspective Calculations

The second implementation serves as an example of the Waldorf philosophy, which integrates mathematical concepts while emphasizing art and experiential learning (Goldshmidt, 2017; Hallam et al., 2015). The activity aimed to strengthen spatial reasoning, creativity, and the understanding of geometric principles through an artistic and practical approach. It enabled gifted students to develop their abstract thinking, creativity, and problem-solving skills, and encouraged them to design complex patterns using geometric shapes. By calculating perspective angles (Figure 3a), the students not only enhanced their spatial reasoning skills but also deepened their understanding of geometric principles, thereby reinforcing various components of mathematical thinking. Furthermore, the students learned that calculating perspective angles can add depth and dimension to shapes in artistic designs (Figure 3b).

Figure 3a

Art and Mathematics Works Created by Students

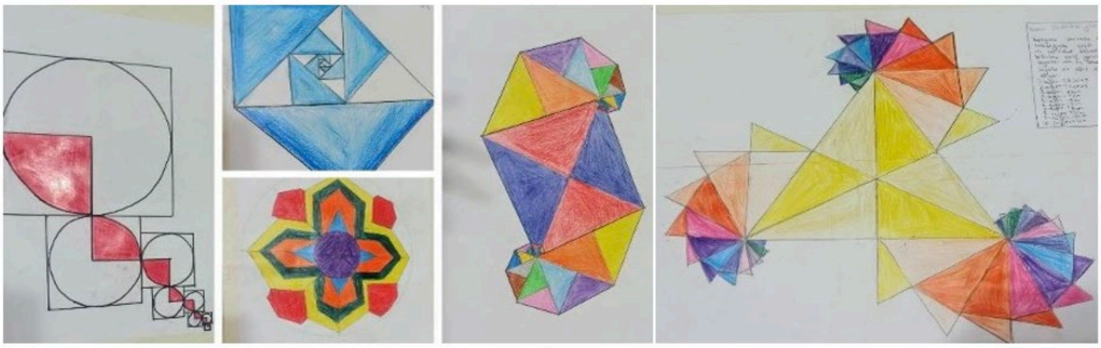
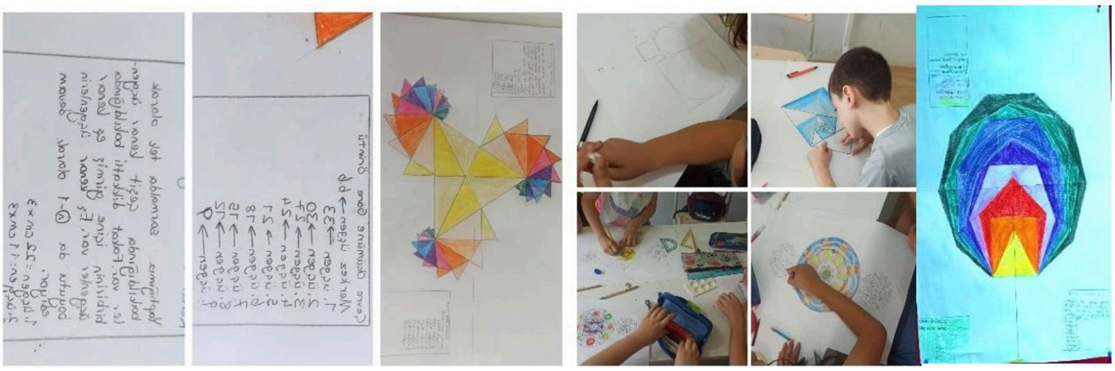


Figure 3b

Mathematical Calculations Related to Students’ Artistic Works



At the end of this cycle, the “Odd Number Pattern” problem from Cai’s (2003) mathematical thinking test was assigned as a task, and the students’ solutions were collected for analysis. The findings obtained from the evaluations of the results and students’ progress guided the planning of the next cycle.

Implementation 3: Mathematics in Nature and the Human Body / Fibonacci Numbers and Patterns

The third implementation was developed based on the emphasis in Waldorf pedagogy (Steiner, 1996; Taplin, 2024) on the holistic relationship that a child establishes with nature and their body. This activity combines the principle of integration with nature and mathematical exploration by focusing on examining the Fibonacci sequence in nature. Gifted students investigated Fibonacci patterns found in various natural objects such as flower petals, tree rings, and snail shells, as well as the golden ratio in the human skeleton. These activities aimed not only to enable a deep understanding of mathematical concepts through real-world implementations but also to contribute to the development of analytical thinking skills and to raise awareness of the relationship between mathematics and nature.

Figure 4

Exploring Mathematics in Nature and Patterns in the Human Skeleton

At the end of this cycle, the “Pizza Ratio” problem from Cai’s (2003) Mathematical Thinking Test was assigned to the students as a task, and their solutions were collected for analysis. The findings obtained and the students’ progress were used to develop and implement the next action plan.

Implementation 4: Mathematics in Daily Life / Market Shopping and Picnic Budgeting

The fourth implementation reflects the practical and hands-on learning approach emphasized in Waldorf education (Rawson, 2024). The activities were integrated into everyday life scenarios-such as market shopping and picnic budgeting-to help students better understand the proportional relationships between the whole and its parts. Gifted students created budgets, planned their expenditures, and visualized these data using graphs (Figure 5). These activities fostered the development of students' critical thinking and data interpretation skills by encouraging them to analyze real-life scenarios through mathematical models and visual representations. This approach enabled gifted students to contextualize mathematical concepts, enhance their thinking skills, and effectively evaluate complex situations.

Figure 5

Market Shopping and Budget Calculation for a Picnic

On the fourth day of this cycle, the "Problem Posing" task from Cai's (2003) Mathematical Thinking Test was administered, and the students' solutions along with the problems they created were collected for analysis.

Data Analysis

The data of the study were examined using content analysis. Content analysis is based on the grouping of similar concepts and themes, which are then organized and interpreted in a manner that is understandable to the reader (Yıldırım & Şimşek, 2018). The data from the problem-solving activity conducted for the needs analysis were subjected to content analysis by both researchers according to the protocol presented in Appendix 1. The analysis results were tabulated (Table 2 and Table 3). Based on the analysis, comparisons were made regarding similarities and differences, and the agreement between coders was calculated. To ensure the reliability of the evaluation process, Cohen's Kappa analysis (κ = 0.98) was performed, indicating a very high level of consensus between the researchers (Cohen, 1960).

To assess the effects of the activities following the implementation of the action plans, the content analysis of the written solutions to the tasks in the "Mathematical Thinking Test" was conducted by the researchers according to the analysis protocol of Mason et al. (2010) (see Appendix 4). This protocol focuses on identifying key processes of mathematical thinking, specifically specializing, generalizing, conjecturing, and convincing. By utilizing this framework, the researchers were able to categorize the students' cognitive transitions and the depth of their reasoning during the tasks. The analysis results were presented in a table (Table 5). Mutual evaluations were carried out for this analysis, and to ensure consistency, Cohen's Kappa statistic (κ = 0.87) was calculated, indicating a strong agreement (Cohen, 1960). At this stage, the triangulation method was applied by evaluating the student solutions, their explanations, and the researchers' observations together; in this way, the integration of different data sources was ensured and researcher bias was minimized (Liamputtong, 2019). For this purpose, the audio recordings of the students' oral explanations were transcribed, and the field notes recorded weekly by the researcher were compiled. These qualitative data sources (transcripts and field notes) were not subjected to independent coding; instead, they were utilized as supplementary evidence to verify, contextualize, and provide deeper insight into the students' written solutions. This approach allowed the researchers to cross-check the students' cognitive processes and ensure that their written performance accurately reflected their mathematical reasoning. Consequently, some direct quotes from the participants' oral and written explanations, along with field notes recorded weekly by the researcher, were presented in conjunction with figure examples. These served as evidence of the students' developmental progress in problem-solving and problem-posing performance. Moreover, to further increase the credibility of the data analysis, an experienced associate professor reviewed the findings, and text excerpts supporting the classifications were added to the analysis tables (Appendix 1 and Appendix 4) (Hallgren, 2012).

Ethical Concerns

In action research, the ethical principles to be considered were classified as follows: preparation of ethical documents (ethics statement, permission request document, and official permission document), negotiations (with school administrators/institution officials, participants, and parents), ensuring confidentiality (of information, data, and identity), safeguarding the participants' right to withdraw from the research, commitment to professional and academic conduct, and protection of well-being (McNiff & Whitehead, 2010). In this study, as part of the ethical precautions, the necessary documents-such as ethics statements, permission letters, and informed consent forms-were prepared and approvals were obtained from the relevant institutions. The research was initiated after obtaining ethical approval from the Selçuk University Ethics Committee (decision number: E.722040, date: 13.03.2024). All stakeholders, including school administration, teachers, students, and parents, were thoroughly informed about the study, and voluntary participation forms were signed. Throughout the research process, all observations were systematically recorded, and the data related to student work were labeled with code names to ensure confidentiality. The student data, coded with different names, were presented to an expert to ensure the reliability of the data analysis. Moreover, to maintain transparency, all data were transferred to a digital environment and stored in an encrypted file for a period of time.

Findings

The main research question formulated to achieve the study's primary aim was: “In light of the students' experiences, how do Waldorf activities develop gifted students’ specializing, generalizing, conjecturing, justifying, and convincing skills?”. At the end of the 12 week process conducted within the framework of Waldorf activities, the findings on the development of gifted students’ mathematical thinking skills are presented in Table 5.

Table 5

Developments in the Mathematical Thinking Skills of Gifted Students

Student	Mathematical Thinking Skills	After Implementation 1 (Hat Average Problem Solving Task)	After Implementation 2 (Odd Number Pattern Problem Solving Task)	After Implementation 3 (Pizza Ratio Problem Solving Task)	After Implementation 4 (Problem Posing Task)
Ali	Specializing	WP	GP	HP	HP
	Generalizing	GP	HP	HP	HP
	Conjecturing	GP	HP	HP	HP
	Justifying and Convincing	WP	WP	WP	GP
Zeynep	Specializing	HP	GP	GP	HP
	Generalizing	HP	GP	GP	HP
	Conjecturing	HP	GP	GP	HP
	Justifying and Convincing	WP	WP	WP	GP
Mehmet	Specializing	GP	GP	HP	HP
	Generalizing	GP	GP	HP	HP
	Conjecturing	HP	HP	HP	HP
	Justifying and Convincing	WP	WP	WP	GP
Elif	Specializing	MP	HP	HP	HP

Student	Mathematical Thinking Skills	After Implementation 1 (Hat Average Problem Solving Task)	After Implementation 2 (Odd Number Pattern Problem Solving Task)	After Implementation 3 (Pizza Ratio Problem Solving Task)	After Implementation 4 (Problem Posing Task)
	Generalizing	HP	HP	HP	HP
	Conjecturing	GP	HP	HP	HP
	Justifying and Convincing	WP	WP	WP	GP

Note. WP :Weak Performance, MP :Moderate Performance, GP:Good Performance, HP:Higy Performance

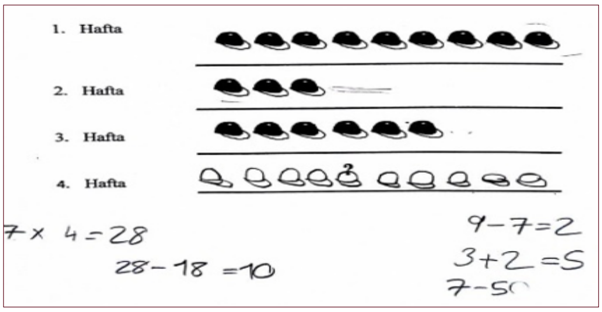
The table 5 provides an overview of the performance levels of four gifted students across four implementations, assessing their skills in specializing, generalizing, conjecturing, and justifying and convincing. Overall, there is a noticeable improvement in most areas over time. Initially, some students exhibited weak performance, particularly in justifying and convincing, but by the final implementation, this category had improved to a good level across all participants. Similarly, the skills of specializing, generalizing, and conjecturing show progressive enhancement from the first to the later tasks. These results suggest that the sequential interventions were effective in fostering the mathematical thinking abilities of the students.

Developments in Specializing Skills

In terms of specializing skills, the most notable development was observed in the increase in students' ability to transfer their concrete experiences with specific examples to mathematical problems in order to reach general conclusions. Ali's performance in the first implementation exemplifies this phenomenon. In solving the first problem (Figure 6), he focused solely on arithmetic operations related to the hat average problem and disregarded the contextual aspects of the problem.

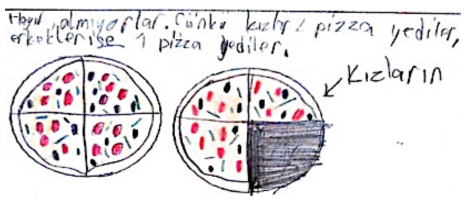
Figure 6

Ali's Problem Solving Performance



However, following the picnic budgeting activities, he was able to utilize this experience to specialize the problem. His statement-"We created similar tables and graphs in our activities. I will follow a similar approach in this problem" -demonstrates that he was able to apply insights gained from previous experiences to understand the new problem situation. This development was also reflected in his solution to the Pizza Ratio problem.

> To solve this problem, I will try several options. First, let me assume that the amount of pizza received by each girl is equal to the amount received by each boy. Accordingly: "2/7 = 1/3" It turned out that this assumption was incorrect, meaning they are not equal. This implies that in the pizza distribution, the girls have to share the fourth slice among themselves. If we represent this visually, it would appear as follows:

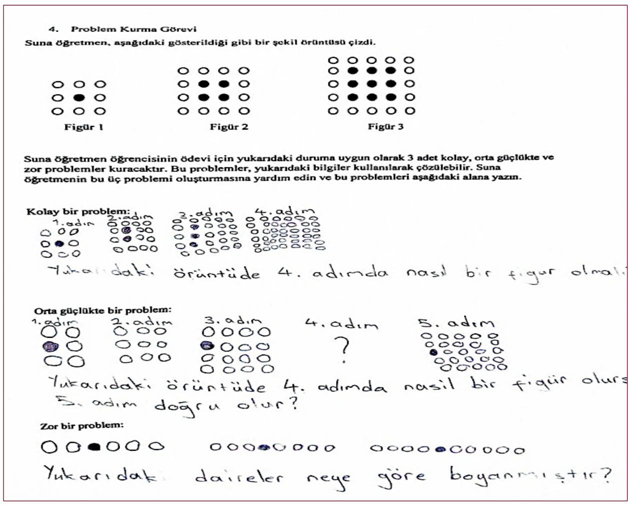


> It is clearly evident that the girls receive less pizza. (Pizza Ratio Task – HP)

Similarly, Zeynep transferred her experience from the budget calculation in the fourth implementation to problem posing by stating, “Actually, when we calculated the budget for the picnic, we were essentially posing a problem. The solution turned out to be easy.” This indicates that the development of her specializing skill influenced her ability to pose problems. At the beginning of the action research process, students were asked to pose a problem in the final step of the “Square Sequence” question. However, Zeynep was not even able to attempt problem posing then. In the problem posing task at the end of the fourth implementation, she demonstrated the performance shown in Figure 7.

Figure 7

Zeynep’s Problem Posing Performance

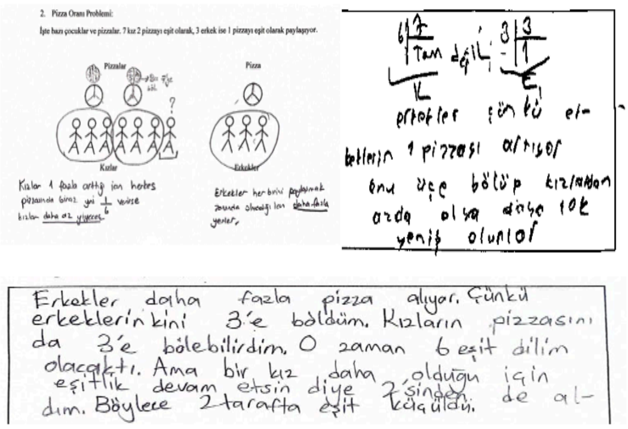


At the end of the third implementation, in the Pizza Ratio problem-solving task, Mehmet was able to develop solutions for the given problem situation by emphasizing the length ratios between the bones in our skeletal structure and the calculation of the golden ratio, thereby exploring different problem scenarios.

> We did an activity about the lengths of our bones and the proportions between them. I can actually solve this by using the example of the golden ratio in our body. There's ratio and proportion involved here. If I look at it this way, I can solve the problem. That way, I would conclude that the girls ate less pizza. (Pizza Ratio Task – HP)

Figure 8

Mehmet’s Problem Solving Performance



These findings support the effectiveness of the mathematics teaching process, underpinned by the Waldorf philosophy, in enhancing students’ specializing skills. Moreover, several statements from the researcher’s notes indicate that the students have made progress in this area. A few examples of these statements are as follows:

> Zeynep's mathematical thinking skills have improved significantly, particularly in integrating art and mathematics. By applying mathematics in artistic contexts and recognizing natural structures, her ability to define problems has been strengthened. Meanwhile, Mehmet's mathematical thinking skills have notably advanced in the context of artistic designs, enabling him to provide quick and creative solutions. Essentially, this demonstrates that his problem comprehension has developed-he is now able to specialize a problem for definition and do so based on correct assumptions.

Developments in Generalizing Skills

The most significant improvement in generalizing skills has been observed in the performance of pattern recognition and formula derivation. Ali's evolution in the odd number pattern problem is particularly remarkable in this regard. The student, who initially struggled to identify the pattern in the square sequence problem, achieved an algebraic generalization such as "y = 2x – 1" in the odd number pattern problem after participating in activities based on the integration of art and mathematics. When asked to explain his solution, he said:

> Let me first check if there is a pattern between the number of bell rings and the guests… Odd numbers cannot be divided by even numbers.
>
> 1.bell = 2 × 1 – 1
>
> 2.bell = 2 × 2 – 1
>
> 3.bell = 2 × 3 – 1
>
> 4.bell = 2 × 4 – 1
>
> It keeps continuing like this. That means it's a formula. We multiply the bell number by 2 and subtract 1. Accordingly, when 99 guests enter, the calculation would be: 2 × 50 − 1 = 99, which means that the bell will ring 50 times. (Odd Number Pattern – HP)

Such explanations indicate that the mathematics teaching process, supported by the Waldorf approach, is effective in developing Ali's generalizing skills. In particular, the statement, "I recalled the patterns in artistic designs… once I noticed that the increment was consistently 2, I established a general rule," highlights the role of interdisciplinary approaches in strengthening mathematical generalization skills.

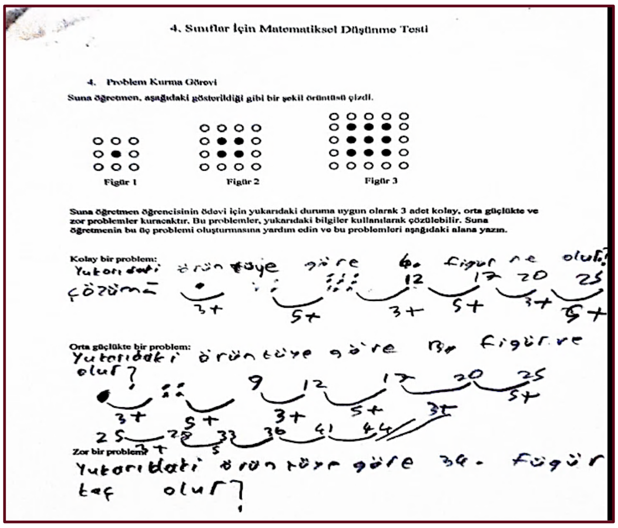
Mehmet's problem posing performance demonstrates the gradual development of his generalization process. Inspired by studies with fractal patterns, Mehmet designed a continuation problem. While constructing his problem, he used expressions such as:

> Just like the Fibonacci spiral in nature. We examined it in plants. The leaves increase by specific numbers. I tried to set up a problem similarly. In art activities, an artist creates new patterns by doubling the number of sides of the previous shape at each step...

Using these ideas, he posed a problem as shown in Figure 9. Notably, before the action research process began, during the Square Sequence task, he had not even attempted to pose a problem. This example illustrates that Mehmet is capable of creatively applying mathematical concepts by generalizing them.

Figure 9

Mehmet's Problem Posing Performance


4. Sınıflar İçin Matematiksel Düşünme Testi

4. Problem Kurma Görevi

Suna öğretmen, aşağıdaki gösterildiği gibi bir şekil örüntüsü çizdi.

Figür 1 Figür 2 Figür 3

Suna öğretmen öğrencisinin ödevi için yukarıdaki duruma uygun olarak 3 adet kolay, orta güçlükte ve zor problemler kuracaktır. Bu problemler, yukarıdaki bilgiler kullanılarak çözülebilir. Suna öğretmenin bu üç problemi oluşturmasına yardım edin ve bu problemleri aşağıdaki alana yazın.

Kolay bir problem:

Orta güçlükte bir problem:

Zor bir problem:

Some of the researcher's field notes also indicate improvements in generalizing skills. For instance, it was noted that "Elif was able to define broader phenomena from specific cases through the reasoning she demonstrated in multiplication and area calculations."

Developments in Conjecturing Skills

The development of the students' skills in formulating assumptions has become evident in the processes of hypothesizing and testing. In her third implementation task, Zeynep asked, "Do the number of leaves on plants conform to this rule?" regarding the Fibonacci sequence. This question, which involves making assertions, indicates the advancement of her scientific inquiry skills. This situation demonstrates how the natural curiosity of gifted students supports their processes of mathematical discovery.

Mehmet's approach to the hat average problem clearly demonstrates the impact of the multiplication and area calculation game in the first implementation on his mathematical conjecturing abilities. Drawing on the experiences he acquired during the game, Mehmet formed the basic conjecture that calculating an average requires an even distribution while solving the problem. He transformed this conjecture into concrete strategies by stating, "If I know the total number of bell rings (7×4=28), then I can reach the solution… I need to perform subtraction to find the missing one (28–18=10)." His remark, "I tried different approaches at first, but as I learned in the game, I realized that the most logical solution is to calculate the total and then divide equally," demonstrates that he successfully applied the conjecture-testing skills he developed during the game to this mathematical problem. This evidence provides concrete support for the effectiveness of game-based learning environments, as part of the Waldorf-supported mathematics teaching process, in enhancing students' abilities to form strategic conjectures and systematically test them in mathematical problem solving.

Ali's development in the ability to form assumptions is a result of the activities based on the integration of art and mathematics in the second implementation. His approach in the odd number pattern problem clearly demonstrates this progress: his statement, "Every time the bell rings, two more people arrive; it seems to form a pattern," shows that he formed an assumption based on his observations. Using the experiences he acquired during the design of geometric patterns, Ali predicted that the numerical sequence would continue as "1, 3, 5, 7," and he transformed this assumption into a concrete hypothesis by stating that "the bell number is twice the number minus one." This process illustrates how the pattern analysis skills he developed in artistic activities paved the way for creating assumptions about numerical sequences. It is evident that while forming this assumption, Ali referred to the recurring structures in the geometric motifs he had encountered in previous activities.

As noted in the researcher's notes, Ali discovered the relationship in the numerical sequence by recalling the reduction ratio in artistic fractals (n/2), while Elif remarked, "In perspective, all lines converge at a single point. In this numerical sequence (1, 3, 5, 7...), all increases follow a single rule: two is added each time. Just as lines converge at a single point, these numbers always adhere to the same mathematical rule." This explanation demonstrates that she applied the art concept of "everything has an order" to mathematics as well. These statements provide evidence of how the interdisciplinary approach enriched their ability to form assumptions.

Limitations in the Development of Justifying and Convincing Skills

Throughout the study, the development of students' justifying and convincing skills remained slower and more limited compared to other components of mathematical thinking. For instance, in the solution of the hat average problem during the first implementation, it was observed that students' abilities to justify their answers were quite restricted. Ali reached the correct result by performing the calculation (7×4=28), yet he was unable to explain why he chose that particular mathematical operation. Most students were content with superficial statements such as "Because it needs to be done that way," and they failed to present their solution strategies in a logical manner.

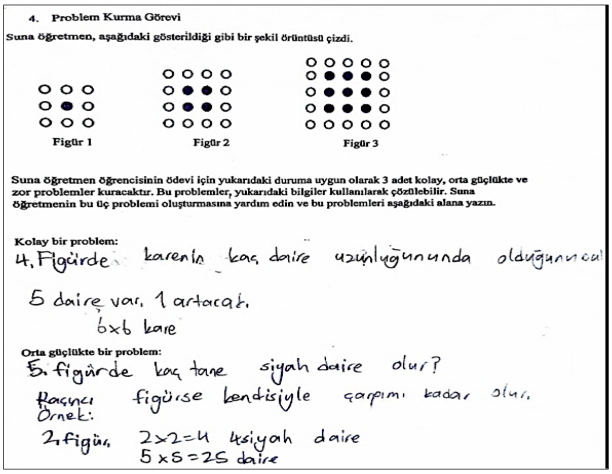
In the odd number pattern problem during the second implementation, students succeeded in identifying the pattern; yet, they struggled to justify the underlying rule. Mehmet remarked that "It goes as 1, 3, 5, 7," but he could not explain why this pattern conforms to the rule "2x – 1." Similarly, although Zeynep correctly extended the pattern, her explanation did not go beyond general statements like "this rule always applies."

In the pizza ratio problem solutions from the third implementation, students performed the calculations correctly, yet they did not present their results in a convincing manner. Elif accurately calculated the pizza distribution between girls and boys but was satisfied with a simple explanation such as "I shared it equally." None of the students provided detailed explanations of their calculation methods or clarified why they selected a particular approach.

Moreover, instances of slow and limited development were also observed. In the fourth implementation, when students were asked to transform a visual pattern containing one black square in Figure 1, four in Figure 2, and nine in Figure 3 into a mathematical problem, they demonstrated more advanced justifying skills compared to previous cycles. Ali, moving beyond the simple proposition that "The number of squares in the nth figure is n^2," formulated a problem that linked the validity of this rule to the increase in side lengths and added an additional inquiry: "In which figure does it become equal to its own product?" (Figure 10).

Figure 10

Ali's Problem Posing Performance


4. Problem Kurma Görevi

Suna öğretmen, aşağıdaki gösterildiği gibi bir şekil örüntüsü çizdi.

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Kolay bir problem:

4. Figürde karenin kaç daire uzunluğununda olduğunu bul

5 daire var. 1 artacak.

6x6 kare

Orta güçlükte bir problem:

5. figürde kaç tane siyah daire olur?

Kaçıncı figürse kendisiyle çarpımı kadar olur.

Örnek:

2. figür, 2x2=4 4 siyah daire

5x5=25 daire

While adapting the pattern to computer game levels, Mehmet developed a problem structure that, by eliminating possible alternative patterns (such as an arithmetic sequence like 2, 5, 8), opened a discussion on why square numbers are more appropriate. As seen in the examples based on problem posing, it is evident that students are now capable of constructing problems that emphasize not only the final result but also the underlying mathematical processes and logical connections. It can be argued that problem posing tasks have contributed to the development of the, albeit limited, justifying skills of gifted students. In particular, the inclusion of “why” and “how” questions in the problems they formulated reflects this maturation in the depth of their mathematical thinking.

Discussion

Changes in Specializing, Generalizing, and Conjecturing Skills

This action research demonstrated that by harnessing the concepts of play, art, nature, and practical implementations emphasized in the Waldorf educational approach, gifted students’ mathematical thinking skills-specifically in specializing, generalizing, and conjecturing-experienced significant development.

The solutions and explanations provided by Ali and Mehmet for the “Pizza Ratio” task (Figures 6 and 8), as well as Zeynep’s performance on the “Problem Posing” task (Figure 7), demonstrate that the problem-solving experiences they acquired through real-life implementations (picnic budgeting) and nature-based content enabled them to apply these experiences more effectively to new problem situations. In other words, through these tasks, the students showed improvements in their ability to develop solutions based on different examples—a skill defined as specializing—which in turn reinforces their capacity to understand new problem situations. These findings support Rawson’s (2024) conclusion that linking everyday life with academic content through the Waldorf approach provides students with meaningful learning experiences. Moreover, exposure to activities integrated with nature has been shown to promote progress in logical reasoning and strategy formation (Taplin, 2024). Field note analyses further reveal that the students’ integration of art and mathematics played a significant role in the development of their specializing skills. This supports the idea that various activities, such as exploring natural patterns and participating in artistic tasks, contribute to the multifaceted problem-solving and problem-posing abilities of gifted students (Piske & Stoltz, 2021). In fact, these types of activities designed to enhance mathematical thinking have increased the students’ abilities to analyze specific situations and foster their productivity through personalization, thereby boosting their capacity to analyze and simplify complex problems (Mason et al., 2010).

Ali’s progress in solving the “Odd Number Pattern” problem indicates that his generalizing skills have begun to meet the expectations. In this solution, he was able to translate his conjectures into a mathematical expression (equation). Similarly, in Mehmet’s problem-posing task, it is evident that the mathematical activities conducted in both nature- and art-based contexts have contributed significantly to the development of his generalizing skills (Figure 9). In other words, Mehmet was able to generate similar problems by generalizing from examples found in the Fibonacci spiral and artistic activities. This demonstrates that the integration of creative and aesthetic teaching practices, such as those in art, can enhance a child’s creative potential in mathematics (Huchingson & Huchingson, 1993; Goldshmit, 2017; Taplin, 2024). According to the researcher’s field notes, the behavior observed in Zeynep during the multiplication and area calculation game—where she reached broader conclusions through reasoning—can also be interpreted as an improvement in generalizing skills. Indeed, Kodsi (2022) and Shank (2016) report that mathematical games promote multifaceted development in students’ mathematical abilities. Therefore, it can be assumed that a portion of the improvements in mathematical thinking skills is attributable to mathematical games.

Zeynep’s inquiry at the end of the professional integrating mathematics with nature and the human body through Fibonacci numbers and patterns, Mehmet’s hypotheses following the multiplication and area calculation game, and Ali’s conjectures after the art and mathematics integration activities—all of which were transformed into hypotheses and further developed—can be regarded as indicators of the growth in their conjecturing skills (Brown & Stillman, 2017; Komatsu et al., 2019; Mason et al., 2010). Moreover, the researcher’s field notes indicate that as Ali and Elif tested their hypotheses using various conjectural approaches, they demonstrated an increased capacity to identify relationships among the variables and make more systematic generalizations. All these findings demonstrate that artistic activities foster progress in logical reasoning and strategy formation. Additionally, the success of holistic learning experiences obtained through artistic and creative presentations in subjects such as mathematics, history, or science serves as further evidence of their effectiveness (Aljabreen, 2020; Lutzker, 2024). Furthermore, although Mehmet initially struggled to understand the problem and establish relationships among data, after the gamified implementation he was able to correctly formulate mathematical relationships in the hat average problem, generalize them, analyze patterns, and create a general rule. This can be cited as evidence of his development in hypothesis formation.

Overall, these findings indicate that the Waldorf philosophy nourishes the unique talents of gifted students and enhances their intrinsic commitment to learning, as reflected in their mathematical thinking skills. The differences observed before and after the intervention confirm the effectiveness of this intervention, as demonstrated in previous studies (Huchingson & Huchingson, 1993; Piske & Stoltz, 2021).

Challenges in the Process of Justifying and Convincing

This study also revealed that there was at least limited progress in the skills of justifying and convincing. However, this development was relatively smaller compared to other mathematical skills. For justification, after the implementations, Ali defended his solution for the hat average problem with a limited perspective by stating, "It needs to be done that way." In the pizza ratio problem, Elif stated that the distribution was equal but encountered difficulties in explaining the reasoning behind this equality. The challenges that students face in mathematical justification and convincing are not unique to this study; they are frequently highlighted in the literature (Evans et al., 2022; Stylianides et al., 2022). Research shows that even when students are successful in problem solving, they struggle to justify and formally validate their solutions (Mora et al., 2022). This issue is observed among the general student population as well as among gifted students in this study.

There may be several reasons for the limited development observed in the skills of justification and convincing. First, the literature emphasizes that justification and proof are among the most challenging components of mathematical reasoning (Mora et al., 2024; Stylianides et al., 2022). Although gifted students are generally proficient in rapid problem-solving and pattern recognition, they often experience difficulty in systematically validating their reasoning and constructing mathematical arguments (Leikin, 2011). More often than not, they rely on intuitive or experiential approaches to support their results and do not sufficiently focus on formal proof processes.

Second, the pedagogical nature of the Waldorf approach may have influenced these outcomes. Waldorf pedagogy is highly effective in promoting experiential learning, imagination, and holistic thinking; however, it tends to prioritize intuitive understanding over formal logical structures (Goldshmidt, 2017). As a result, students may not frequently encounter activities that require systematic justification or proof. This finding aligns with prior research suggesting that in art- and play-based learning environments, intuitive or aesthetic approaches may be emphasized at the expense of formal proof processes (Komatsu & Jones, 2022; Oberski et al., 2007).

Third, the internal motivation of students to justify their solutions may serve as an important mediating factor. Unless students feel the need for proof or justification to support their solutions, their ability to develop convincing arguments remains weak (Komatsu & Jones, 2022; Lannin, 2005; Mora et al., 2022). Lannin (2005) noted that students actively engage in justification processes only when they feel the need to convince others or to clarify their own thinking. Although the Waldorf-based activities in this study provided opportunities for exploration and expression, they may not have sufficiently created a sense of cognitive conflict or need that would prompt students to engage in justification processes.

Finally, although the students in this study were identified as gifted, their age (9–10 years) may not yet correspond to the cognitive developmental stage required for high-level skills such as abstract reasoning, logical thinking, and structured proof. According to Piaget's theory of cognitive development, children typically enter the formal operational stage around the age of 11, during which abilities such as hypothesizing, systematic testing, and drawing logical conclusions begin to emerge (Inhelder & Piaget, 1958). Therefore, students in this age group may not yet be cognitively prepared for formal processes such as mathematical justification and proof. Leikin (2011) also emphasized that although gifted students may possess advanced skills in pattern recognition and generating solutions, they may still face difficulties in formal proof and reasoning processes due to their developmental stage. In this context, it can be concluded that the limited progress observed in the study may be related not only to the pedagogical approach but also to the students' level of cognitive development.

Nevertheless, Ali's written statements and verbal explanations in his problem-posing task (Figure 10), along with Mehmet's incorporation of "why" and "how" questions in the problems he created, indicate that there was some development in their justifying and convincing skills. Therefore, the limited success in fully developing this skill may be attributed to the students' frequent resistance to the processes of justification and convincing. This situation underscores the necessity for instructional approaches that assist students in explaining why and how they apply mathematical concepts. In mathematics education, including for gifted students, expanding proof and validation processses is essential for the advancement of mathematical thinking. This situation suggests that such skills can be improved through targeted instructional strategies such as structured proof-writing activities, opportunities for peer discussion, and step-by-step scaffolding of justification processes (Ball et al., 2003; Kurniawan et al., 2022).

Recommendations

Building on these findings, several pedagogical recommendations can be proposed. First, problem-solving and problem-posing tasks should be designed in ways that require students to justify their solutions and explain their reasoning, thereby addressing the relative weakness observed in justifying and convincing skills. Second, greater emphasis may be placed on integrating mathematics with art in order to make abstract concepts more concrete and to strengthen students' representational and conceptual understanding through drawings, models, and other artistic forms. Third, the connection between mathematics and everyday life may be reinforced through thematic, real-life activities, such as shopping or recipe measurement tasks, particularly when teaching concepts such as ratio and proportion. Finally, professional development opportunities may help equip educators with the pedagogical tools needed to implement Waldorf-inspired and experiential mathematics teaching practices more effectively, thereby fostering gifted students' creativity and mathematical thinking.

Limitations

When interpreting the findings of this study, several limitations must be taken into account. The research employed Mason et al.'s (2010) classification of mathematical thinking skills. While this structured approach is useful, employing different categorizations or theoretical perspectives could yield different results in terms of solutions and verbal expressions, thereby affecting cross-study comparability. The study predominantly focused on pattern recognition and proportional reasoning, and it remains unclear whether similar methods would be equally effective for other mathematical topics. Future research should aim to comprehensively understand the impact of the Waldorf approach by covering a broader range of mathematical concepts. Additionally, the small sample of gifted students limits the generalizability of the findings, and studies involving larger and more diverse samples would enhance the robustness of the results.

Conclusion

Taken together, the findings suggest that the Waldorf philosophy can support the development of gifted students' mathematical potential and enhance their engagement with learning, as reflected in the development of their mathematical thinking skills. The clearest gains were observed in specializing, generalizing, and conjecturing, and the differences identified before and after the intervention indicate the potential effectiveness of the intervention, in line with previous studies (Huchingson & Huchingson, 1993; Piske & Stoltz, 2021).

At the same time, progress in justifying and convincing remained comparatively limited. This pattern suggests that although Waldorf-based, art-, play-, and nature-oriented learning environments are effective in supporting intuitive, creative, and relational forms of mathematical thinking, they may not, by themselves, be sufficient for the full development of formal justification and proof-related competencies.

Accordingly, mathematics education for gifted students may benefit from combining rich experiential learning opportunities with more explicit support for explanation, justification, and proof. Structured proof-writing activities, opportunities for peer discussion, and step-by-step scaffolding of justification processes may be particularly valuable for extending the gains observed in this study (Ball et al., 2003; Kurniawan et al., 2022).

CRediT authorship contribution statement

N. Demirci: Conceptualization, Methodology, Validation, Formal Analysis, Investigation, Resources, Writing – Original Draft, Writing – Review & Editing; E. Ergül: Conceptualization, Methodology, Validation, Formal Analysis, Writing – Original Draft, Writing – Review & Editing, Supervision.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

Ethics Approval and Consent to Participate

This research obtained ethical committee approval through decision number E.722040, adopted at the Selçuk University Ethics Committee meeting on March 13, 2024.

Declaration of AI Usage Statement

The authors affirm that no AI tools were employed in the preparation of this article.

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Appendices

Appendix 1: Square Sequence Problem – Mathematical Thinking Performance Assessment Criteria

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Specializing	Lists the numbers but cannot see the pattern. Example: "1, 3, 6, it goes like that."	Notices the amount of increase. Example: "2, then 3, then 4 is being added."	Calculates all terms correctly. Example: "6th number: 15+6=21."	Performs concrete modeling. Example: "I made a triangle with 21 Lego pieces."
Generalizing	Cannot explain the rule. Example: "I don't know."	States the rule in simple language. Example: "We add one more each time."	Expresses the rule as a formula. Example: "n×(n+1)÷2."	Explains the formula in different ways. Example: "Like half of a rectangle."
Conjecturing	Cannot make a prediction. Example: "?"	Predicts the next term. Example: "The next should be 21."	Predicts more than one term. Example: "7th number 28, 8th number 36."	Discovers the characteristics of the pattern. Example: "Odd-even numbers alternate."
Justifying and Convincing	Cannot explain the solution. Example: "It just came to my mind."	Shows the calculations. Example: "I did 15+6=21."	Solves using more than one method. Example: "Both addition and the formula yielded the same result."	Establishes mathematical reasoning. Example: "Why does the formula work?"
*Problem-Posing	The student only asks for the next step in the pattern. Example: "What number comes after 1, 3, 6?"	The student applies the pattern to a simple real-life situation. Example: "In a market, on Monday 1, on Tuesday 3, on Wednesday 6 customers arrived. How many customers will arrive on Thursday?"	The student integrates the pattern into a complex problem situation. Example: "In a garden, each day one more tree than the number added the previous day is planted. If on day 1, 1 tree is planted, on day 2, 3 trees, on day 3, 6 trees, how many trees will there be in total by the end of day 5?"	The student creates an original and creative scenario by linking the pattern to different mathematical concepts. Example: "A software developer adds as many new features to his implementation each day as the number of features added the previous day. On day 1, 1 feature is added; on day 2, 3 features; on day 3, 6 features. a) How many features will be added on day 7? b) Why is this pattern different from the Fibonacci sequence? c) Explain the advantages of this growth model."

Note. *The problem posing task has been added to the table in Appendix 1 to maintain the integrity of the analysis, and it is a concept separate from the mathematical thinking skills outlined by Mason et al. (2010).

Appendix 2: Mathematical Thinking Test

The Hats Averaging Problem

Alya is selling hats for the Mathematics Club. This Picture shows the number of hats Alya sold during the first three weeks. How many hats must Alya sell in Week 4 so that the average number of hats sold is 7? Show how you found your answers.

The Odd Number Pattern Problem

Ahmet is having a party.

- The first time the doorbell rings. 1 guest enters.
- The second time the doorbell. 3 guests enter.
- The third time the doorbell rings. 5 guests. Enter.
- The fourth time the doorbell rings. 7 guests enter.

Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that enteredon the previous ring.

How many guests will enter on the 10th ring?

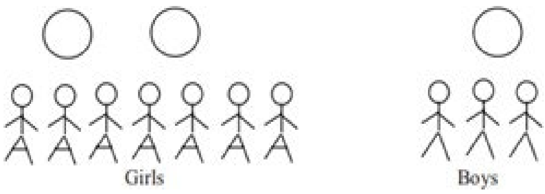
In the space below, write a rule or describe in words how to find the number of guests that entered on one each ring.

99.guests entered on one of the rings. What ring was it? Explain or show how you found yours answers.

The Pizza Ratio Problem

Here are some children and pizzas. 7 girls share 2 pizzas equally and 3 boys share 1 pizza equally.

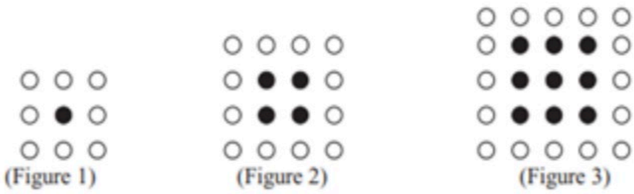
Does each girl the same amount as each boy? (Explain or show how you found your answers.),



If each girl does not get the same amount as each boy, who gets more? (Explain or show how you found your answers.)

The Problem Posing Task

Mr Aylin drew the following figures in a pattern, as shown below.



For this student's homework, he wanted to make up three problems based on the situation: an easy problem, a moderate problem, and a difficult problem. The problems can be solved using the İnformation in the situaiton.

Help Ms Aylin make up three problems and write these problems in the space below.

- The Easy problem:
- The Moderately problem:
- The Difficult problem:

Appendix 3: Semi-Structured Interview Form

Section 1: Instructions and Personal Information

Hello! I would like to chat with you a bit about the math problems we worked on today. The purpose of this interview is to better understand how you think and how you solve problems. This is not a test; it doesn't matter if your answers are 'right' or 'wrong.' What is truly valuable to me is your thought process. Our conversation will last approximately 30 minutes, and I will be audio-recording what you say so that I don't forget any details. These recordings will be used only for research purposes, and your identity will be kept strictly confidential. Thank you for participating!

Student Pseudonym (Rumuz):

Age:

Date:

Section 2: Interview Questions

1.What are your thoughts on the problem? How does it relate to other problems you have encountered previously?

2.Can you elaborate on the method you used to solve the problem? Please discuss the strategies you employed, the solution methods applied, and your thought process during the resolution.

3.Do you have an alternative approach for solving the problem? If so, could you describe your suggested solution? If not, please explain your reasoning.

4.Is the solution you found for the problem valid? How can you assess its correctness?

Appendix 4: Evaluation Criteria for the Mathematical Thinking Performance of Subsequent Implementation Tasks

The Hats Averaging Problem

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Specializing	Applies the concept of "average" without truly understanding it. Example: "If Alya sells more hats, the average will increase."	Understands basic concepts but omits important details. Example: "It seems that Alya will sell more."	Correctly understands most aspects of the problem analysis. Example: "A total of 15 hats were sold, and 10 are needed in week 4."	Conducts a comprehensive analysis in every aspect. Example: "18 hats were sold in the first 3 weeks, and 10 are necessary in week 4."
Generalizing	Does not propose a general solution. Example: "Since there are few hats, everyone buys few."	Makes partial generalizations. Example: "There should be an equal distribution, but I mixed up the numbers."	Identifies relationships between examples. Example: "28 divided by 4 equals 7, which gives the average."	Clearly identifies all relationships within the problem. Example: "28 hats over 4 weeks yield the average."
Conjecturing	Cannot form correct hypotheses. Example:	Offers unsubstantiated hypotheses. Example: "It	Forms valid hypotheses supported by analysis.	Forms hypotheses that are well-supported by logical analysis.

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
	"Maybe everyone buys the same amount."	seems that Alya's sales are increasing."	Example: "10 hats should be sold in week 4."	Example: "The total should be 28."
Justifying and Convincing	Lacks proper verification during the control phase. Example: "I did not check the results."	Follows logical steps but overlooks some errors. Example: "My calculations might not be completely accurate."	Provides a logical approach with verification. Example: "I checked the calculations; it should be 10."	Meticulously verifies all results. Example: "I checked all the calculations."

The Odd Number Pattern Problem

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Specializing	Does not understand basic concepts. Example: "Only 1 guest arrived at the first bell; I did not count the others."	Partially recognizes the pattern. Example: "1, then 3, then 5 guests."	Correctly analyzes most aspects. Example: "At the 10th bell, 19 guests arrive."	Conducts a comprehensive analysis. Example: "The number increases by 2 at each bell, so at the 10th bell, 19 guests arrive."
Generalizing	Does not propose a general solution. Example: "I don't know how many people arrived at the other bells."	Partial generalization. Example: "It increases by 2 each time."	Defines the pattern. Example: "It goes as 1, 3, 5, 7…."	Describes all relationships in the problem. Example: "The number of guests at the nth bell equals 2n-1."
Conjecturing	Incorrect hypotheses. Example: "The same number of guests should come every time."	Unsupported hypotheses. Example: "It increases by 2, but I can't keep up as the numbers get bigger."	Forms valid hypotheses supported by analysis. Example: "The number of guests at the nth bell is 2n-1."	Forms hypotheses supported by logical analysis. Example: "For the 100th bell, 199+1=200, 200/2=100."
Justifying and Convincing	Lack of verification. Example: "I did not check the results."	Logical steps but with errors. Example: "The results should be correct."	Logical approach with verification. Example: "I found 99 for the 50th bell, which is correct."	Provides rigorous verification. Example: "I checked all the steps."

The Pizza Ratio Problem

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Specializing	Does not understand the basic concept of sharing. Example: "Girls get more pizza because there are many of them."	Recognizes inequality but omits details. Example: "Girls will take more, but I don't know how."	Correctly analyzes most aspects. Example: "Girls take 2/7 and boys 1/3 of the pizza."	Conducts a comprehensive analysis. Example: "Each girl gets 2/7 and each boy 1/3 of the pizza."
Generalizing	Does not propose a general solution. Example: "Because there is little pizza, everyone eats little."	Partial generalizations. Example: "Equal sharing, but what about the last slice?"	Identifies relationships between examples. Example: "Since 2/7 < 1/3, girls get less."	Defines all relationships in the problem. Example: "Since 1/3 > 2/7, boys get more."
Conjecturing	Vague hypotheses. Example: "Maybe everyone takes an equal amount."	Unsubstantiated hypotheses. Example: "It seems that girls eat more."	Valid hypotheses and analysis. Example: "Girls are taking less pizza."	Hypotheses supported by logical analysis. Example: "Because there are more girls, their share is smaller."

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Justifying and Convincing	Lack of verification. Example: “I did not check the results.”	Logical steps but with errors. Example: “My calculations might not be completely accurate.”	Logical approach with verification. Example: “I checked the calculations; girls get less.”	Rigorous verification. Example: “I checked all the calculations.”

The Problem Posing Task

Mathematical Thinking Skills	Weak Performance	Moderate Performance	Good Performance	High Performance
Specializing	Does not recognize the basic pattern. Example: “I see the numbers 1, 4, 9, but I couldn’t establish the relationship.”	Partially recognizes the pattern. Example: “The numbers 1, 4, 9 look like square numbers.”	Analyzes the pattern correctly. Example: “It goes as $1^2=1$, $2^2=4$, $3^2=9$.”	Conducts an in-depth analysis. Example: “Square numbers increase geometrically.”
Generalizing	Cannot create a general rule. Example: “The numbers are increasing, but I can’t explain how.”	Makes a simple generalization. Example: “In each figure, the square of the previous number is taken.”	Expresses the mathematical rule. Example: “In the nth figure, there are n^2 squares.”	Expresses the rule in different ways. Example: “$n\times(n+0)=n^2$.”
Conjecturing	Makes random guesses. Example: “The next number could be 12.”	Provides limited hypotheses. Example: “The 4th figure should have 16 squares.”	Makes logical hypotheses. Example: “The 5th figure will have 25 squares.”	Makes comprehensive hypotheses. Example: “This rule is valid for all positive integers.”
Justifying and Convincing	Cannot explain the solution. Example: “It just came to my mind.”	Offers a partial explanation. Example: “Because $4\times4=16$.”	Provides a logical explanation. Example: “The square of the side length is taken.”	Provides a mathematical proof. Example: “The area of a square with side length n is n^2.”