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ANALYZING ALGEBRAIC THINKING USING "GUESS MY NUMBER" PROBLEMS

Barba Patton
Assoc. Prof., University of Houston-Victoria, USA
pattonb@uhv.edu

Estella De Los Santos
Assoc. Prof., University of Houston-Victoria, USA
delossantose@uhv.edu

The purpose of this study was to assess student knowledge of numeric, visual and algebraic representations. A definite gap between arithmetic and algebra has been documented in the research. The researchers' goal was to identify a link between the two. Using four "Guess My Number problems, seventh and tenth grade students were asked to write numeric, visual, and algebraic representations. Seventh-grade students had significantly higher scores than tenth-grade students on visual representation responses. There were no significant differences between the seventh and tenth grade students' responses on the numeric and algebraic representation. The researchers believed that the semi-concrete and visual models, such as used in this study, may provide the link between numeric and algebraic concepts for many students.

Key Words: algebraic thinking, adolescent studies, learning, numeric, visual and algebraic representations

INTRODUCTION

The National Council of Teachers of Mathematics (NCTM) has set expectations for middle and high school algebra. "In grades 6-8 all students should represent, analyze, and generalize a variety of patterns with tables, graphs, words, and when possible, symbolic rules" (NCTM, 2000, p. 222). In the United States (U. S.), students are in the seventh grade at about the ages of 12 and 13. The typical U. S. middle school would have students only in grades 6-8. Piaget's cognitive development theory suggests most of these students would be in a semi-concrete stage which is the stage of cognitive development between the concrete (hands-on) and the abstract (Fitch, 2001; Ormrod, 2003; Santrock, 2005; Santrock, 2008; Babal, 2009). Shayer's and Adey's (1981) findings suggested that poor achievement in high school science for the majority of students be attributed to the fact that the students had not yet reached Piaget's abstract or formal stage of development. In the U. S., students in the tenth grade would be about 15-16 years of age and should be in the abstract stage of cognitive development however, the literature is creating doubt this is in truth an actuality. A typical U. S. high school would have students in grades 9-12. In the 2000 NCTM standards it is stated, "In grades 9-12 all students should use symbolic algebra to represent and explain mathematical relationships" (p.296).

Lee (2001) and Blanton and Kaput (2005) advocated that students should be involved in algebraic activities which involve working with a variety of algebraic materials. They further believe that students should be able to use modeling and problem solving using an assortment of algebraic methods. Popper (1979, 1992) believed that learning takes place when there is a problem and one attempts to solve it, survives thus creating changes in the world and in the learner. The learner may not even be aware that the changes have taken place.

Students seem to do well in arithmetic, but experience difficulties with algebraic concepts. The NCTM (2000) advocates, that connections be made between arithmetic and algebraic concepts. All too long students have relied on memorized facts and algorithms to solve lower-level thinking problems. When algebra is introduced, the student is expected and required to use higher level thinking skills. Many times students are expected to build their own bridge in connecting the arithmetic and the algebraic concepts.

The purpose of the study was to analyze students' knowledge of numeric, visual, and algebraic representations. Four "Guess My Number" type problems were used to analyze seventh- and tenth-grade students' knowledge of the three representations. The research questions investigated were:

RQ1: Is there a significant difference between seventh-grade and tenth-grade students' knowledge of numeric representations in algebra?

RQ2: Is there a significant difference between seventh-grade and tenth-grade students' knowledge of visual representations in algebra?

RQ3: Is there a significant difference between seventh-grade and tenth-grade students' knowledge of algebraic representations in algebra?

The researchers studied numeric, visual, and algebraic representations for four "Guess My Number" problems. The numeric representations were numbers and computations used to solve the given problems. Visual representations were semi-concrete models of algebraic expressions. A sketch of a square was used to represent a number and tally marks were used to represent additions to that number. The algebraic representations required use of variables, operations on variables and the ability to simplify algebraic expressions.

Review of Literature

Algebraic thinking is defined as “the use of any of a variety of representations that handle quantitative situations in a relational way” (Kieran, 1996, p. 4). Swafford and Langrall (2000) promoted algebraic thinking as the ability to operate on an unknown quantity as if the quantity was known, in contrast to arithmetic reasoning which involves operations on known quantities. Driscoll (1999) said that algebraic thinking could be thought of as the “capacity to represent quantitative situations so that relations among variables become apparent” (p.1). The researchers’ purpose was to analyze the use of numeric, visual and algebraic representations for the quantitative situations that were presented to the students.

The success of using multiple representations to teach algebraic concepts has been documented in the research. Allsop (1999) found eighth-grade students, at-risk of failing mathematics, learned how to solve one-variable equations through a combination of instruction methods. Methods used were direct instruction, specific learning strategies, and problem solving. The instruction was developmentally appropriate and followed Piagetian methods by beginning with concrete, then representational and finally abstract. It was concluded use of multiple representations helped students develop problem-solving abilities or higher level thinking skills (Piez & Voxman, 1997).

Yerushalmy and Shternberg (2001) demonstrated the use of visual representation as a model for building the symbolic representation when learning the algebraic concept of *function*. They concluded this type of learning formed a strong foundation for students’ knowledge of the concept of function. Smith and Phillips (2000) found middle school students had strong algebraic knowledge in the areas of linear functions and constant rate of change. They also found the students could analyze functions in tabular, graphical, and symbolic representations; analyze with the aid of a graphing calculator; understand equivalent representations; and understand beginning exponential and quadratic relationships.

Chappell and Stutchens (2001) used concrete models in the algebra classroom to create connections when teaching polynomial concepts. Crossfield (1997) found concrete models helped second-year algebra students to improve number sense. Students were able to make connections between concrete observations, numeric evidence, and patterns.

According to Nathan and Koedinger (2000), informal strategies provided intuitive learning that led to student understanding of formal methods when solving equations. One advantage of using multiple strategies is that the weakness of one strategy may be offset by the strengths of another.

Algebra has been in existence since 1800 BC; however, it has not been in common practice until about the last three to four hundred years. Algebra emerged later than arithmetic, thus ushering in a belief that since arithmetic was earlier, it should be taught earlier. As a result of this, many believe that algebra requires the student to be developmentally ready. The concept of “developmentally ready” relates to the thought that, until a certain stage of development, a student cannot use ideas and representations which are typically found in algebra. However, Carraher, Schliemann, Brizuela and Earnest (2006) found that 9-10 year old students were quite capable of using the algebraic ideas and representations. They also found that even in early grades “algebraic notation can play a supportive role in learning mathematics” (p. 88). They further state that, rather than to try to put algebraic meaning to the existing mathematics, mathematics should include algebraic thinking from the beginning of formal instruction. If this is done, the students will have powerful tools such as symbolic notation, number lines, and graphs at their command.

While there are many reasons given in mathematics studies to postpone integrating algebra into the curriculum, Carraher et al. (2006) found more compelling reasons to make it an integral part of the early mathematics curriculum. Their work revealed that teaching the abstract and the concrete should be interwoven rather than taught distinctly. This interweaving of algebra and concrete mathematics can be most fruitful for the students. For example, algebraic notation (tables, graphs) can provide a means of conveying patterns more clearly.

Some studies in the 1970’s, 1980’s and 1990’s seemed to indicate that young children were not capable of learning algebra as the student did not have cognitive ability to handle such concepts (Collis, 1975; Filloy & Rojano, 1989; Kuchemann, 1981; Herscovics & Linchevski, 1994; Linchevski, & Herscovics, 1996). However, later studies found that young students were able to handle algebraic concepts (Carpenter & Franke, 2001; Carpenter & Levi, 2000; Schifter, 1999). They also found that the students were able to use the algebraic notation without difficulties. The need for the transition period from concrete to abstract has been shown to be eliminated. Blanton and Kaput (2005) believe that the integration of algebra in the primary grades is an alternative that builds "the conception development of deeper and more complex mathematics into students' experiences from the very beginning" (p. 413).

Yackel's (1997) work confirmed that the use of activities can encourage students to move beyond numerical reasoning. These activities can be used in elementary grades to facilitate the development of foundations for algebraic reasoning.

Since students are being introduced to algebraic concepts at a younger age than in the past, the researchers believe that it is important for algebra teachers to make meaningful connections between arithmetic and algebra with concrete methods such as the use of visual representations. The researchers used this study to determine if there was a significant difference in seventh-grade and tenth-grade students' knowledge of numeric, visual and algebraic representations. This study supported the students' success of using multiple-representations to demonstrate algebraic expressions. The visual representations appeared to bridge the gap between the numeric and algebraic concepts.

METHOD

Subjects

This investigation was a study using an experimental design with two groups. Group one consisted of 76 tenth-grade students and group two consisted of 69 seventh-grade students. In the United States (U.S.), students in the tenth grade are about 15-16 years of age and seventh-grade students are about 12-13 years of age. The tenth-grade students, who participated in this study, attended a high school that has students in grades 9-12. The seventh-grade students, who participated in this study, attended a middle school that has students in grades 6-8.

Instruments

The assessment instrument was four researcher developed "Guess My Number" problems. A pilot study, with different subjects, was conducted and modifications were made as deemed appropriate. Problems One and Two remained identical to the two problems used in the pilot study. Problems Three and Four from the pilot study were modified to avoid the use of large numbers. Large numbers required the use of many tally marks in the visual representations and were difficult to assess; therefore, these two problems were adjusted for the current study. Each group was given one problem as an example. They were given instructions concerning the recording of their representation of the problem numerically, visually, and algebraically on the researcher-designed form. During the delivery of the instructions, the groups were carefully monitored. The researchers feared that the results might become tainted by lack of recording knowledge rather than lack of content knowledge.

The numeric representation involved the use of arithmetic to represent algebraic expressions. The visual representation involved using a square for the variable and tally marks for amounts added to the variable. The algebraic representation was symbolized with the variable ‘n’ and numbers.

For example, one problem asked students to choose a number and then add 5 to the number. If the student chose ‘3’, the numeric representation was explained to be “3 + 5 = 8”. Students were asked to record the visual representation; they were instructed to draw a square followed by five tally marks to represent the number plus five. Next students were asked to write the algebraic representation ‘n + 5’. Students recorded their responses on tables as the researchers read the problem three times, once for each representation.

Problem One had the following five steps:
1) Choose a number
2) Multiply the number by 2
3) Add 2 to the result
4) Divide by 2
5) Subtract 1

The final answer should have been the same as the number that was chosen in Step 1.

Problem Two had the following six steps:
1) Choose a number
2) Add 5
3) Double the result
4) Subtract 4
5) Divide by 2
6) Subtract the original number

In Problem Two the result should always have been ‘3’.

Problem Three and Problem Four from the pilot study both required the use of large numbers in certain steps. The use of large numbers required a lot of tally marks for the visual representation, so both problems were modified. For example, Step 3 of Problem Three of the pilot study problem, asked the student to “Multiply the result by 4”. This step was modified to read, “Multiply the result by 3”. Steps 4 and 5 were changed so the final answer remained one greater than the original number. Problem Three used in the current study read as follows:

1) Choose a number
2) Add 15
3) Multiply the result by 3
4) Subtract 6
5) Divide by 3
6) Subtract 12

In Problem Three, the final result should be one greater than the number chosen in Step One.

Problem Four of the pilot study asked the student to choose a number that was a multiple of 7 and in Step 3 the student was asked to “Add 49”. The problem used in the current study was changed to ask the student to choose a multiple of 5, since multiples of 5 are easier to identify. The remaining steps of Problem Four were changed so that the final answer was one-fifth of the original number. Problem Four read as follows:

1) Choose a number
2) Multiply the result by 3
3) Add 15
4) Divide the result by 5
5) Subtract 3
6) Divide by 3

RESULTS

The researchers made assessments of student knowledge in the areas of numeric, visual, and algebraic representations. The first problem had five steps so was worth five points. Problems Two, Three, and Four each had six steps so were worth six points each. Means were calculated and compared for the two groups. Students obtained one point for each step of a problem that was correct. For example, Problem Four had six steps; therefore, the point range for Problem Four was from zero to six points.

Problem One

The researchers analyzed the results for Problem One even though this problem was worked on an overhead together with the students. Figures 1 and 2 are examples of students’ responses to the problem. The problem in Figure 1 was chosen because it illustrates some of the very common errors in the students’ work. The “Choose a number” and “Multiply the number by 2” steps did not create any problems for the students in the numeric, visual or algebraic representations. However when the students were asked to add 2 it became apparent that they did not grasp the visual representation. Many of the students did not appear to realize that the square (which was the symbol designated to be used) represented the number which they had chosen earlier in the problem. Many just added two squares which is an indication that they did not make the connection that the original square represented their chosen number and that additional tally marks represented one each. The only case when the students adding two additional squares would have been correct is when the student had chosen 1 as their original number. None of the students in the study chose 1 as their number. As a result of the error in Step 3, Steps 4 and 5 were also incorrect.

The student's work shown in Figure 1 did not indicate any difficulties with the numeric representation with Problem One; but did have some inaccuracies in the algebraic representations. Step 3 has "2n + 2" correct; however in Step 4 when asked to "Divide by 2", the student incorrectly responds with "2n" instead of "n+1". This shows a lack of understanding of the distributive property that many algebra students have. Many students also put nothing for "2/2" instead of "1". The thinking here is that the quotient is "0" instead of "1". The researchers have seen this error many times in their observations and instruction of students.

Figure 1. Student A's work on Problem 1 (incorrect responses)

Problem 1- steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	10	□	n
Multiply the number by 2	20	□ □	2n
Add 2 to the result	22	□□□□	2n+ 2
Divide by 2	11	□□	2n
Subtract 1	10	□	n=1

Figure 2 is an example of one student's correct response to Problem One. It was interesting to see that in the visual representations, this students substituted triangles; instead of the squares that were given during the instruction phase. This shows a clear understanding of the concept on the student's part.

Figure 2. Student B's work on Problem 1 (correct responses)

Problem 1- steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	100	△	n
Multiply the number by 2	200	△ △	2n
Add 2 to the result	202	△ △ | |	2n + 2
Divide by 2	101	△ |	n + 1
Subtract 1	100	△	n

The researchers expected means to equal 5.00 for each of the representations; however, this was not the case. The tenth-grade students had a higher mean (M=5.00 or 100%) than the seventh-grade students (M=4.84 or 96.8%) on numeric representations, but the difference was not significant. The means were the same (M=4.54 or 90.8%) for both groups on algebraic representations. The seventh-grade students (M=4.65 or 93.0%) scored significantly higher [$F(1,143)=26.704$, $p<.01$] than the tenth-grade students (M=3.12 or 62.4%) on visual representations.

Table 1. Problem 1—Means

Group		Numeric	Visual	Algebraic
Tenth-grade	Mean	5.00	3.12	4.54
n=76	Std. Dev.	.000	2.197	1.051
Seventh-grade	Mean	4.84	**4.65	4.54
n=69	Std. Dev.	.720	1.174	1.208

** p < .01

Problem Two

On Problem Two, the seventh-grade students scored higher than the tenth grade students on all three representations. Students did not have difficulties with the numeric representation on this problem. However, the students had difficulties with the visual representation and did not seem to realize that the square represented their chosen number. The example in Figure 3 illustrates this error. The students' work shows nothing in Step 1, in the place of a square to represent the chosen number. The student left Step 1 blank, which does illustrate choosing the number "0"; but cannot be generalized to other chosen numbers. The student uses squares instead of tallies to "Add 5" in Step 2. The illustration should have one square and five tally marks.

Under the algebraic representation column many students were successful on the first two or three steps but most students either left the remaining steps blank or had incorrect responses. The example in Figure 3 shows success in the algebraic representation column up to Step 3 where the student writes, "2(n+5)". In Step 4, the student is asked to "Subtract 4" from "2(n+5). The student is unable to use the distributive property of multiplication over addition in order to "Subtract 4". The student leaves the remaining steps blank.

Figure 3. Student C's work on Problem 2 (incorrect responses)

Problem 2 Steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	0		n
Add 5	5		n+5
Double the results	10		2(n+5)
Subtract 4	6		
Divide by 2	3		
Subtract the original number	3		

Figure 4 is an example of one student's correct response to the problem. The student's work shows understanding of the distributive property of multiplication over addition in both the "Double the results" and "Divide by 2" steps. The student had no errors in Problem 2.

Figure 4. Student D's work on Problem 2 (correct responses)

Problem 2 - steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	3		n
Add 5	8		n+5
Double the results	16		2n+10
Subtract 4	12		2n+10-4=2n+6
Divide by 2	6		$\frac{2n}{2}+\frac{6}{2}=n+3$
Subtract the original number	3		3

The seventh-graders (M=5.90 or 98.3%) scored higher on numeric representations than the tenth-graders (M=5.71 or 95.2%); however the difference was not significant. The seventh-graders (M=3.19 or 53.2%) scored significantly higher [F(1,143)=20.541, p<.01] on visual representations than the tenth-graders (M=1.63 or 27.2%). The seventh-grade students (M=3.01 or 50.2%) also had significantly higher scores [F(1,143)=20.022, p<.01] on the algebraic representations for Problem Two than the tenth-grade students (M=1.79 or 29.8%).

Table 2. Problem 2—Means

Group		Numeric	Visual	Algebraic
Tenth-grade	Mean	5.71	1.63	1.79
n=76	Std. Dev.	1.056	2.190	1.878
Seventh-grade	Mean	5.90	**3.19	**3.01
n=69	Std. Dev.	.489	1.920	1.345

** p < .01

Problem Three

On Problem Three, the seventh-grade students again scored higher than the tenth-grade students on all three representations. The students did not have any problems with any of the steps on the numeric representation or with the first two steps on the visual or algebraic representations. The visual representation and algebraic representation in Step 3 proved to be difficult for the students. On the visual, some of the students were not able to multiply correctly. As shown in Step 3 of Figure 5, the student multiplied the variable by 3; but multiplied the constant by 2. In Step 4, the student actually added 3 tallies instead of subtracting 6. The student did not appear to understand Steps 5 and 6 as there was only one square drawn.

On the algebraic representation for Problem Three, the same student illustrated Steps 1-5 correctly; but did not "Subtract 12" from "n+13" correctly in the last step. The student wrote that the difference was "n+12" instead of "n+1".

Figure 5. Student E's work on Problem 3 (incorrect responses)

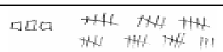
Problem 3 – steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	15		n
Add 15	30		n + 15
Multiply the result by 3	90		nnn + 45
Subtract 6	84		nnn + 39
Divide by 3	28		n + 13
Subtract 12	16		n + 12

Figure 6 is an example of one student's correct response to Problem Three. The student's work shows a thorough understanding of all of the symbolic concepts as well as all of the algebraic concepts.

Figure 6. Student F's work on Problem 3 (correct responses)

Problem 3 – steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number	4	□	x
Add 15	19	□ ||||| ||||| |||||	x+15
Subtract 6	51	□ ||||| ||||| ||||| □ ||||| ||||| |||| □ ||||| ||||	3x+45−6 39
Divide by 3	17	□ ||||| ||||| |||	x+13
Subtract 12	5	□+1	x+1

The seventh-graders (M=5.84 or 97.3%) scored significantly higher [F(1,143)=4.142, p<.05] on numeric representations than the tenth-graders (M=5.42 or 90.3%). The seventh-graders (M=2.12 or 35.3%) also scored significantly higher [F (1,143)=17.125, p<.01] on visual representations than the tenth-graders (M=1.04 or 17.3%). The seventh-grade students (M=2.10 or 35%) had higher scores on algebraic representations for Problem Three than the tenth-grade students (M=1.95 or 32.5%); however the difference was not significant.

Table 3. Problem 3—Means

Group		**Numeric**	**Visual**	**Algebraic**
Tenth-grade	Mean	5.42	1.04	1.95
n=76	Std. Dev.	1.602	1.669	2.737
Seventh-grade	Mean	*5.84	**2.12	2.10
n=69	Std. Dev.	.633	1.449	1.374

*p < .05; **p < .01

Problem Four

The seventh-grade students scored higher than the tenth-grade students on all three representations of the most difficult problem, Problem Four. In Figure 7, Student G's work was representative of the students who did not answer the questions correctly. The numeric representation was correct. On the visual, most of the students were able to do the first three steps correctly but they were not able to illustrate division by 5 in Step 4; therefore the remaining steps were also incorrect. The student's work in Figure 7 is an illustration of these errors.

The same student also was unsuccessful with Step 4 of the algebraic representation for Problem Four. The student was unable to "Divide the result by 5". While the student was able to divide the constant by 5, the student was unable to show division by 5 of the constant. Again the students had difficulty with the distributive property of multiplication over addition. This problem was slightly more difficult than the others and as a result proved to be more than many of the students could work correctly.

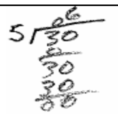
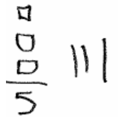
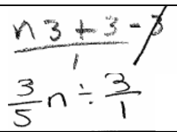
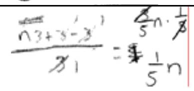
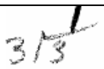
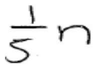
Figure 7. Student G's work on Problem 4 (incorrect responses)

Problem 4-Steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number. Calculations will be easier if you select a multiple of 5 i.e. 5,10,15,20…	25		N
Multiple the result by 3	75		n n n
Add 15	75 + 15 = 90		nnn + 15
Divide the result by 5	18 5)90		nnn + 3
Subtract 3	18 − 3 = 15		nnn
Divide by 3	3)15 5	1	1

Figure 8 is an example of one student's correct response to the problem. This student's work showed a clear understanding of algebraic concepts such as the distributive property of multiplication over addition and algebraic computation with fractions.

Figure 8. Student H's Work on Problem 4 (correct responses)

Problem 4 - steps	*Numeric Representation*	*Visual Representation*	*Algebraic Representation*
Choose a number. Calculations will be easier if you select a multiple of 5 i.e. 5,10,15,20…	5		n
Multiple the result by 3	5 × 3 = 15		n·3+15
Add 15	15 + 15 = 30		$\frac{n3+15}{5}$ | $\frac{3}{5}n + 3$

Divide the result by 5			
Subtract 3			
Divide by 3			

The seventh-grade students had a higher mean (M=5.83 or 98.8%) than the tenth-grade students (M=5.61 or 93.5%) on numeric representations, but the difference was not significant. The seventh-grade students (M=1.64 or 27.3%) scored significantly higher [$F(1,143)=26.746$, $p<.01$] than the tenth-grade students (M=0.61 or 10.2%) on visual representations. The seventh-grade students (M=1.87 or 31.2%) also scored higher than the tenth-grade students (M=1.40 or 23.3%) on algebraic representations; but the difference was not significant.

Table 4. Problem 4—Means

Group		Numeric	Visual	Algebraic
Tenth-grade	Mean	5.61	.61	1.40
n=76	Std. Dev.	1.266	1.144	1.986
Seventh-grade	Mean	5.83	**1.64	1.87
n=69	Std. Dev.	.641	1.260	1.464

** p < .01

DISCUSSION

The lack of transition between arithmetic and algebra has been documented in the research (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994; Linchevski, & Herscovics, 1996; Kiran, 1996; Chappell & Strutchens, 2001). The findings from this study support the research and show that semi-concrete models are beneficial in making connections between concrete and abstract concepts. In this study the visual models were used to connect the numeric, and algebraic concepts.

This study showed that both groups of students did well with the numeric representation with all mean scores above ninety percent (90%). These data further support the literature that students generally do well on arithmetic or numeric skills (Yackel, 1997; Blanton & Kaput, 2005; Carraher, et. al., 2006).

The tenth-grade students did better on the algebraic representations than they did on the visual representations on all four problems. Piaget's cognitive development theory (Fitch, Ormrod, Santrock, 2005; Santrock, 2008) suggests that students of this age are in the formal stage of cognitive development and should be abstract thinkers. However, these findings may be that most of these tenth-grade students are not truly abstract thinkers, may have forgotten the visual representations or may not have wanted to think at a concrete level. Peer pressure may also have been a factor (Sayer &Adey).

The seventh-grade students scored higher on visual representations than algebraic representations on Problems One, Two and Three, but had higher scores on algebraic representations than visual representations on Problem Four. The seventh-grade students had significantly higher scores than the tenth-grade students on the visual representation part of all four problems. It may be that the seventh grade students are working with visual representations and that the tenth-grade students are not working with visual representations. According to Piaget's cognitive development theory (Fitch, Ormrod, Santrock, 2005; Santrock, 2008; Babal), most seventh-grade students are in the semi-concrete and only a few are in the abstract stage of development. This is a possible explanation as to why seventh-grade students did better on visual or concrete representations than on algebraic or abstract representations.

The seventh-grade students had higher scores than the tenth-grade students on all parts of Problems Two, Three, and Four; although the differences were not significant. The researchers observed that the seventh-grade students appeared more cooperative than the tenth-grade students who participated in the study.

Implications

The implications for future studies include using replication, a larger sample and using random samples. Replication of the current study is needed with students in other regions, since this study was conducted in south Texas with an assessable sample. Using random sampling techniques and more students are necessary to be able to make further generalizations.

Implications for instruction concern the learning environment, the use of instructional materials, and instructional methods related to the instruction of algebra. The current study is consistent with research that shows that the learning environment must be rich with a variety of algebraic materials (Lee, 2001; Blanton & Kaput, 2005). Concrete models need to be used in the Algebra classroom in order for the students to make connections between numeric and algebraic concepts (Chappell & Stutchens, 2001; Crossfield, 1997). The instructional methods used in the Algebra classroom must show relationships between arithmetic and algebra (Kieran, 1996; Swafford & Langrall, 2000; Driscoll, 1999; Nathan & Koedinger, 2000). The findings support the research that multiple representations need to be used in teaching algebraic concepts (Allsop, 1999; Piez & Voxman, 1997; Yerushalmy & Shternberg, 2001; Smith & Phillips, 2000; Carraher, et al., 2006). The need for a transition from concrete to abstract or numeric to algebraic is essential in order for students to understand algebra concepts.

Based on the results of this study, the researchers believe that the visual representations used helped some students to bridge the gap between numeric and algebraic concepts. The researchers advocate that visual representations be used in all math classrooms. Although the researchers know that the visual representations will not be a panacea for all students, it is a teaching method which should be utilized as it will be helpful to some. Students have different learning styles and as mathematics educators it is our purpose to try to help or guide as many students as possible to be successful in learning algebra.

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