A REFINED SHEAR DEFORMATION PLATE THEORY FOR STATIC AND FREE VIBRATION ANALYSIS OF FUNCTIONALLY GRADED PLATES

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ABSTRACT

In this research, an efficient shear deformation plate theory for a functionally graded plate has been investigated by the use of the new four variable refined plate theory. Unlike any other theory, the number of unknown functions involved is only four, as against five in case of other shear deformation theories. The theory account for higher-order variation of transverse shear strain through the depth of the plate and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Based on the present higher-order shear deformation plate theory, the equations of the motion are derived from Hamilton's principal. The plate faces are assumed to have isotropic, two-constituent material distribution through the thickness, and the modulus of elasticity, Poisson's ratio of the faces, and thermal expansion coefficients are assumed to vary according to a power law distribution in terms of the volume fractions of the constituents. The validity of the present theory is investigated by comparing some of the present results with those of the classical, the first-order and the other higher-order theories. The influences played by the transverse shear deformation, aspect ratio, side-to-thickness ratio, and volume fraction distribution are studied. Numerical results for deflections and stresses of functionally graded plate are investigated.

Keywords: Functionally graded material, Static Analysis, Vibration Analysis , Modeling, Bending.

INTRODUCTION

In recent years, astonishing advances in science and technology have motivated researchers to work on new structural materials. Functionally graded materials (FGMs) are classified as novel composite materials which are widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical, and shipbuilding industries. Due to smoothly and continuously varying material properties from one surface to the other, FGMs are usually superior to the conventional composite materials in mechanical behavior.

The classical plate theory, based on the Kirchhoff hypothesis, is inaccurate for analyzing the distribution of displacements and stresses in FG plates. The inaccuracy is due to neglecting the effects of transverse shear in the FG plates. In order to take into account this effect, a number of first-order shear deformation theories have been developed. However, since in these theories the transverse shear strains are assumed to be constant in the thickness direction, shear correction factors have to be incorporated to adjust the transverse shear stiffness. The accuracy of solutions of the first-order shear deformation theory will be strongly dependent on predicting better estimates for the shear correction factors. It has been shown that the classical and first-order shear deformation theories are inadequate to predict the accurate solutions of FG plates.

In this article, for the first time, the new four variable refined plate theory is developed for static of FG sandwich plates. The thickness-to-side ratio on the through-the-thickness deflections, in-plane displacements, and axial stress distributions are studied in detail. Numerical results for deflections and stresses are investigated. Numerical examples are presented to illustrate the accuracy and efficiency of the present theory by comparing the obtained results with those computed using various other theories.

PROBLEM FORMULATION

Consider the case of a uniform thickness, rectangular FG sandwich plate composed of three microscopically heterogeneous layers referring to a rectangular coordinates (x, y, z). The top and bottom faces of the plate are at $z=\pm h/2$, and the edges of the plate are parallel to axes x and y. The sandwich plate is composed of three elastic layers, namely, ''Layer 1'', ''Layer 2'', and ''Layer 3'' from bottom to top of the plate (Fig. 1). The volume fraction of the FGMs is assumed to obey a power-law function along the thickness direction:

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$$V^{(1)} = \left(\frac{z-h_1}{h_2-h_1}\right)^k,$$

k	Theory	1-0-1	3-1-3	2-1-2	1-1-1

$z \in [h_1, h_2]$ (1a)

$$V^{(2)} = 1, \quad z \in [h_2, h_3] \tag{1b}$$

$$V^{(3)} = \left(\frac{z-h_4}{h_3-h_4}\right)^k, \quad z \in [h_3, h_4] \tag{1c}$$

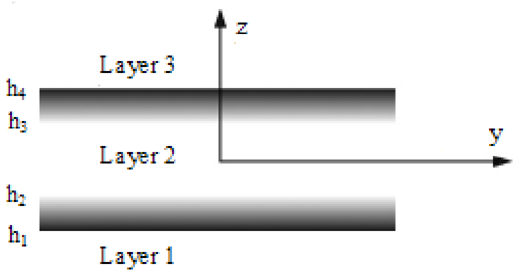

Fig.1. The material variation along the thickness of the FG sandwich plate.

HIGHER-ORDER PLATE THEORY

The displacements of a material point located at (x, y, z) in the plate may be written as

$$u = u_0(x,y) - z\frac{\partial w_0}{\partial x} + \Psi(z)\,\theta_x, \quad v = v_0(x,y) - z\frac{\partial w_0}{\partial y} + \Psi(z)\,\theta_y, \quad w = w_0(x,y) \tag{2}$$

where, u, v, w are displacements in the x, y, z directions, u_0, v_0 and w_0 are midplane displacements, θ_x and θ_y rotations of the y_z and x_z planes due to bending, respectively. $\Psi(z)$ represents shape function determining the distribution of the transverse shear strains and stresses along the thickness.

$$\Psi(z) = z\left(1 - \frac{4z^2}{3h^2}\right)$$

(3)

REFINED PLATE THEORY FOR FUNCTIONALLY GRADED PLATES

Unlike the other theories, the number of unknown functions involved in the present refined theory (RPT) is only four, as against five in case of other shear deformation theories

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.

(ii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x, y only.

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.

(iv) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

NUMERICAL RESULTS

Table I contains the dimensionless center deflection $\bar{w}$ for an FG sandwich plate subjected to mechanical and static loads. The deflections are considered for k = 0, 1, 2, 3, 4, and 5 and different types of FG sandwich plates. It shows that the effect of shear deformation is to increase the deflection. For a FG plate, the deflections increase as the core thickness decreases whereas k increases. The maximum deflections occur for an FG plate without core thickness (i.e., the (1-0-1) FG plate) and this irrespective of the value of k. It can be observed that the results obtained by the present refined plate theory (with four unknown functions) are identical to those of SSDPT (with five unknown functions).

Table I: Comparison of nondimensional center deflections $\bar{w}$ for different FG sandwich square.

0	Present	0.808168	0.808168	0.808168	0.808168
	SSDPT	0.796783	0.796783	0.796783	0.796783
1	Present	1.077690	1.059613	1.050672	1.025367
	SSDPT	1.062840	1.045026	1.036213	1.011263
2	Present	1.137297	1.120582	1.111353	1.082911
	SSDPT	1.121608	1.105175	1.096094	1.068091
3	Present	1.157693	1.143856	1.135420	1.107475
	SSDPT	1.141655	1.128080	1.119793	1.092312

Figures 3 contain the plots of the axial stress $\bar{\sigma}_x$ through-the-thickness of both symmetric and nonsymmetric FG plate ($k = 1.5$) using various shear deformation theories. the present theory and TSDPT are coincided.

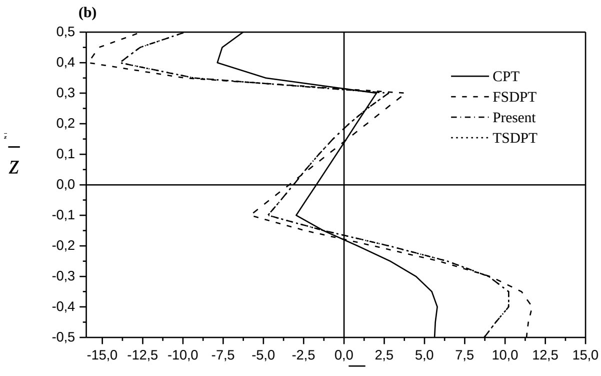

Fig. 3. Comparison of the variation of nondimensional axial stress $\bar{\sigma}_x$ across the thickness of nonsymmetric FG sandwich plates (2-2-1), ($k = 1.5$) by applying different shear deformation theories.

CONCLUSION

In this paper, a new four variable refined plate theory (RPT) has been developed to investigate for the first time the thermomechanical behavior of simply supported FG sandwich plates. Unlike any other theory, the theory presented gives rise to only four governing equations resulting in considerably lower computational effort when compared with the other higher-order theories reported in the literature having more number of governing equations. In general, the fully ceramic plates give the smallest deflections and transverse shear stress. The gradients in material properties play an important role in determining the response of the FGM plates. The mixture of the ceramic and metal with continuously varying volume fraction can eliminate interface problems of sandwich plates and thus the stresses distributions are smooth. All comparison studies demonstrated that the deflections and stresses obtained using the present refined theory (with four unknowns) and other higher-order shear deformation theories (five unknowns) are almost identical. Hence, it can be said that the proposed theory RPT is accurate and simple in solving the static behavior of FG plates.

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