

Research Article

## Effect of Some Factors on Solution Sensitivity in Determination of Temperature Field by Finite Difference Method for the Drying Process of Yarn Bobbins

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**Abstract:** In this study, a mathematical model for the drying process of dyed wool yarn bobbins by passing hot air through them has been presented. The presented mathematical model reduced the drying problem to a nonlinear, nonstationary heat convection problem including the convective effect caused by forced convection, together with the effective thermophysical properties of the wool yarn bobbins. The finite difference method was used for solving the mathematical model.

**Keywords:** Drying, Finite difference method, Mathematical model, Yarn bobbin

### İplik Bobini Kurutulması Prosesinde Sıcaklık Alanının Sonlu Farklar Yöntemi İle Belirlenmesinde Bazı Faktörlerin Çözüm Hassaslığına Etkisi

**Özet:** Bu çalışmada, boyanmış yün iplik bobinlerinin içerisinde basınçlı sıcak hava geçirilerek kurutulması işlemi için bir matematiksel model ortaya konmuştur. Sunulan matematiksel model, kurutma problemini, içerisinde yün iplik bobinin efektif termofiziksel özellikleriyle birlikte, zorlanmış taşınım etkisinden kaynaklanan konvektif terimi barındıran nonlinear, nonstasioner bir ısı taşınım problemine indirgemıştır. Matematiksel modelin çözümü için sonlu farklar metodu kullanılmıştır. Bu yöntemde uygulanan algoritmadaki uzay ve zaman adımlarının ve nonlineerliğin ( $k_e(T)$ ,  $C_{ve}(T)$ ) sıcaklık alanına etkisi incelenmiştir.

**Anahtar Kelimeler:** Kurutma, Sonlu farklar metodu, Matematiksel model, İplik bobini

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## 1. Introduction

Drying of yarn bobbins has a great importance for the textile industry. It is important to perform this process in a short time with minimum energy consumption in terms of efficiency. For this reason, mathematical models have been emphasized for many years for drying processes. These models generally focus on simultaneous solutions of heat and mass transfer equations.

Ribierio and Ventura (1995) conducted an experimental study in which the drying process was carried out by passing hot air both through the inside and outside of the yarn bobbins [1]. Smith and Farid (2004), in their experimental work, have obtained correlations that allow the drying times of materials to be determined, taking into account the moving boundary theory for cylindrical geometries [2]. In a study by Hussain and Dinçer (2003), a two-dimensional numerical analysis of heat and moisture transfer during the drying of a cylindrical object was performed using the finite difference approach [3]. Barati and Esfahani (2011) considered heat and mass transfer by conduction and convection in order to model the drying process and developed an analytical solution for the mathematical model they created [4].

This study considers a system in which hot air is passed through only the inside of the yarn bobbins to dry them. Then, the mathematical model which is reduced the physical process to the heat transfer problem was written and the finite difference method was used for the solution of the model. In the thermophysical properties ( $k_e(T)$ ,  $C_{ve}(T)$ ,  $P_e(r, T)$ ) the Runge principle was used to minimize the errors resulting from the method and the solution technique [5]. Accordingly, the solution obtained in the low density mesh is compared with the solutions obtained in the increasingly dense meshes. By changing the values of the effective volumetric heat capacity ( $C_{ve}(T)$ ) in the phase conversion region ( $\Delta T = T_s - T_b$  value and different  $T_b$  and  $T_s$  values for the same  $\Delta T$ ), the effect of the solution of the problem has been investigated. Finally, the problem is solved by taking constant  $k_e(T)$  to investigate the effect of the effective thermal conductivity on the temperature field.

## 2. Materials and Methods

The wool yarn bobbins were dried with hot air in the test setup. It passed through the yarn bobbins and the bobbins were dried from only the inside to the outside due to the pressure difference. The volumetric flow rate of the hot air is  $500 \text{ m}^3/\text{h}$  while the temperature and the pressure are  $80^\circ\text{C}$  and 1 bar (effective) respectively. The yarn bobbins used in the experiments have an inner diameter of 35 mm and a length of 150 mm (Fig. 1).

The wrapped yarn is made up of holes made from polyethylene material that allow air to pass over it. During the tests, 7 thermocouples were placed in the bobbin at even intervals to measure the temperature of the yarn bobbin section (Fig. 2). Akyol et al. (2013) have shown that the temperature change during the length of the bobbin is not important in the experimental study [6]. Therefore, in this study, it is considered that the heat transfer in the bobbin is performed in one dimension in the radial direction.



Figure 1. Wool yarn bobbins subjected to drying process

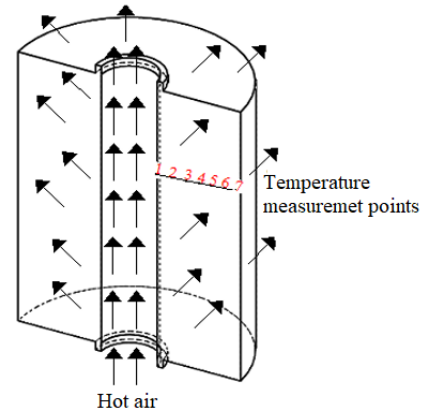


Figure 2. Schematic view of the yarn bobbin used in the experiments and temperature measurement points

When the mathematical model is written, it is thought that the heat transfer mechanism is realized as follows: Air mass, moving at a specific temperature under a certain pressure has thermal (internal) and kinetic energies. Thermal and kinetic energies are absorbed from the inner surface through the interstices between the yarns by the bulk fluid movement into the bobbin and enter the bobbin volume through spaces between the yarns. Energy can also pass through surfaces by molecular processes. This can happen in two ways: Conduction and mass diffusivity. However, during the passage of the fluid through the material, work is done by pressure and friction forces. Hot air passing through spaces between yarns, collides with yarns and water molecules and transfers some of the energy to them. During the movement, by taking up the evaporating water molecules, the increasing amount of moisture continues to move towards the outer surface while warmed water molecules also undergo forced diffusion [7]. As a result, the mathematical model is expressed as follows:

$$C_{ve} \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( k_e r \frac{\partial T}{\partial r} \right) + P_e(r, T) \frac{\partial T}{\partial r} \quad (1)$$

Here;  $t$ ,  $T$ ,  $r$ ,  $C_{ve}$ ,  $k_e$  respectively; time, temperature, radial coordinate, effective volumetric heat capacity, effective thermal conductivity.  $P_e(r, T)$  is the function of temperature and coordinate and can be expressed as:

$$P_e(r, T) = (c_{ph}\rho_h + c_{pb}\rho_b)V_a \quad (2)$$

Where  $c_{ph}$ ,  $c_{pb}$ ,  $\rho_h$ ,  $\rho_b$ , and  $V_a$  are the specific heat capacity of the dry air and water vapor, density of the dry air and water vapor, velocity of the fluid in the porous body, ie, in the yarn bobbin respectively. The mathematical model that expresses the physical process is in the form of Eq. (1). The direct

problem is solved by the finite difference method in the initial and boundary conditions as follows:

$$r_{in} < r < r_{out}; \quad 0 < \tau < \tau_m$$

$$T(r, 0) = T_i; \quad T(r_{in}) = f_1(\tau); \quad T(r_{out}) = f_2(\tau)$$

### 3. Results and Discussion

Firstly, six different situations (Case 1, 2, 3, 4, 5 and 6) have been considered to examine the effect of time and space step on study. Then four different situations (Case 7, 8, 9 and 10) have been taken into account to examine the effect of the  $C_{ve}$  (T) change character on the temperature field and drying time. Finally, Case 11 has been considered in order to examine the effect of the change in  $k_e$ (T) on the temperature range and drying time. First, the model is solved by determining constant time and constant space steps. The time step was chosen as  $\tau=300$  s and the space step  $h=0.0095$  m (Case 1). First, to examine the effect of the space step, the space step is reduced to half value at all points and the model is solved (Case 2). Afterwards, for the effect of the time step, the time step has been reduced to half its value throughout the entire process (Case 3). Finally, the model is solved in various time and space steps to study the combined effect of these two parameters (Case 4, 5, 6). The test duration is 12900 s for all cases.

**Table 1.** The time and space step values considered in the study for cases 1, 2 and 3

Case	Time step $\tau$ , s	Space step h, m
1	300	0.0095
2	300	0.00475
3	150	0.0095

**Table 2.** The time and space step values considered in this study for different time and space step ranges in Case 4

Time step $\tau$ , s	Space step h, m
100 (between 0-200 s)	0.00475 (between $r=0.033-0.0425$ m)
200 (between 200-600 s)	0,0095
300 (between 600-12900 s)	(between $r=0.0425-0.090$ m)

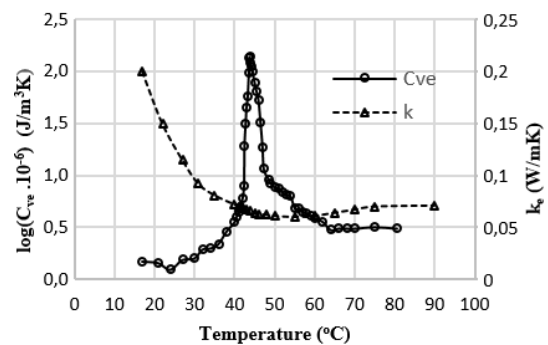
**Table 3.** The time and space step values considered in this study for different time and space step ranges in Case 5

Time step $\tau$ , s	Space step h, m
100 (between 0-200 s)	0.00475 (between $r=0.033-0.052$ m)
200 (between 200-600 s)	0,0095
300 (between 600-12900 s)	(between $r=0.052-0.090$ m)

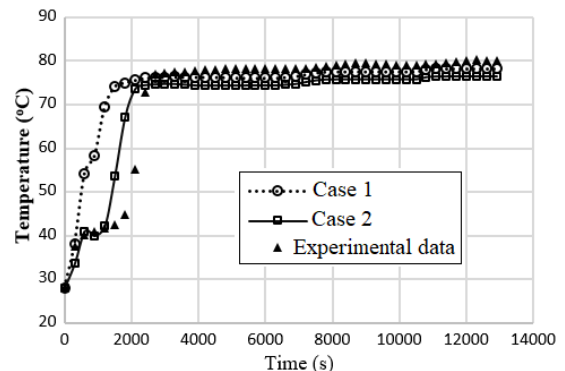
**Table 4.** The time and space step values considered in this study for different time and space step ranges in Case 6

Time step $\tau$ , s	Space step h, m
100 (between 0-200 s)	0.00475 (between $r=0.033-0.052$ m)
200 (between 200-600 s)	0,0095
300 (between 600-4200 s)	(between $r=0.052-0.090$ m)
100 (between 4200-4500 s)	
300 (between 4500-5700 s)	
100 (between 5700-7200 s)	
300 (between 7200-12900 s)	

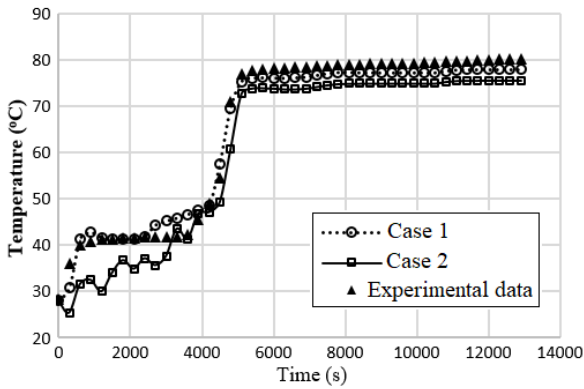
The examined cases are given in Tables 1-4. The convective term  $P_e$ , which includes the convection effect of the air, is the function of the temperature. When  $P_e$  values are determined, three different temperature regions are considered. These; before evaporation, evaporation, and after evaporation.  $P_e$  values used in this study were taken as  $130W/m^2K$  before evaporation,  $66W/m^2K$  in evaporation region and  $503W/m^2K$  after evaporation. In addition, the effective volumetric heat capacity ( $C_{ve}$ ) including phase conversion factor and the effective thermal conductivity ( $k_e$ ) including both mass transport and thermal conductivity are shown in Figure 3. These two thermophysical properties were taken from [8].



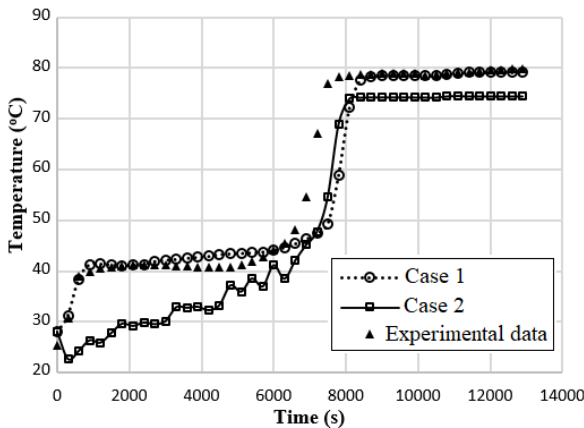
**Figure 3.** Variation of the effective volumetric heat capacity and the effective thermal conductivity with temperature for wool yarn bobbins (Akyol, 2007)



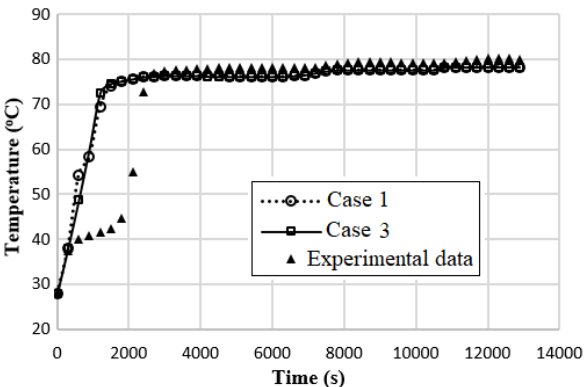
**Figure 4.** Comparison of experimental results and model results (Case 1 and 2) for temperature measurement point  $r = 0.0425m$



**Figure 5.** Comparison of experimental results and model results (Case 1 and 2) for temperature measurement point  $r = 0.0615\text{m}$



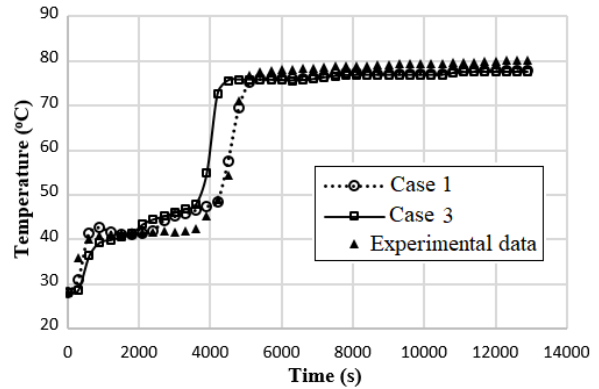
**Figure 6.** Comparison of experimental results and model results (Case 1 and 2) for temperature measurement point  $r = 0.0805\text{m}$



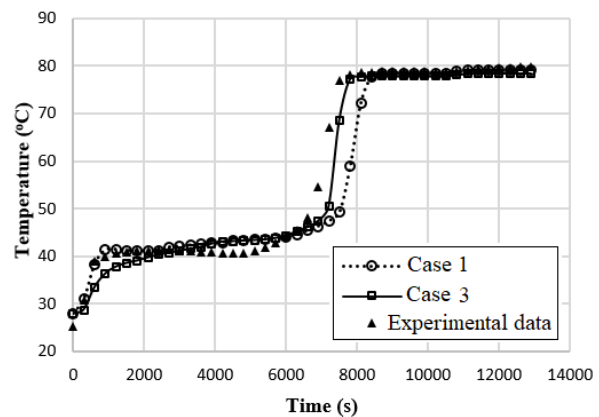
**Figure 7.** Comparison of experimental results and model results (Case 1 and Case 3) for temperature measurement point  $r = 0.0425\text{m}$

Comparison of cases 1, 2 and experiment results (Figure 4-6) shows the effect of space steps on the temperature field. Compared with the experimental results, the model results show that the model has a poor result in the outer surface regions, even though there is some improvement in the areas close to the inner surface as a result of lowering the space step at all points, so it is not a good way to reduce the space step at all points. After the space step, the time step was reduced by using the initial value of the space step to investigate the effect of the time step on the model results (Case 3). The results were

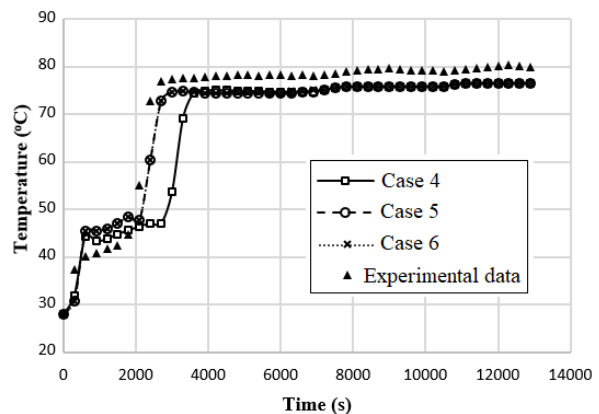
compared with the experimental results.



**Figure 8.** Comparison of experimental results and model results (Case 1 and Case 3) for temperature measurement point  $r = 0.0615\text{m}$



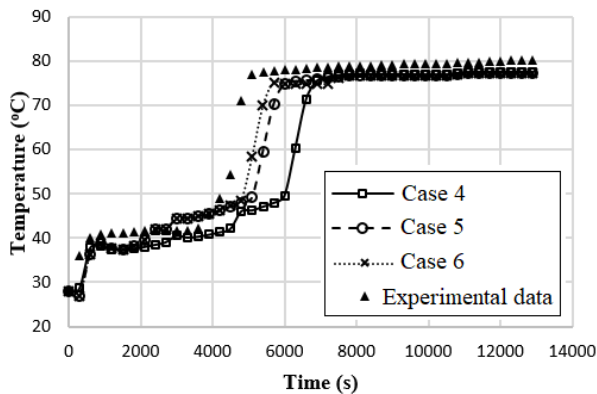
**Figure 9.** Comparison of experimental results and model results (Case 1 and Case 3) for temperature measurement point  $r = 0.0805\text{m}$



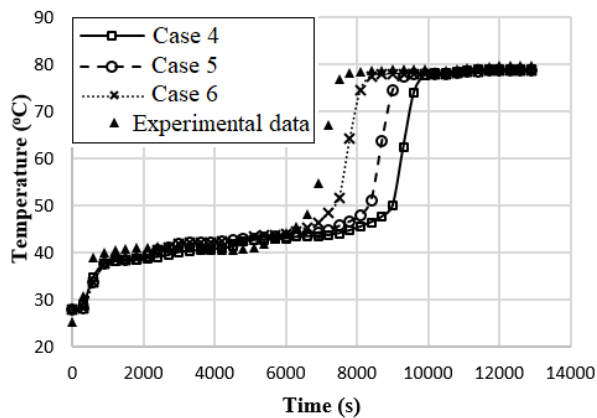
**Figure 10.** Comparison of experimental results and model results (Case 4, 5 and 6) for temperature measurement point  $r = 0.0425\text{m}$

In Figures 7-9, the comparison of Case 1, Case 3 and experiment results shows the effect of time step values on the temperature field. Reducing the time steps to half the value of the whole process did not make any significant difference according to Case 1. The model results show that it is not a good way to select the time steps at small values throughout the entire process. When the temperature field is examined, it is seen that the biggest faults are in the vicinity of the inner

surface where the temperature changes sharply at the beginning of the process, and at the inlet and outlet regions to the evaporation temperatures. Case 4, Case 5 and Case 6 (different values of  $h$  and  $\tau$ ) were solved by the finite difference method and the results were compared with the experimental results (Figures 10-12) to minimize the errors caused by the numerical solution method. The best result was obtained for Case 6. Analyzes demonstrate that when these problems are solved, the time step must be reduced in regions and times where the temperature gradient is large, and at the same time in the temperature regions where the thermophysical properties change sharply. That is, in the mathematical model, the space step must be reduced in the regions close to the inner surface and different  $\tau$  values should be selected throughout the process.



**Figure 11.** Comparison of experimental results and model results (Case 4, 5 and 6) for temperature measurement point  $r=0.0615m$



**Figure 12.** Comparison of experimental results and model results (Case 4, 5 and 6) for temperature measurement point  $r=0.0805m$

The effective volumetric heat capacity ( $C_{ve}$ ) which contains the phase conversion factor, varies depending on temperature. At the beginning of the drying process, the  $C_{ve}$  values progressing with small changes show a large increase in the evaporation region. After reaching a certain peak, the  $C_{ve}$  values decrease to the end of the evaporation region. The area

under the evaporation region of the  $C_{ve}$ -T graph shows the energy of the amount of water evaporating. As long as the conditions of the evaporating water do not change, the area under the graph must be constant. Five different  $C_{ve}$  values were used in this study. The first used  $C_{ve}$  values were taken from Akyol's (2007) study. Other  $C_{ve}$  values are determined using a rectangular geometry provided that the area in the evaporation region remains constant (Fig. 13). Also the evaporation temperature ranges have been changed. The  $C_{ve}$  values generated in this way are solved by using the model in the drying problem. Results are compared to experimental data.

The following conditions have been taken into account for the effect of the  $C_{ve}$  (T) change character on the temperature field and drying time. The values in Case 6 are used for  $h$  and  $\tau$  values in Case 7-10.

For Case 7, the evaporation beginning temperature and the evaporation ending temperature was determined as  $T_b=40^\circ C$  and  $T_s=43^\circ C$  respectively. In this case, the temperature difference is  $\Delta T=T_s-T_b=3^\circ C$ . For Case 8, the evaporation beginning temperature and the evaporation ending temperature was determined as  $T_b=42^\circ C$  and  $T_s=45^\circ C$  respectively. In this case, the temperature difference is  $\Delta T=T_s-T_b=3^\circ C$ .

For Case 9, the evaporation beginning temperature and the evaporation ending temperature was determined as  $T_b=42^\circ C$  and  $T_s=48^\circ C$  respectively. In this case, the temperature difference is  $\Delta T=T_s-T_b=6^\circ C$ . For Case 10, the evaporation beginning temperature and the evaporation ending temperature was determined as  $T_b=41^\circ C$  and  $T_s=46^\circ C$  respectively. In this case, the temperature difference is  $\Delta T=T_s-T_b=5^\circ C$ .

Comparison of Cases 7 and 8 shows that the evaporation temperature range ( $\Delta T$ ) is the same and the difference in evaporation starting ( $T_b$ ) and ending temperatures ( $T_s$ ) has an effect on the solution of the problem.

Comparison of Cases 8 and 9 shows that different evaporation temperature intervals are effective in solving the problem when the initial evaporation temperature is the same. Case 10 was established by choosing close to the phase conversion temperatures of the experiment.

When the results are examined (Figures 14-16), the experimental results and the model results show good agreement in some regions, but in near-surface regions these temperatures show considerable differences in some time periods. When  $T_b$  and  $T_s$  values for the same  $\Delta T$  were low (Case 7), the evaporation duration of the model decreased. If  $\Delta T$  increases, the duration of evaporation increases. Also, the results show that when the phase conversion temperatures taken in the model and observed in the experiment are very close (Case 10), the experimental and model results are in a good agreement with each other.



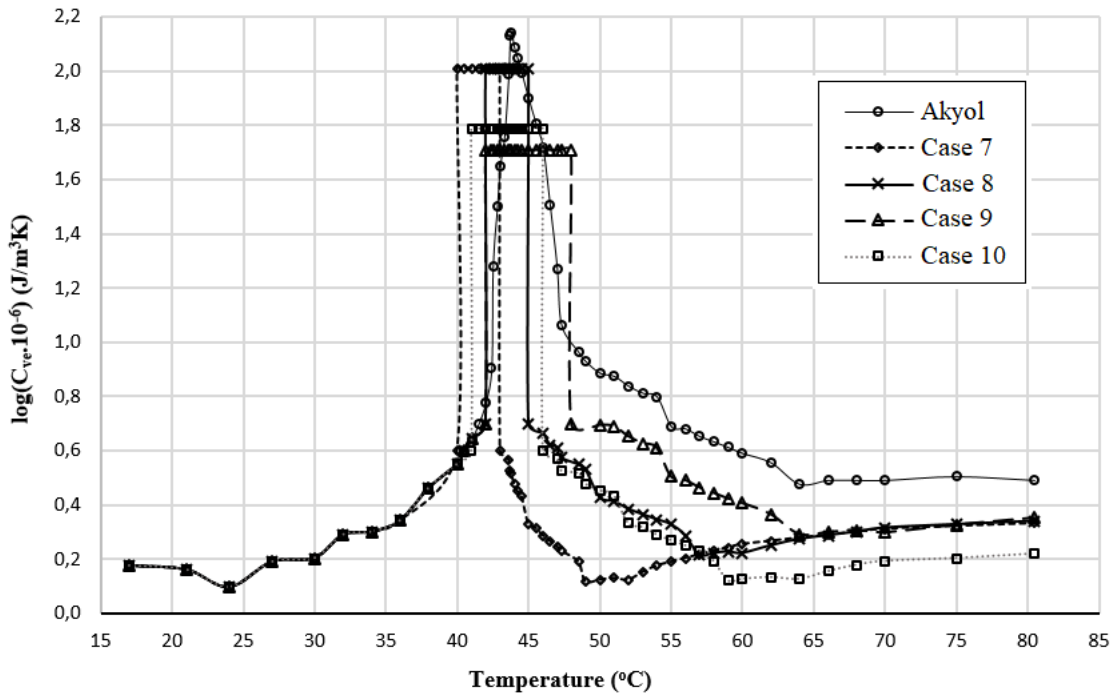


Figure 13. Variation of the effective volumetric heat capacity with temperature with different evaporation intervals for the yarn bobbin

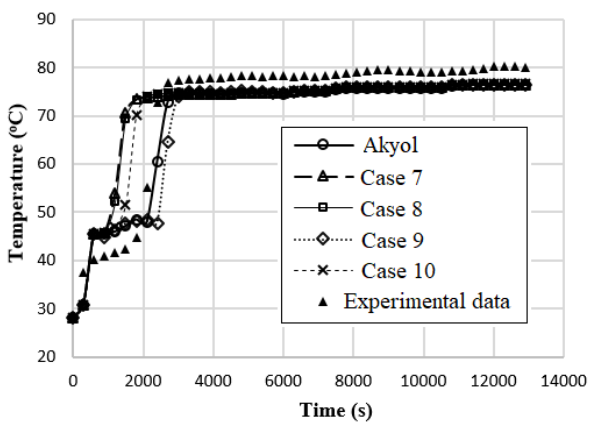


Figure 14. Comparison of experimental results and model results (Case 7-10) for temperature measurement point  $r=0.0425m$

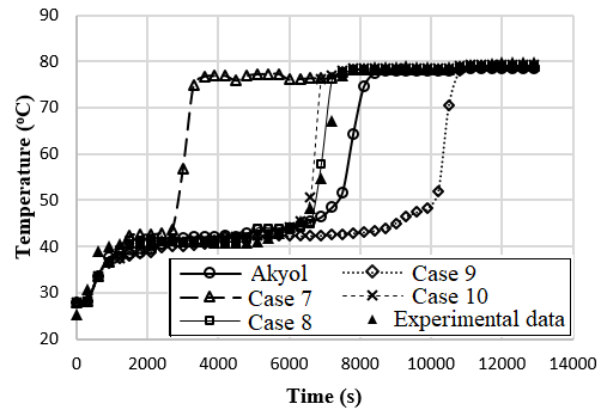


Figure 16. Comparison of experimental results and model results (Case 7-10) for temperature measurement point  $r=0.0805m$

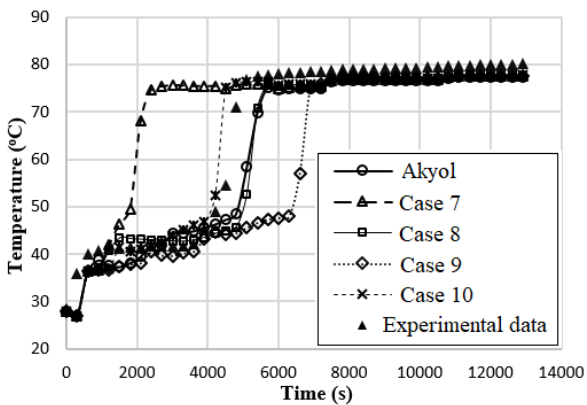


Figure 15. Comparison of experimental results and model results (Case 7-10) for temperature measurement point  $r=0.0615m$

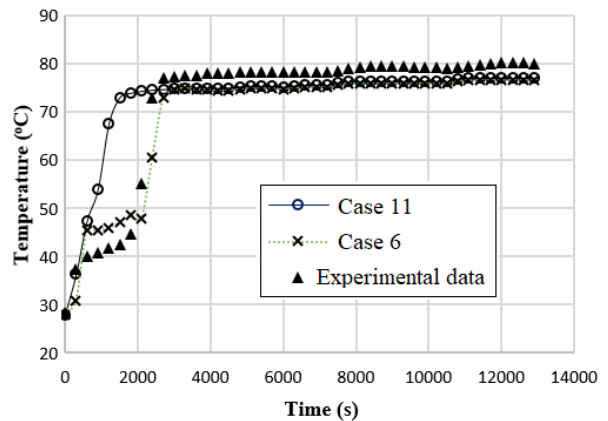
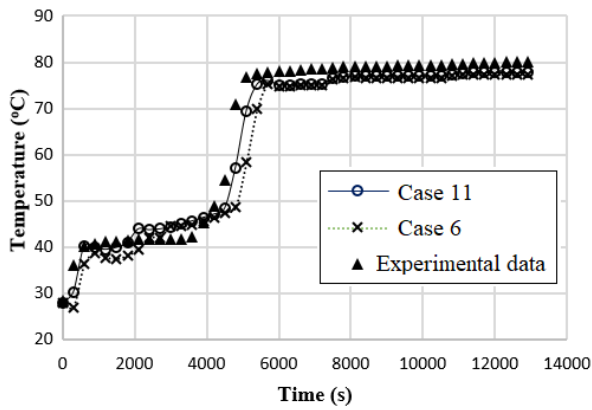
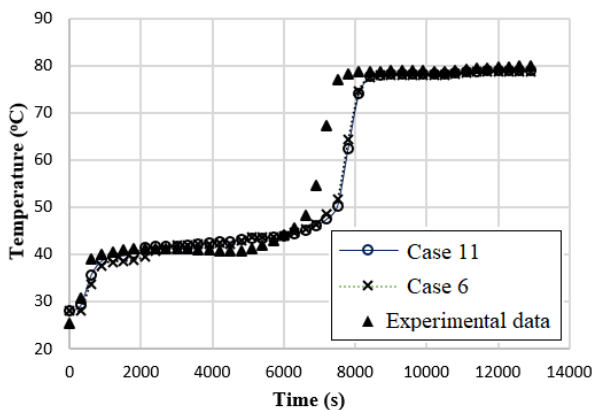


Figure 17. Comparison of experimental results and model results (Case 6 and Case 11) for temperature measurement point  $r=0.0425m$



**Figure 18.** Comparison of experimental results and model results (Case 6 and Case 11) for temperature measurement point  $r=0.0615m$

After examining the effect of the effective volumetric heat capacity on the temperature field, the partially linearized heat transfer problem is solved to investigate the effect of the effective thermal conductivity on the temperature field. The following situation has been taken into account in order to examine the effect of the change in  $k_c(T)$  on the temperature range and drying time. The effective thermal conductivity was taken as  $k=0.06$  W/mK for Case 11 and for  $h$ ,  $\tau$  and  $C_{ve}$  expressions, Case 6 values are used. The results are compared with Case 6 and experimental results (Figures 17-19).



**Figure 19.** Comparison of experimental results and model results (Case 6 and Case 11) for temperature measurement point  $r=0.0805m$

When the model results are examined, it is seen that the results are very close to the results obtained by constantly taking the model results solved according to the temperature of the heat conductivity. This shows that the effect of the thermal conductivity on the temperature field is very small and the thermophysical property can be taken as a constant for such problems when the pressurized hot air is passed through the wool yarn bobbins.

As a result, when the effect of time and space step is examined; the selection of the time step over the whole time interval and the space step at a constant value for all measuring points caused the model to perform poorly. Thus, a small selection of the space step at the temperature measurement points (the inner parts of the yarn bobbin), in which the phase conversion takes place quickly and a large selection at points farther from the temperature measurement points, is deemed appropriate.

The time step was chosen to be large within the time period when the phase conversion was intense. With the completion of the phase conversion, the temperature values in the bobbin begin to increase sharply. For this reason, the problem has been solved by reducing the time step value in this period. It is seen that the model gives better results.

It has been observed that the evaporation profile geometry (rectangular or not) has no effect on the solution of the problem when  $C_{ve}(T)$  is selected. The key criterion for selecting  $C_{ve}(T)$  is  $T_b$  and  $T_s$ , which is seen from the model results. Finally, the model results show that the effect of  $k_c(T)$  is very small in problems where this kind of convection effect is great.

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