Corporate Tax Payoff in A Game Theory Approach

Oyun Teorisi Yaklaşımı ile Kurumlar Vergisi Ödemeleri

ERSİN KIRAL¹, Can MAVRUK²

ABSTRACT
This article investigates corporate tax payoffs in a repeated game theory approach. An application of mixed strategy infinitely repeated game for corporate tax payoffs is provided with four payment types included under new draft tax procedural law. The Folk Theorem is used to find SPNE of infinitely repeated game strategies. The results demonstrate that: (1) the solution set of zero sum game is linear and that of variable sum game is trapezoidal; (2) Grim-trigger and Tit-for-tat of defection both are not appropriate strategies; (3) in pure and mixed strategy equilibrium, TRA needs high audit rates in order to force CTP to point of indifference; (4) CTP will prefer evading tax indefinitely and if possible request settlement when get caught for evading tax; and (5) the number of qualified tax inspectors and audit rates must be increased to break the courage of CTP evading tax.

Keywords: Repeated games, business tax, tax evasion, tax law

1. INTRODUCTION
Development of repeated games starts with Shapley (1953) who defined the model of stochastic games and investigated several models of continuing actions. Luce and Raiffa (1957) mentioned repeated games using prisoner’s dilemma with discount factor. Aumann (1959, 1960, 1961) studied the relation between repetition and cooperation and he showed that non-cooperative infinitely repeated game and cooperative one-shot game both have the same solution and Aumann (1966) introduced the model of repeated games with incomplete information. Using Bayesian approach Harsanyi (1967) formulated games of incomplete information consistently for the first time. Following Harsanyi (1967), Aumann and Maschler (1967-1968) developed the model of repeated games with incomplete information.

Nayyar (2009) analyzed tax payoffs based on cost and benefit in symmetric and asymmetric games. At each time period t=1, 2, 3, … both players may not defect. If we employ the discussion to our case, the cost of not defecting for corporate tax payer (CTP) is a revenue for Turkish Revenue Administration (TRA). In addition, not auditing in this case, TRA saves the audit cost. Therefore, TRA income will always be greater than CTP cost of being honest by a difference of audit cost plus no defection cost. If we assume that players are risk-neutral and that no audit rate is greater than...
no evasion rate in any period, we have an asymmetry case for the set of tax payoffs. Based on the history of audit rates and tax declaration rates, a symmetric game is unlikely to happen in this study because probabilities of both players not defecting are not equal.

The aim of this article is to investigate applicability of strategies of infinitely repeated games to corporate tax payoffs. In infinitely repeated Prisoner’s dilemma, researchers expect that Grim-trigger be a subgame-perfect Nash equilibrium (SPNE) and Tit-for-tat be a Nash equilibrium but not an SPNE. Do we get same results with a similar analysis?

In this article, we consider pure strategy and mixed strategy zero sum game and variable sum game as a stage game repeatedly played between CTP and TRA. We construct strategy profile matrix forms of payoffs for pure and mixed strategies. Throughout this article only three unavoidable taxes for corporations are included in all strategy profile matrix forms: corporate tax (CT), advance corporate tax (ACT) and value added tax (VAT). These three taxes are used to evaluate payoff functions and expected payoffs of the players based on their strategies. Audit rate of indifference (ARI) and evasion rate of indifference (ERI) are derived from equality of the expected payoffs. In equilibrium, TRA must be indifferent between A and NA to find CTP’s equilibrium mixed strategy. If we reverse, in equilibrium, CTP must be indifferent between E and NE to find TRA’s equilibrium mixed strategy.

The main objective of this paper is to investigate corporate tax payoff in a repeated game theory approach under NDTPL. To this end, present value or discounted sum of payoffs in pure and mixed strategies are calculated and The Folk Theorem is applied to check for subgame-perfect Nash equilibrium. The main question is “can we find a sufficiently large discount rate so that strategy is an SPNE?”

The remainder of the article is constructed as follows. In section 2, literature related to our study is provided. In section 3, repeated game is defined. In section 4, strategies of players are described. In section 5, Nash equilibrium is defined. In section 6, the game model is defined. In section 7, the theory of the games is given. In section 8, the data is provided. In section 9, an application is provided. In section 10, possible outcomes of the NDTPL and the results of the application are discussed. In section 11, article is concluded.

2. LITERATURE SUMMARY

Arrow and Honkapohja (1985) investigated repeated games of incomplete information in which players may not possess some of the relevant information about the one-shot game. According to the author, the repetition enables players to infer and learn about the other players from their behavior, and therefore there is a subtle interplay of concealing and revealing information: concealing, to prevent the other players from using the information to your disadvantage; revealing, to use the information yourself, and to permit the other players to use it to your advantage.

Aumann and Maschler (1966) introduced the model of repeated games with incomplete information analyzing long-term interactions in which some or all of the players do not know which stage game \( G \) is being played. The game \( G = G^k \) depends on a parameter \( k \); at the start of the game a commonly known lottery \( q(k) \) with outcomes in a product set \( S \) is performed and player \( i \) is informed of the \( i \)-th coordinate of the outcome.

Hart (2006) showed the main scientific contributions Robert J. Aumann has made to the areas of repeated games, knowledge, rationality and equilibrium and perfect competition. The author considered repeated games as long-term interactions with interdependent stages, reactions to past experience and future impact of choices. The author used the Folk Theorem to represent the set of Nash equilibrium payoffs geometrically.

Hinriches (1969) analyzed tax evasion in a game theory approach by constructing a zero sum game matrix and a variable sum game matrix. TP was a row player and government was a column player in both matrices where the government enforced a high tax rate 80% and a low tax rate 20%. He found an optimum solution in each game and found general conditions of evasion and audit.

Matsushima (2014) investigated an infinitely-repeated prisoners’ dilemma with imperfect monitoring and considered the possibility that the interlinkage of the players’ distinct activities enhances implicit collusion. The author showed a necessary and sufficient condition for the existence of a generous tit-for-tat Nash equilibrium.

Nayyar (2009) analyzed two models that involve repeated interaction in an environment where some information is private. The author characterizes the
equilibrium set of infinitely repeated game in which one of the players does a favor to the other player and assumes that the cost of doing a favor is less than the benefit to the receiver so that, always doing a favor is the socially optimal outcome. The author shows that the equilibrium set expands with discount factor and finds sufficient conditions under which equilibria on the Pareto frontier of the equilibrium set are supported by efficient payoffs.

3. PLAYERS AND ACTIONS

This is a two person game in which players are the agents of the both sides. The agent of corporation is the accountant, director, CEO or the owner. The agents of corporation are considered as one person, which is defined as dependency by Fukofuka (2013). The agent of the government is tax auditor, tax administration, bank or tax revenue administration. The agents of government are also considered as one person. A game of incomplete information is played between the two persons in which CTP as the first player starts the game by declaring an income to TRA based on history of audits. TRA as the second player responds to this move by reviewing past financial activities (tax payments, bank transactions, inputs, outputs etc.) of the CTP to decide whether declaration is substantially less than expected declaration. If declared tax is substantially less than expected tax then TRA will audit. If not, TRA will not audit. At a Nash equilibrium, as the agent of the government, TRA wants to collect more tax to maximize tax payoff and as the agent of the corporation, CTP do not want to pay actual tax owed in order to minimize tax payoff.

We assume that once corporation is established, both players cooperate in the first year, i.e. the CTP declares what is expected by the TRA and TRA does not audit. Therefore, both cooperate at (no evasion, no audit). In this case the CTP is considered to be honest. If the CTP decides not to declare what is expected by the TRA, then it is considered as deviation to defection, i.e. tax evasion. In this case, the response of the TRA will also be deviation to defection, i.e. audit. Tax evasion is considered as underdeclaration of income. Tax evasion includes underreporting or not reporting sales, nylon invoice (reporting cost that does not exist actually) and reporting donations which cannot be deducted from the tax owed. Evasion cost and audit costs of CTP are theoretically included in variable sum games. Audit costs are considered as administrative audit cost (AAC) and CTP audit cost (TAC) in the game matrix. AAC is measured by the audit cost over per ₺100 revenue collected. TAC is measured by the sum of possible bribery and extra accounting cost in case of an audit. In case of an audit and getting caught for tax evasion, CTP has to pay fines and interests as described in procedural tax law (TPL) and TRA has expenses in tax collection process. In all the strategy profiles of underdeclaration, we assume that CTP declares $X = \sum_{n} X_{n}$, a proportion of $B = \sum_{n} B_{n}$ where $0 \leq X \leq B$ and $B$ is actual income or base. In case of no audit, a tax amount is paid from a declared income as expected, actual or evaded. In case of audit and getting caught (both players defect), in addition to declared tax payment $rX$, evaded tax $r(B - X)$, tax fines $f$ and interest $i$ on evaded tax where $r$ is the tax rate are to be paid. Yavaslar (2015) described three different interest rates imposed in Turkish TPL: (1) default interest; (2) late interest; and (3) deferment interest. Default interest and late interest both are 16.80% annually, and deferment interest is 12% annually. Tax payment schedule with respect to fine rates are given in Appendix A.2.

4. REPEATED GAME

CTP randomize over stage game strategy profiles in every quarter of the year and is evaluated by TRA in every taxation year. If this behavior exists for long period of time, it can be considered as a repeated game. If a corporation exists indefinitely, the game is called an infinitely repeated game.

A finitely repeated game is played over discrete time periods. In each period a finite number of players play a stage game selecting actions independently and simultaneously. The payoff is defined as the sum of the utilities in each stage game for every time period. A repeated game strategy must specify the action of a player in a given stage game given the entire history of the repeated game (Knight, 2017).

A main objective of studying repeated games is to explore the relation between the short term incentives (within a single period) and long term incentives (within the broader repeated game). Conventional wisdom in game theory suggests that when players are patient, their long-term incentives take over, and a large set of behavior may result in equilibrium. Indeed, for any given feasible and “individually rational” payoff vector and for sufficiently large values of $\delta$, there exists some subgame perfect equilibrium that yields the payoff vector as the average value of the payoff stream (OpenCourseWare, 2012).
5. STRATEGIES OF THE PLAYERS

Strategies of players are pure strategy and mixed strategy. The set of pure strategies is mapped into the real numbers by a payoff function $u_i$ of player $i$ and mixed strategy is a convex combination of pure strategies (Nash, 1951). In infinitely repeated pure and mixed strategy games, both players play Grim-Trigger and Tit-for-tat. Strategies are similar to those in infinitely repeated Prisoner’s Dilemma. In Grim Trigger strategy both players cooperate in initial stage ($n=0$), cooperate at time $n$ if the previous history $(0,1,2,...,n–1)$ is cooperation and defect if a player played at least one $E$ or at least one $A$ in the history. In tit-for-tat strategy both players cooperate at initial stage and a player’s action is the other player’s previous action at each of the following stages.

5.1. CORPORATE TAX PAYER STRATEGIES

CTP has two strategies: (1) be honest (NE) at probability $1–q$ and pay all taxes based on actual income; (2) evade tax (E) at probability $q$ and pay less tax than expected to minimize cost. CTP is uncertain whether TRA will conduct a tax audit or not on the tax return. However the CTP knows about audit rates in past years.

5.2. TURKISH REVENUE ADMINISTRATION STRATEGIES

TRA has two strategies: (1) audit (A) at probability $p$ when declared income is substantially less than expected income; (2) do not audit (NA) at probability $1–p$ when expected income is greater than or equal to declared income. TRA is also uncertain whether CTP is honest or tax evader. However TRA knows about tax evasion rates in past years.

6. NASH EQUILIBRIUM

An equilibrium of a noncooperative game is a profile of strategies, one for each player in the game, such that each player's strategy maximizes his expected utility payoff against the given strategies of the other players (Nash, 1950). In a zero-sum game, Nash equilibrium, if exists, can be found using minmax strategy. In a variable sum game Nash equilibrium, if exists, can be found by any outcome that is best response for both players.

According to the Folk Theorem, the set of Nash equilibrium outcomes of the repeated game $G^*$ is precisely the set of feasible and individually rational outcomes of the one-shot game $G$ (Hart, 2006) and the set of equilibrium points is the solution of the game (Nash, 1951).

Hitzig Z., Hoffman M., and Yoeli E. (2013) defines pure Nash equilibrium as follows: A strategy profile $s \in S$ is a pure Nash equilibrium if $\forall i$ and $\forall s_i \in S_i$,

$$U_i(s_i, s_{-i}) \geq U_i(s'_i, s_{-i})$$

7. THE MODEL

Let $G=(S_i,u_{ijm})$ be a two person game for players $i=1, 2$ and for payment type $m=1, 2, 3, 4$ for variable sum game. Here $G$ is a one-shot simultaneous game. $S_i$ is a finite set of actions for player $i$ and $u_{ijm}$ is player $i$’s stage game utility function or payoff in $j$ th stage of variable sum game for payment type $m$ where $S = S_1 \times S_2$ is a finite set of strategy profiles. Elements of $S_i$ are referred to as actions (Kalai, Samet and Stanford, 1988) and an action pair of $S$ is called an outcome (Abreu and Rubinstein, 1988). Player $i$’s stage game utility functions in $j$ th stage of zero sum game in case of settlement and variable sum game are denoted by $u_i^0$ and $u_i$ respectively. Here $S_1=\{(A, NA)\}$ and $S_2=\{(E, NE)\}$ are pure strategy sets of player 1 and player 2 respectively and $S=\{(A, E), (A, NE), (NA, E), (NA, NE)\}$ where $(NA, NE)$ is cooperation and $(A, E)$ is defection for both players. We consider an infinitely repeated game $G^*$ (supergame of $G$), each play of which consists of an infinite repetition of plays of $G$. In each stage of $G^*$ (each play of $G$) the players are assumed to know the outcomes of all previous stages. The choices of the players in $G^*$ are referred to as “strategies”. Regardless of the play of any previous strategy profiles, player $i$ plays strategy $S_i$ in any stage Nash profile.

Present value or discounted sum of payoffs is

$$G^* = \sum_{j=0}^{\infty} \beta^j u_{ijm}$$

respectively where $0<\beta<1$. Average payoffs is

$$\overline{G}^* = (1-\beta) \sum_{j=0}^{\infty} \beta^j u_{ijm}$$

The payoff can also be calculated as the limiting average of payoffs using

$$\overline{G}^* = \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} u_{ijm}$$

This behaves like the limiting case of (1) as $\beta$ approaches to 0 (N.Megiddo, 1994).
8. THEORY

An infinitely repeated game is a combination of an initial stage and subsequent stages repeated indefinitely. Each subsequent stage game payoff is discounted constantly at a positive discount rate depending on an interest rate of the corresponding year. Based on the strategy of the players, present value is calculated by the sum of these subsequent stage game payoffs. Present and future payoffs can be estimated using Grim-trigger regardless of strategy history and using Tit-for-tat based on previous time. Finally, existence of Nash equilibrium and sub-game-perfect Nash equilibrium (SPNE) are checked according to the Folk Theorem.

9. TAX PAYOFF

For different taxes and fines imposed, we introduce a general payoff function (G) for both zero-sum game and variable sum game. Using this function for a finite number \( n \geq 1 \) of taxes we construct a base payoff function \( (u_{ij0}) \) for zero-sum game payoff matrix and also we generate sub-functions \( (u_{ijm}) \) of \( G \) based on payment schedule for the variable sum game matrix for \( 41 \leq q \leq 4 \) quarters. Each sub-function is based on evaded tax, penalty, interest, audit cost and evasion cost where available. The general representative payoff function of TRA from defection \{E, A\} is

\[
G = T_n + ACT_q + I - R_n - C_1
\]

where

\[
T_n = D_n + CVAT_n
\]

\[
D_n = \sum r_nX_n \quad \text{is the sum of declared taxes and}
\]

\[
X_n \quad \text{is declared income for each tax.}
\]

\[
CVAT_n = \sum r_n(B_n - X_n)(1 + f + i_n) \quad \text{the sum of ARs from evaded CT and evaded VAT, fines and late interests}
\]

9.1. Zero Sum Game

If CTP plays strategy 1 (S1), regardless of TRA strategy, CTP payoff is the base tax amount which is equal to the declared tax amount: \( \sum r_nB_n \). In this case, game value is the declared tax amount \( v = \sum r_nB_n \).

Ignoring \( ACT_q \), \( I \), \( R_n \) and \( C_1 \) in (4), we find base game model (payoff function) for tax payoff

\[
u^0(s_t) = D_n + CVAT_n + ACT_q
\]

and strategy profile matrix form of base payoffs is given in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Base payoff matrix for zero sum game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero sum game</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>TRA</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>NA</td>
</tr>
</tbody>
</table>

To find ARI, we set the expected revenue (ER) in strategy 2 (S2) to be equal to the ER in S1:

\[
p \sum r_nB_n + (1-p) \sum r_nB_n = p \left[ \sum r_nX_n + \sum r_n(B_n - X_n)(1+f+i_n) + \sum r_{act}(B_q - X_q)(f+i_q) \right] + (1-p) \sum r_nX_n
\]

which implies
\[
\sum r_n (B_n - X_n) = p \left( \sum r_n X_n + \sum r_n (B_n - X_n)(1 + f + i_n) + \sum r_{act} (B_q - X_q)(f + i_q) \right)
\]

where \( X_n < B_n \) and \((B_n - X_n) \neq 0\). Solving (6) for \( p \) gives

\[
P = \frac{\sum r_n X_n + \sum r_n (B_n - X_n)(1 + f + i_n) + \sum r_{act} (B_q - X_q)(f + i_q)}{\sum r_n (B_n - X_n)}
\]

If \( \text{TRA} \) satisfies \( p > \frac{\sum r_n X_n + \sum r_n (B_n - X_n)(1 + f + i_n) + \sum r_{act} (B_q - X_q)(f + i_q)}{\sum r_n (B_n - X_n)} \), then \( \text{CTP} \) declaration will approach to \( B \) as proportional to the greatness of \( p \) than the point of indifference. On the other hand, if \( p > \frac{\sum r_n X_n + \sum r_n (B_n - X_n)(1 + f + i_n) + \sum r_{act} (B_q - X_q)(f + i_q)}{\sum r_n (B_n - X_n)} \), then \( \text{CTP} \) declaration will diverge from \( B \) as proportional to the smallness of \( p \) than the point of indifference. Audit rate \( p \) and fine rate \( f \) are inversely proportional to each other in (7).

9.2. Variable Sum Game

We consider pure strategy and mixed strategy variable sum games. Strategy profile matrix form of payoffs are constructed based on payment types. All payoff functions \( u_{ijm} \) enumerated by (8), (9), (10) and (11) are generated from \{\( E, A \}\) as \( \text{CTP} \) payoffs to \( \text{TRA} \).

In this game, audit costs for \( \text{TRA} \) and for \( \text{CTP} \) and evasion cost for \( \text{CTP} \) are included. \( \text{CTP} \) declares \( \text{ACT} \) for every quarter of the fiscal year, \( \text{VAT} \) every month of the fiscal year and \( \text{CT} \) every year. In S2 of \( \text{CTP} \), \( C_1 \) is the audit cost for \( \text{TRA} \), \( c_1 \) is the audit cost and \( k \) is evasion cost for \( \text{CTP} \). In S1 of \( \text{CTP} \), \( \text{CTP} \) pays actual tax amount \( rB \) which costs \( C_0 \) for \( \text{TRA} \) and \( c_0 \) for \( \text{CTP} \). In this case there is no evasion and revenue of \( \text{TRA} \) becomes \( rB - C_0 \). This shows a loss of \( C_0 \) for \( \text{TRA} \). We assume that \( c_0 \leq c_1 \).

Even though all taxes are declared, a tax loss may be caused deliberately or indeliberately for at least one quarter of the fiscal year. Base difference often arises from underdeclaration, disallowable or over-stated expenses recorded to books, an unrecorded invoice or undocumented sales.

9.2.1. Remorse Exemption

Replacing \( f, l \) and \( R_n \) with 0 in (4) gives TRA payoff from defection:

\[
u_{ij1} = T_n + ACT_j - C_i
\]

Table 2 shows strategy profile matrix form of payoffs for correction (COR) and remorse exemption (REM) declarations.

9.2.2. Tax Completion After Statutory Period

\( \text{CTP} \) may select COR together with tax completion after statutory period (ASP). In this case \( \text{CTP} \) will have to pay a fine for tax loss and late interest. Therefore, substituting 0 for \( l \) in (4) gives TRA revenue from \( \text{CTP} \) payoff ASP:

\[
u_{ij2} = T_n + ACT_j - R_n - C_i
\]

Table 3 shows strategy profile matrix form of payoffs for the payment ASP.

Table 2: Game matrix for payoffs from correction and remorse exemption

<table>
<thead>
<tr>
<th></th>
<th>( \text{NE} )</th>
<th>( \text{CTP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{TRA} )</td>
<td>( \sum r_n B_n - C_0 ); ( \sum r_n B_n + c_0 )</td>
<td>( T_n + ACT_j - C_i ); ( T_n + ACT_j + c_i )</td>
</tr>
<tr>
<td>( \text{NA} )</td>
<td>( \sum r_n B_n ); ( \sum r_n B_n )</td>
<td>( D_n ); ( D_n + k )</td>
</tr>
</tbody>
</table>

Table 3: Game matrix for payoffs after statutory period

<table>
<thead>
<tr>
<th></th>
<th>( \text{NE} )</th>
<th>( \text{CTP} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{TRA} )</td>
<td>( \sum r_n B_n - C_0 ); ( \sum r_n B_n + c_0 )</td>
<td>( T_n + ACT_j - R_n - C_i ); ( T_n + ACT_j - R_n + c_i )</td>
</tr>
<tr>
<td>( \text{NA} )</td>
<td>( \sum r_n B_n ); ( \sum r_n B_n )</td>
<td>( D_n ); ( D_n + k )</td>
</tr>
</tbody>
</table>

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9.2.3. Settlement

After an audit starts, settlement (SET) can be requested by CTP before or after tax is assessed. We consider the first. In this case, one of the fine rates of 1, 2 or 3 is imposed. In addition, special irregularity fine is imposed in cases including failure to issue documents in accordance with tax laws, failure to keep daily records in books, failure to comply with accounting standards and chart of accounts and to refrain from giving information during the tax inspection (TRA 2016d). We consider that one of these cases is committed. According to NDTPL, there will be no reduction on actual tax liability even though a settlement is granted by TRA. Hence, in this game late interest reductions does not apply either. Late interest reductions are applied in existing tax procedure law (TRA, 1961) due to reductions on actual tax liabilities. Therefore, \( F_k = 0 \) in (4) implies that TRA revenue from CTP payoff after SET agreement is

\[
T_n + ACT_q - R_n + F - C_i = 0 
\]

and Table 4 shows strategy profile matrix form of payoffs after settlement.

9.2.4. No Settlement

In case settlement requirements are not met, CTP may file: (1) no lawsuit; (2) a lawsuit. This section includes the first only. In this case CTP can request (a) no reduction (NR); (b) reduction (R). The reasons for not meeting settlement requirements include: (a) not joining to settlement meeting; (b) not signing settlement report although joining to meeting or wanting to sign the report with prejudice. In this case tax will be assessed by TRA as proposed on inspection report (TRA, 2007).

If no reduction is requested, \( F_k = 0 \) and \( R_n = 0 \) in (4) implies that TRA revenue from CTP payoff with no settlement (no SET) agreement is

\[
u_{i,j} = T_n + ACT_q + F - C_i
\]

and Table 5 shows strategy profile matrix form of payoffs after no settlement is reached.

If CTP does not file a lawsuit and requests a reduction, the game model is the same model as in (10) and the game matrix is the same matrix as in Table 4.

10. Data

Data is extracted from TRA Activity Reports and TACOM Activity Reports. Declaration rates are calculated for each tax by dividing the declared income by actual income. Tax rates are calculated by dividing requested tax amount to be levied by base difference, and fine rates are calculated by dividing tax fine by tax amount to be levied. Table 6 shows according to results of 2015 audits in Turkey that declared CT rate and declared ACT rate are approximately 58% and 28.5% respectively.

<table>
<thead>
<tr>
<th>Tax type</th>
<th># of TP audited (000)</th>
<th>Declared income X (₺ million)</th>
<th>Base difference B-X (₺ million)</th>
<th>Tax difference (₺ million)</th>
<th>Levied tax (₺ million)</th>
<th>Declared rate X/B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corporate</td>
<td>0.491</td>
<td>38.046</td>
<td>27.709</td>
<td>5.190</td>
<td>5.529</td>
<td>0.579</td>
</tr>
<tr>
<td>VAT</td>
<td>2.104</td>
<td>1229.173</td>
<td>464.869</td>
<td>41.539</td>
<td>57.313</td>
<td>0.726</td>
</tr>
<tr>
<td>Advance</td>
<td>1.066</td>
<td>32.711</td>
<td>81.930</td>
<td>8.874</td>
<td>9.069</td>
<td>0.285</td>
</tr>
</tbody>
</table>

Source: Constructed from (TRA, 2016c)
The 2015 active number of CTPs in Turkey is about 700 thousand (TRA, 2016a). Therefore, the mean annual corporate tax paid per TP is ₺52.87 thousand in 2015. Table 7 shows that CT payoff rate after audit is 0.14.

Table 8 shows that audit rate increased from 4.4% in 2012 to 15.33% in 2013 which is a 246% increase. In 2014 it decreased to 11.62% which is a 24% decrease and in 2015 again increased to 15% which is a 22.5% increase.

Table 9 shows that TRA audit cost per ₺100 collected in 2015 is ₺0.53 after a 33% decrease over the last 10 years (TRA, 2016c).

Table 10 shows the number of active tax auditors in 2015 based on the groups established with legislative decree No.646 (TACOM, 2016). These groups are classified as: (A) Small and middle sized TP auditors; (B) Large scaled TP auditors; (C) Organized tax evasion auditors; (Ç) Hidden capital, transfer pricing and overseas earnings auditors.

Table 11 shows that the number of large scaled TPs per auditor, the number of inspections per auditor, and tax amount collected (₺ million) per auditor all have increased over the last three years. On the other hand, tax amount per inspection and the number of tax auditors have decreased over the last three years.

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**Table 7: Tax audit results according to report evaluation commission**

<table>
<thead>
<tr>
<th>Tax Type</th>
<th>Base Difference Found (₺ billion) (1)</th>
<th>Amount Of Tax To Be Levied (₺ billion) (2)</th>
<th>Tax Fine (₺ billion) (3)</th>
<th>Tax Rates (2)/(1)</th>
<th>Fine Rates (3)/(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT</td>
<td>4.452993319</td>
<td>0.625009452</td>
<td>1.160898911</td>
<td>0.140</td>
<td>1.86</td>
</tr>
<tr>
<td>ACT</td>
<td>6.183449336</td>
<td>0.474823283</td>
<td>0.652303996</td>
<td>0.077</td>
<td>1.37</td>
</tr>
<tr>
<td>VAT</td>
<td>5.781288813</td>
<td>4.490696278</td>
<td>9.970059589</td>
<td>0.778</td>
<td>2.22</td>
</tr>
</tbody>
</table>

Source: TACOM 2015 Activity Report

**Table 8: Inspection rates for large-scale taxpayers**

<table>
<thead>
<tr>
<th>Year</th>
<th>The number of large-scale TP (000)</th>
<th>The number of inspected large-scale TP (000)</th>
<th>Audit rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012</td>
<td>13.288</td>
<td>0.589</td>
<td>%4.43</td>
</tr>
<tr>
<td>2013</td>
<td>13.774</td>
<td>2.111</td>
<td>%15.33</td>
</tr>
<tr>
<td>2014</td>
<td>15.591</td>
<td>1.811</td>
<td>%11.62</td>
</tr>
<tr>
<td>2015</td>
<td>16.735</td>
<td>2.511</td>
<td>%15.00</td>
</tr>
</tbody>
</table>

Source: TACOM 2012-2015 activity reports

**Table 9: Administrative costs/gross revenue collected in percent.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Audit Cost %</td>
<td>0.79</td>
<td>0.76</td>
<td>0.74</td>
<td>0.73</td>
<td>0.82</td>
<td>0.69</td>
<td>0.69</td>
<td>0.69</td>
<td>0.57</td>
<td>0.58</td>
<td>0.53</td>
</tr>
</tbody>
</table>

Source: TRA 2015 activity report (TRA, 2016c)

**Table 10: Active Number of Auditors Breakdown With Respect To Groups**

<table>
<thead>
<tr>
<th>Title</th>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
<th>Group Ç</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax inspector-general</td>
<td>0</td>
<td>155</td>
<td>15</td>
<td>20</td>
<td>190</td>
</tr>
<tr>
<td>Tax inspector</td>
<td>2485</td>
<td>232</td>
<td>41</td>
<td>37</td>
<td>2795</td>
</tr>
<tr>
<td>Assistant tax inspector</td>
<td>1022</td>
<td>44</td>
<td>0</td>
<td>0</td>
<td>1066</td>
</tr>
<tr>
<td>Total</td>
<td>3507</td>
<td>431</td>
<td>56</td>
<td>57</td>
<td>4051</td>
</tr>
</tbody>
</table>

Source: TACOM 2015 activity report
11. APPLICATION

All strategy profile matrix forms in section 9 are calculated in this section. We want to find Nash equilibrium payoffs for finitely and infinitely repeated games based on game strategies. We use discounted sum or present value to calculate payoffs. Before all these, we apply the data to the game strategies to calculate payoffs for zero sum game and variable sum game matrices.

Throughout this section the corporate income tax rate levied on business profits is 19.5% for honest taxpayers and 20% for the others according to NDT-PL. Actual income is $431,000, declared corporate annual income is $250,000 and CTP is audited on 16 December 2016. Therefore tax accrual date is 15 January 2017 which is 30 days after the notification date and also the last day to file a lawsuit in court. Therefore, base difference is $B-\delta X=\$181,000 and declared tax amount is $250,000(0.20) = \$50,000 which is not significantly different from \$52,865 (mean corporate tax paid in 2015). Evaded CT is \$36,200.

Actual income of the third quarter is \$B_3=\$84,382.28, declared AC income is \$X_3=\$24,048.95 and undeclared AC income is \$B_3-\delta X_3=\$60,333.33 in the third quarter. ACT evaded in the third quarter is 0.20(60,333.33) = \$12,066.67. VAT evaded in the 3rd quarter is 0.18(60,333.33) = \$10,860 and VAT on the base difference is 0.18(181,000) = \$32,580. Monthly late interest rate is 1.4%. CTP declaration and payment due dates are April 25 and April 30, 2015 respectively. ACT declaration and payment due dates are the 14th and 17th day of the second month following every quarter, respectively. VAT declaration and payment due dates are 24th of the next month and 26th of the same month respectively.

In zero sum game if CTP plays to be honest and TRA plays no audit, then TRA collects 19.5% CT and 18% VAT, a total of 37.5% of \$431,000. In case of evasion and no audit, tax payoff is 20% CT plus 18% VAT of declared \$250,000 which is \$95,000. In case of audit and evasion, a fine rate of 1 (considered as regular audit) and monthly late interest rate of 0.014 are imposed on CT, VAT and ACT. Strategy profile matrix form or tax payoff matrix given in Table 12 can be considered as a stage game.

### Table 12: Zero sum game payoff (\$000) matrix

<table>
<thead>
<tr>
<th>CTP</th>
<th>E</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRA</td>
<td>247,663</td>
<td>161,625</td>
</tr>
<tr>
<td>NA</td>
<td>95,000</td>
<td>161,625</td>
</tr>
</tbody>
</table>

Pure strategy minmax payoff for TRA is minimum of maximum payoffs of CTP from E and NE. TRA gets maximum of 247,663 from E and maximum of 161,625 from NE and minimum of the two payoffs is 161,625. Pure-strategy maximin payoff for CTP is maximum of minimum payoffs of TRA from A and NA. CTP pays minimum of 161,625 from A and minimum of 95,000 from NA and maximum of the minimum payoffs is 161,625. Therefore, with no probabilities included, (161,625,161,625) at (NE, A) is a Nash equilibrium payoff. So the zero sum game is a pure strategy game. Hence, (161,625,161,625) is a pure-strategy Nash equilibrium payoff. In this equilibrium of repeated game, average payoff of TRA is at least 161,625.

Now we randomize the set of actions. First we calculate TRA's expected payoff from CTP’s mixed strategy. $E\mu^{0,1}_y(A) = 247,663q + 161,625(1-q)$ and $E\mu^{0,1}_y(NA) = 95,000q + 161,625(1-q)$. In equilibrium, TRA wants to randomize if indifferent between A and NA and CTP wants to randomize if indifferent between E and NE. So we set $E\mu^{0,1}_y(A) = E\mu^{0,1}_y(NA)$ : $247,663q + 161,625(1-q) = 95,000q + 161,625(1-q)$.
Solving for \( q \), we find that \( q=0 \) and \( 1-q=1 \). Also 
\[
Eu^0_{2,1}(E) = Eu^0_{2,1}(NE) = 161,625 \]
and 
\[
Eu^0_{2,1}(A) = Eu^0_{2,1}(NA) = 161,625. \]
So minmax payoff of the zero-sum game is \( \mathcal{L}161,625 \), i.e. average payoff from mixed strategy Nash equilibrium is at least 161,625.

An outcome \( u_i \) of \( S \) is individually rational in \( G \) if \( u_i \geq 161,625 \) for both TRA and CTP. The Folk Theorem gives the set of Nash equilibrium outcomes of the infinitely repeated sum zero game in bold straight line segment as shown in Figure 1. So the solution of the zero sum game is \( (u_i, u_j) \leq 161,625 \) for both TRA and CTP. The solution set shows that TRA gets a minimum of \( \mathcal{L}161,625 \) in equilibrium which also implies a minimum average of \( \mathcal{L}161,625 \).

Now we consider finitely repeated game with discount rate \( \beta \) where the stage game is the zero sum game. Regardless of strategy, a finite series of payoffs from cooperation which is \((NA, NE)\) is

\[
161,625+161,625\beta+161,625\beta^2+\ldots+161,625\beta^n \quad (12)
\]
where \( \beta \) is the discount rate at each stage and \( n \) is nonnegative integer.

Figure 2 shows a twice repeated game in which the sums of payoffs after two repeats are given at the end of last stage. \( n=0 \) in (12) gives the initial payoff with empty set of history and \( n=1 \) gives four histories at the first stage which are \((NE, NA)\), \((NE, A)\), \((E, NA)\) and \((E, A)\) and one subgame at each history. Each subgame has \((NE, A)\) as the unique Nash equilibrium. For example, first subgame at history \((NE, NA)\) in Figure 2 has \( (323,250, 323,250) \) as the unique Nash equilibrium. At each history, \((NE, A)\) is played. Therefore, we have \((NE, A)\) as a unique SPNE. Hence, the repeated game has a unique SPNE.

Now we consider infinitely repeated game with discount rate \( \beta \) where the stage game is the zero sum game. Discounted sum or present value from cooperation is 
\[
G^*=161,625+161,625\beta+161,625\beta^2+\ldots+161,625/(1-\beta) \]
and average payoff from cooperation is 
\[
\mathcal{L}G^*=161,625(1+\beta+\beta^2+\ldots)=\mathcal{L}161,625.
\]
Now suppose that both players use grim trigger strategy and that the history is cooperation, i.e. E or A has never been played by any player. More precisely (NE, NA), (NE, NA),..., (NE, NA). If (NE, NA) is played in this period, then from now on (NE, NA) will be played forever. Therefore, present value of next time is $161,625 + 161,625\beta + 161,625\beta^2 + \ldots = 161,625/(1–\beta)$.

On the other hand if any player defects, then each player will defect forever. In this case present value at next time is $247,663 + 247,663\beta + 247,663\beta^2 + \ldots = 247,663/(1–\beta)$. If both players play (NE, NA) in this period, then the payoff for each player will be $161,625 + 161,625\beta/(1–\beta)$. If TRA plays A while CTP plays NE, then TRA gets $161,625 + 247,663\beta/(1–\beta)$ and CTP pays $161,625 + 247,663\beta/(1–\beta)$. Stage game matrix at the given history is given in Table 14.

### Table 14: (Grim, Grim) discounted sums of cooperation history starting from this period

<table>
<thead>
<tr>
<th></th>
<th>CTP</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRA</td>
<td>A</td>
<td>$247.663 + 247.663\beta/(1–\beta)$</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>$95.000 + 247.663\beta/(1–\beta)$</td>
</tr>
</tbody>
</table>

### Table 15: (Grim, Grim) discounted sums of defection history starting from this period

<table>
<thead>
<tr>
<th></th>
<th>CTP</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRA</td>
<td>A</td>
<td>$247.663 + 247.663\beta/(1–\beta)$</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>$95.000 + 247.663\beta/(1–\beta)$</td>
</tr>
</tbody>
</table>
Now we check if \((A, E)\) is a Nash equilibrium: TRA will not benefit from not auditing an evading CTP, but an audited CTP will benefit to deviate from defection to cooperation. Therefore, \((A, E)\) is a Nash equilibrium if and only if 
\[
247.663 + 247.663 \beta/(1-\beta) \geq 161.625 + 247.663 \beta/(1-\beta) \quad \text{or} \quad \beta \leq 1.
\]
So Grim-Trig is an SPNE of the infinitely repeated game.

Now for both players we consider tit-for-tat strategy. We want to check whether this strategy is an SPNE or not. If \((NA, NE)\) is played in this period (time \(n\)), then it will also be played in the next period \((n+1)\) and on. Therefore, discounted sum starting from next period will be 
\[
(161.625, 161.625) + \beta(161.625, 161.625) + \ldots + \beta^n(161.625, 161.625).
\]
If \((NA, E)\) is played in this period, then strategies in the next period and on will be \((A, NE)\), \((NA, NE)\), \((A, NE)\), \ldots. Therefore, discounted sum starting from the next period will be 
\[
(161.625, 161.625) + \beta(95.000, 95.000) + \beta^2(161.625, 161.625) + \ldots = \frac{161.625}{1-\beta^2} + \frac{95.000 \beta}{1-\beta^2} + \frac{161.625 \beta^2}{1-\beta^2}.
\]
If \((A, NE)\) is played in this period, then strategies in the next period and on will be \((A, NE)\), \((A, NE)\), \ldots. Therefore, discounted sum starting from the next period will be 
\[
(161.625, 161.625) + \beta(95.000, 95.000) + \beta^2(161.625, 161.625) + \ldots = \frac{161.625}{1-\beta^2} + \frac{95.000 \beta}{1-\beta^2} + \frac{161.625 \beta^2}{1-\beta^2}.
\]
If \((A, E)\) is played in this period, then \((A, E)\) will be played thereafter. Therefore discounted sum starting from the next period will be 
\[
247.663 + \frac{247.663}{1-\beta}.
\]
Now if we start from this period, game matrix of discounted sums at the given history is given in Table 16.

Table 16: (Tit-for-tat, Tit-for-tat) discounted sums starting from this period

<table>
<thead>
<tr>
<th>CTP</th>
<th>TRA</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>247.663 + \beta \frac{247.663}{1-\beta}</td>
<td>161.625 + \beta \frac{95.000 \beta}{1-\beta^2} + \frac{161.625 \beta^2}{1-\beta^2}</td>
</tr>
<tr>
<td>NA</td>
<td>95.000 + \frac{161.625 \beta + 95.000 \beta^2}{1-\beta^2}</td>
<td>161.625 + \beta \frac{161.625}{1-\beta}</td>
</tr>
</tbody>
</table>

As for variable sum game, game matrices in section 9.2 apply in this part. According to Law No. 6183 as explained in TPL article 112, late interest will be calculated for complete months only (Tax Inspectors Foundation, 2016). There are eight complete months from April 30, 2016 to January 15, 2017. Late interest rate on CT is 8\% and late interest amount is $4,054.40. For ACT there are thirteen complete months from November 17, 2015 to January 15, 2017. Late interest rate on ACT is 13\% and late interest amount is $5,777.52. Throughout this section, all amounts of interest besides \(C_0\) are fixed, and tax liability is the sum of all taxes, fines and late interests. Game matrices are calculated for cases of REM and COR, ASP and COR, SET and NOSET. In the game matrix ACT is underdeclared only for the third quarter of the last fiscal year and hence CT is underdeclared. If CTP plays S1 and TRA plays to audit, then CTP will pay 37.5% of $431,000 which is $161,625 and TRA will collect $161,625 minus the audit cost \(C_0=161,625(0.53)/100 = $856.61\), which is approximately $860. If CTP plays S1 and TRA plays not to audit, then TRA will collect the actual tax which is $161,625. If CTP plays S2 and TRA plays not to audit, then TRA will collect what CTP declares which is 38% of $250,000. If CTP plays S2 and TRA plays to audit, then TRA will collect all fines and interests imposed on both underdeclared ACT and underdeclared CT.
In remorse exemption a COR for CT needs to be submitted because of tax loss from ACT. Hence both REM and COR can be selected at the same time. In this case tax to be levied and fines to be imposed are calculated in ₺ as follows:

\[
\begin{array}{cccc}
\text{CT} & 36,200 & C_i & 931.78 \\
\text{CT late interest} & 4,054.40 & C_o & 856.61 \\
\text{VAT} & 32,580 & \text{ACT late interest} & 2,196.13 \\
\text{VAT late interest} & 5,777.52 & & \\
\end{array}
\]

\[
\begin{array}{cc}
\text{TR} & 174,876.27 \\
\text{Tax payoff} & 175,808.05 + c_1 \\
\text{Tax due} & 80,808.05 \\
\end{array}
\]

Strategy profile matrix form for remorse payment is given in Table 17.

Best response strategies determine Nash equilibrium for each payment type of the variable sum games. First, we determine TRA's best response to each of the CTP's possible strategies without including probabilities. Honest tax payoff of the CTP would pay TRA ₺160,768 or ₺161,625 and the better one is ₺161,625. If the CTP plays E, then TRA's payoffs would be either ₺174,876 or ₺95,000 and greater one ₺174,876 is the best response. The second step is to determine the CTP's best response to each of the TRA's possible strategies. If the TRA decides to audit, the CTP payoff will be either ₺161,625+c_o or ₺175,808+k+c_1 and the best response is ₺161,625+c_o. If TRA does not audit, CTP payoff will be either ₺161,625 or ₺95,000+k and the best response is ₺95,000+k if k<66,625 or ₺161,625 if k>66,625. Any outcome that is a best response for both players is a pure-strategy Nash equilibrium. Therefore, pure-strategy Nash equilibrium payoff is (161.625,161.625) if k>66,625. Otherwise, there is no pure-strategy Nash equilibrium in REM payment type of the game.

According to The Folk Theorem , the set of Nash equilibrium outcomes of the infinitely repeated game for REM is the dotted triangular area in the irregular quadrilateral in Figure 3.

Table 17: Pure strategy matrix for remorse exemption

<table>
<thead>
<tr>
<th>M 1</th>
<th>CTP</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>NE</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>q</td>
<td>1 - q</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRA</td>
<td>A</td>
<td>p</td>
<td>174,876; 175,808 +k+c_1</td>
<td>160.768; 161.625+c_o</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>1 - p</td>
<td>95,000; 95,000+k</td>
<td>161.625; 161.625</td>
</tr>
</tbody>
</table>

Figure 3: Feasible set of payoffs for remorse exemption for k=0 and c0=c1
ERSİN KIRAL, Can MAVRUK

**CTP’s mixed strategy with NE at probability 1−q and E at probability q gives expected payoffs of TRA to be Eu_{1j}(A)=174,876q+160,768 (1−q) and Eu_{1j}(NA)=95,000q+161,625 (1−q) for the actions A and NA respectively. Simplifying gives 160,768+14,108q=161,625−66,625q gives q=0.011 and 1−q=0.989. Similarly CTP’s expected payoffs are Eu_{2j}(E)=(175,808+k+c_1) p+(1−p)(95,000+k) and Eu_{2j}(NE)=(161,625+c_1)p+(1−p)161,625. Simplifying gives (80808+c_1)p+95,000+k and 161,625+pc_0 respectively. Equality of these expected payoffs, Eu_{2j}(E)=Eu_{2j}(NE), gives 66,625−(80808+c_1−c_0)p or p=(66,625−k)/(80,808+c_1−c_0). We assume that c_1≥c_0 because CTP’s audit cost in case of audit is greater than or equal to audit cost in case of no audit. If p is to be between 0 and 1, we must have 0<k<66,625. If CTP’s audit costs are equal, then

\[ p = \frac{(80,808−k)}{88,080} = 0.8245−k/88,080 \quad (11) \]

So p depends on k which can only be known by CTP but 0<k<66,625 implies 0<p<0.8245. However CTP payoff cannot be determined by the value of p alone. The values of c_1, c_0 and k all known by CTP must also be known by researcher. Mixed strategy profile matrix for variable sum game is given in Table 18.

If q is very small then k is very small. So, we can take k=0. Therefore, p=0.8245 from (11). Hence, Eu_{2j}(E)=0.8245(175,808+c_1)+0.1755(95,000) and Eu_{2j}(NE)=0.8245(161,625+c_1) + 0.1755(161,625).

In equilibrium: Eu_{2j}(E)=Eu_{2j}(NE)=161,625 if c_1=c_0. Also expected payoffs of TRA are in equilibrium: Eu_{1j}(A)=Eu_{1j}(NA)=160,918. Therefore, ((A, 0.8245; NA, 0.1755), (E, 0.0011; NE, 0.9989)) is a mixed strategy Nash equilibrium and (160,918,161,625) is a mixed strategy Nash equilibrium payoff. So minmax payoff is €160,918, i.e. average payoff from mixed strategy Nash equilibrium is at least €160,918.

In case of tax completion after statutory period CTP voluntarily submits an e-declaration on January 15, 2017 before an audit is imposed or before referral commission receives the audit report. CTP selects both ASP and COR. Strategy profile matrix form for ASP and COR is given in Table 19.

First we check for pure strategy Nash equilibrium. Using the same discussions from the previous payment type we find that only pure strategy Nash equilibrium is (161,625,161,625) if k<66,625. Otherwise there is no pure strategy Nash equilibrium in ASP payment type of the game. Next, we check for mixed strategy Nash equilibrium. Expected payoffs of TRA are Eu_{1j}(A)=215,128q+160,768 (1−q) and Eu_{1j}(NA)=95,000q+161,625 (1−q) for the actions A and NA respectively. In equilibrium we must have Eu_{1j}(A)=Eu_{1j}(NA):54,360q+160,768=161,625−66,625q. This implies that q=0.0071 and 1−q=0.9929. Similarly expected payoffs of CTP are (216,181+k+c_1)p+(95,000+k)(1−p) and (161,625+c_1)p+161,625 (1−p) for the actions E and NE respectively. Simplifying gives (121,181+c_1)p+95,000+k and 161,625+pc_0 respectively. Equality of these expected payoffs, Eu_{2j}(E)=Eu_{2j}(NE), gives 66,625−(121,181+c_1−c_0)p or p=(66,625−k)/(121,181+c_1−c_0). If CTP’s audit costs are equal, then p=(66,625−k)/(121,181)=0.5498−k/121,181. Using the same discussion from the previous payment type, 0<k<66,625 implies 0<p<0.5498. Expected payoffs of CTP are Eu_{2j}(E)=0.5498(216,181+c_1)+0.4502(95,000) and Eu_{2j}(NE)=0.5498(161,625+c_1)+0.4502(161,625). In equilibrium: Eu_{2j}(E)=Eu_{2j}(NE)=161,625 if c_1=c_0. Also expected payoffs of TRA are in equilibrium: Eu_{1j}(A)=Eu_{1j}(NA)=161,150. Therefore, ((A, 0.5498; NA, 0.4502), (E, 0.0071; NE, 0.9929)) is a mixed strategy Nash equilibrium and (161,150,161,625) is a mixed strategy Nash equilibrium payoff. So minmax payoff is 161,650.

Applying the Folk Theorem for ASP, we get the set of Nash equilibrium outcomes of the infinitely repeated game G as trapezoidal convex set with (215.128, 216.181+c_1) on the upper right vertex of the shape in Figure 3.

---

**Table 18:** Mixed strategy matrix for remorse exemption

<table>
<thead>
<tr>
<th></th>
<th>CTP</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>NE</td>
</tr>
<tr>
<td></td>
<td>0.011</td>
<td>0.989</td>
</tr>
<tr>
<td>TRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0&lt;p&lt;0.8245</td>
<td>174,876; 175,808 + k + c_1</td>
</tr>
<tr>
<td>NA</td>
<td>0.1755&lt;1−p&lt;1</td>
<td>95,000; 95,000+k</td>
</tr>
</tbody>
</table>

---

**Figure 3.** Nash equilibrium outcomes of the infinitely repeat game G as trapezoidal convex set with (215.128, 216.181+c_1) on the upper right vertex of the shape.
For settlement case tax payoff in accordance with both TPL and NDTPL are calculated. CTP requests a settlement within 30 days after receiving penalty notice. Assumptions for this payment type are that (1) settlement is requested before the assessment; (2) one third of the tax loss is reduced before the settlement; and (3) one of the cases of special irregularity fine is committed. TRA sets an appointment day, both agents meet on this date for settlement and they reach the agreement on tax fines. One third of the tax loss is reduced from the tax loss, and only 1/10 of the remaining is imposed, which is 1/15th of the tax loss overall. Since accrued ACT is not paid in the statutory period and it cannot be reduced from the tax calculated over annual return, it will be cancelled. However, late interest will be imposed from the official due date up to settlement date. Settlement can be requested after a regular audit, an ex-officio audit or a tax fraud audit.

After a regular audit, tax is calculated from the books and records in accordance with TPL 29. Strategy profile matrix form for settlement payment is given in Table 16.

Using the same discussions from the previous payment types we find that only pure strategy Nash equilibrium is \((161.625,161.625)\) if \(k>66,625\). Other wise there is no pure strategy Nash equilibrium in SET payment type of the game. Next, we check for mixed strategy Nash equilibrium. Expected payoffs of TRA are \(\text{Eu}_{1j3}(A)=190,180q+160,768(1-q)\) and \(\text{Eu}_{1j3}(NA)=95,000q+161,625(1-q)\) for the actions A and NA respectively. In equilibrium we must have \(\text{Eu}_{1j3}(A)=\text{Eu}_{1j3}(NA)\) where \(q=0.009\) and \(1–q=0.991\). Similarly expected payoffs of CTP are \((191,193+k+c_1)p+(95,000+k)(1–p)+161,625(1–p)\) for the actions E and NE respectively. Simplifying gives \((96,193+c_1)p+95,000+k\) and \(161,625+pc_0\) respectively. Equality of these expected payoffs, \(\text{Eu}_{2j3}(E)=\text{Eu}_{2j3}(NE)\), gives \(66,625–k=(96,193+c_1–c_0)p\) or \(p=(66,625–k)/(96,193+c_1–c_0)\). If CTP’s audit costs are equal, then \(p=(66,625–k)/(96,193)=0.6926–k/96,193\). Using the same discussion from the previous payment type, \(0<k<66,625\) implies \(0<p<0.6926\). Expected payoffs of CTP are \(\text{Eu}_{2j3}(E)=0.6926(191,193+c_1)+0.3074(95,000)\) and \(\text{Eu}_{2j3}(NE)=0.6926(161,625+c_0)+0.3074(161,625)\). In equilibrium: \(\text{Eu}_{2j3}(E)=\text{Eu}_{2j3}(NE)=161,625\) if \(c_0=c_1\).

Also expected payoffs of TRA are in equilibrium: \(\text{Eu}_{1j3}(A)=\text{Eu}_{1j3}(NA)=161,030\). Therefore, \(((A, 0.6926; NA, 0.3074), (E, 0.009; NE, 0.991))\) is a mixed strategy Nash equilibrium and \((161,030, 161,625)\) is a mixed strategy Nash equilibrium payoff. So minmax payoff is 161.030.

Applying the Folk Theorem for ASP, we get the set of Nash equilibrium outcomes of the infinitely repeated game as trapezoidal convex set with \((190.180, 191.193+c_1)\) on the upper right vertex of the shape in Figure 3.

### Table 19: Mixed strategy matrix for after statutory period payment

<table>
<thead>
<tr>
<th></th>
<th>CTP</th>
<th>TRA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>E</strong></td>
<td><strong>NE</strong></td>
</tr>
<tr>
<td></td>
<td>0.0071</td>
<td>0.9929</td>
</tr>
<tr>
<td><strong>TRA</strong></td>
<td>0.5498</td>
<td>215.128; 216.181+k+c_1</td>
</tr>
<tr>
<td></td>
<td>0.4502</td>
<td>95.000; 95.000+k</td>
</tr>
<tr>
<td></td>
<td>160.768; 161.625 + c_0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>161.625; 161.625</td>
<td></td>
</tr>
</tbody>
</table>

### Table 20: Mixed strategy matrix for settlement payment

<table>
<thead>
<tr>
<th></th>
<th>CTP</th>
<th>TRA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>E</strong></td>
<td><strong>NE</strong></td>
</tr>
<tr>
<td></td>
<td>0.009</td>
<td>0.991</td>
</tr>
<tr>
<td><strong>TRA</strong></td>
<td>0.6926</td>
<td>190.180; 191.193+k+c_1</td>
</tr>
<tr>
<td></td>
<td>0.3074</td>
<td>95.000; 95.000+k</td>
</tr>
<tr>
<td></td>
<td>160.768; 161.625 + c_0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>161.625; 161.625</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4: Twice Repeated Variable Sum Game For Settlement

Figure 4 shows the sum of tax payoffs for SET payment type of twice repeated mixed strategy game. At the end of two stages, the first two lines show CTP tax payoff and the third line shows TRA tax payoff.

Now for both players we consider mixed strategy tit-for-tat for settlement. Let (A, E) be played this period and thereafter. (A, E) payoff is 

\[ G^* = \left( 0.009 \times 190.180, 0.6926 \times (191.193 + k + c_1) \right) \]

Hence, discounted sum for (A, E) starting from next period is

\[ G^* = \left( \frac{1.712}{1 - \beta}, \frac{132.420 + 0.6926(k + c_1)}{1 - \beta} \right) \]

and average payoff is \( \bar{G}^* = (1.712, 132.420 + 0.6926(k + c_1)) \). (NA, E) payoff is \( (0.009 \times 95.000, 0.3074 \times (95.000 + k)) \) and (A, NE) payoff is \( (0.991 \times 160.768, 0.6926 \times (161.625 + c_0)) \). For alternating strategies (A, NE), (NA, E), (A, NE), (NA, E)… starting from next period

\[ G^* = \left( 855 \beta, \frac{159.321}{1 - \beta^2}, \frac{29.203 + 0.3074k}{1 - \beta^2}, \frac{111.941 + 0.6926c_0}{1 - \beta^2} \right) \]

\[ \bar{G}^* = \left( \frac{855 \beta}{1 - \beta^2}, \frac{159.321}{1 - \beta^2}, \frac{29.203 + 0.3074k}{1 - \beta^2}, \frac{111.941 + 0.6926c_0}{1 - \beta^2} \right) \]

For alternating (NA, E), (A, NE), (NA, E), (A, NE)… starting from next period

\[ G^* = \left( \frac{855 \beta}{1 - \beta^2}, \frac{159.321}{1 - \beta^2}, \frac{29.203 + 0.3074k}{1 - \beta^2}, \frac{111.941 + 0.6926c_0}{1 - \beta^2} \right) \]

\[ \bar{G}^* = \left( \frac{855 \beta}{1 - \beta^2}, \frac{159.321}{1 - \beta^2}, \frac{29.203 + 0.3074k}{1 - \beta^2}, \frac{111.941 + 0.6926c_0}{1 - \beta^2} \right) \]

Discounted sums starting from this period for (A, E), (A, NE), (NA, E) and (NA, NE) are

\[ \left( \frac{1.712}{1 - \beta}, \frac{132.420 + 0.6926(k + c_1)}{1 - \beta}, \frac{132.420 + 0.6926(k + c_1)}{1 - \beta} \right) \]
\[
\left(\frac{159.321 + 855\beta}{1 - \beta^2} + \frac{159.321\beta^2}{1 - \beta^2}, 111.942 + 0.6926c_0 + \frac{(29.203 + 0.3074k)\beta}{1 - \beta^2} + \frac{(111.942 + 0.6926c_0)\beta^2}{1 - \beta^2}\right)
\]

\[
\left(\frac{855\beta^2}{1 - \beta^2} + \frac{159.321\beta}{1 - \beta^2} + 855, (29.203 + 0.3074k)\beta^2 + (111.942 + 0.6926c_0)\beta + 29.203 + 0.3074k\right)
\]

and \[
\left(\frac{160.170 + 160.170\beta}{1 - \beta}, \frac{49.684 + 49.684\beta}{1 - \beta}\right)
\]

respectively. \(\overline{G^*}\) is same as before.

\[
49.684 + \frac{49.684\beta}{1 - \beta} \geq \left(\frac{(29.203 + 0.3074k)\beta^2}{1 - \beta^2} + \frac{(111.942 + 0.6926c_0)\beta}{1 - \beta^2} + 29.203 + 0.3074k\right)
\]

or

\[
\beta \leq \frac{20.481 - 0.3074k}{62.258 + 0.6926c_0}
\]

If (NA, E) is a Nash equilibrium, then we must have \(\beta \geq \frac{20.481 - 0.3074k}{62.258 + 0.6926c_0}\).

Table 21: Mixed strategy matrix for no settlement payment

<table>
<thead>
<tr>
<th></th>
<th>E</th>
<th>CTP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0054</td>
<td>0.9946</td>
</tr>
<tr>
<td>TRA</td>
<td>0.4194</td>
<td>252.494; 253.839 + k + c_1</td>
</tr>
<tr>
<td></td>
<td>0.5806</td>
<td>95.000; 95.000 + k</td>
</tr>
<tr>
<td></td>
<td></td>
<td>161.625; 161.625</td>
</tr>
</tbody>
</table>

Therefore, Tit-for-tat is an SPNE only if

\[
\beta = \frac{20.481 - 0.3074k}{62.258 + 0.6926c_0}
\]

According to inspection rates for large-scale TPs in Table 8, CTP knows from the past year (history of audit rates) that audit rate is 15% and that even lower before that. Theoretically, CTP expects one audit in about seven years based on 2015 audit rate. In this case TRA audits once in every seven years. In other words, TRA does not defect for the first year and then defects for six years in a row. Assume that TRA repeats this indefinitely. Getting caught in this period for evading tax after an audit, CTP requests a settlement each time with reductions in fines and interests. In this case the limiting average for TRA is

\[
\overline{G^*_L} = \lim_{n \to \infty} \frac{(247.663+95,000*6+247,663+95,000*6+...+247,663+95,000*6)}{n} = \$116,809.
\]

In this article we consider regular, ex-officio and fraud audits. In regular audit for the case of no lawsuit filed, first we assume that no reduction is requested within 30 days after receiving the notification. Even though unlikely to happen this possibility is not ignored in our application. Second, we consider that reduction is requested and granted. Strategy profile matrix form for no settlement is given in Table 17.

The only pure strategy Nash equilibrium is (161.625, 161.625) if \(k > 66,625\). For mixed strategy equilibrium we must have \(\text{Eu}_{1j4}(A) = \text{Eu}_{1j4}(\text{NA})\): \(91,726q + 160,768 = 161,625 - 66,625q\).

This implies that \(q = 0.0054\) and \(1 - q = 0.9946\). Also \(\text{Eu}_{1j4}(E) = \text{Eu}_{1j4}(\text{NE})\): \(253,839 + k + c_1\) or \((158,839 + c_c)_p + (161,625 - k)/(158,839 + c_c)_p = (66,625 - k)/(158,839 + c_c)_p\).

If CTP's audit costs are equal, then \(p = (66,625 - k)/(158,839 - 0.4194 - k/158,839). Using the same discussion from the previous payment type, \(0 < k < 66,625\) implies \(0 < p < 0.4194\). Expected payoffs of CTP are \(\text{Eu}_{1j4}(E) = 0.4194(253,839 + c_c) + 0.5806(95,000 + k)\) and \(\text{Eu}_{1j4}(\text{NE}) = 0.4194(161,625 + c_c) + 0.5806(161,625)\).

In equilibrium: \(\text{Eu}_{1j4}(E) = \text{Eu}_{1j4}(\text{NE}) = 161,625\) if \(c_c = c_1\). Also expected payoffs of TRA are in equilibrium: \(\text{Eu}_{2j4}(E) = \text{Eu}_{2j4}(\text{NE}) = 161,625\).

Therefore, ((A, 0.4194; NA, 0.5806), (E, 0.0054; NE, 0.9946)) is a mixed strategy Nash equilibrium and (161,625, 161,625) is a mixed...
strategy Nash equilibrium payoff. So minmax payoff is 161.264.

Applying the Folk Theorem for no settlement payoff, we get the trapezoidal set of Nash equilibrium outcomes of the infinitely repeated game $G'$ with 

$$(252.494, 253.839 + k + c)$$

on the upper right vertex of the shape in Figure 3.

Now for both players we consider mixed strategy tit-for-tat for no settlement. Play (NA, NE) this period and (NA, NE) thereafter. (NA, NE) payoff is (0.9946*161.625, 0.5806*161.625). Therefore, discounted sum for (NA, NE) starting

\[ G^* = \left( \frac{513}{1 - \beta^2}, \frac{159.900 \cdot 55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{1}{\beta^2}, \frac{1}{\beta^2}, \frac{67.817 + 0.42c_0}{1 - \beta^2} \right). \]

\[ G^* = \left( \frac{513 \beta}{1 - \beta^2}, \frac{159.900 \cdot 55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{1}{\beta^2}, \frac{1}{\beta^2}, \frac{67.817 + 0.42c_0}{1 - \beta^2} \right), \]

and

\[ \overline{G}^*_L = (0.513, 55.157 + 0.58k). \]

For alternating (NA, E), (A, NE), (NA, E), (A, NE)… starting from next period

\[ G^* = \left( \frac{513 \beta}{1 - \beta^2}, \frac{159.900 \cdot 55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{1}{\beta^2}, \frac{1}{\beta^2}, \frac{67.817 + 0.42c_0}{1 - \beta^2} \right), \]

and

\[ G^* = \left( \frac{513 \beta}{1 + \beta}, \frac{159.900 \cdot 55.157 + 0.58k \cdot \beta^2}{1 + \beta}, \frac{1}{\beta^2}, \frac{1}{\beta^2}, \frac{67.817 + 0.42c_0}{1 + \beta} \right), \]

\[ \overline{G}^*_L = (159.900, 67.817 + 0.42c_0). \]

Discounted sums starting from this period for (A, E), (A, NE), (NA, E) and (NA, NE) are

\[ \left( 1.364 \frac{1}{1 - \beta}, 106.460 + 0.42( k + c_1), \frac{106.460 + 0.42( k + c_1)}{1 - \beta} \right), \]

\[ \left( 159.900 \frac{513 \beta}{1 - \beta^2}, 159.900 \frac{55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{86.237 + 0.42c_0 + 55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{67.817 + 0.42c_0}{1 - \beta^2} \right), \]

\[ \left( 513 \frac{513 \beta}{1 - \beta^2} + 159.900 \frac{55.157 + 0.58k \cdot \beta^2}{1 - \beta^2}, \frac{513 \cdot 55.157 + 0.58k \cdot \beta^2}{1 - \beta^2} + 67.817 + 0.42c_0 \right), \]

and

\[ \left( 160.752 \frac{98.839}{1 - \beta}, 98.839 \frac{98.839}{1 - \beta} \right) \]

respectively. $\overline{G}^*$ is same as before. (NA, NE) is a Nash equilibrium if

\[ 98.839 \frac{98.839}{1 - \beta} \geq \left( \frac{55.157 + 0.58k}{1 - \beta^2}, \frac{67.817 + 0.42c_0}{1 - \beta^2} \right) + 55.157 + 0.58k \]

or $\beta \leq \frac{43.682 - 0.58k}{0.42c_0 - 31.022}$. If (NA, E) is a Nash equilibrium, then we must have $\beta \geq \frac{43.682 - 0.58k}{0.42c_0 - 31.022}$.

Therefore, Tit-for-tat is an SPNE only if $\beta = \frac{43.682 - 0.58k}{0.42c_0 - 31.022}$. 
12. RESULTS AND DISCUSSIONS

In variable sum game, four payment types make a trapezoidal convex set of payoffs on which the payoff of the defection strategy is the only moving vertex when \( k=0 \) and \( c_0=c_r \). Using best response strategy value in the Folk Theorem, we found a trapezoidal set of Nash equilibrium payoffs for each payment type. In pure strategy of the variable sum game, after cooperation in the first year, defection strategy cannot be an individually rational outcome indefinitely. Besides, this outcome cannot be feasible because it does not satisfy Nash equilibrium condition. Regardless of pure game strategies, minimum payoff satisfying Nash equilibrium condition is from cooperation. We applied two strategies of infinitely repeated game to search for a set of feasible and individually rational payoffs and a sufficiently large discount rate. First we used Grim-Trigger strategy for two different histories to calculate the payoffs and found that it is an SPNE. Limiting average of payoffs in both Grim-trigger strategies are the same as those in pure strategy of zero sum game. In practice, indefinite audit of TRA will not make sense because seeing this action CTP will cooperate. In this case, punishment will not be possible and TRA will have to face audit cost indefinitely. On the other hand indefinite tax evasion is not profitable for CTP because it costs about \( \$86 \) thousand more in fines and interest given that a regular audit is imposed. Therefore, Grim-trigger strategy of defection is not an appropriate strategy. The same can be said for Tit-for-tat strategy of defection. Only one of the alternating strategies of tit-for-tat and cooperation gives a feasible average payoff. Tit-for-tat cooperation is a Nash equilibrium and an SPNE of the infinitely repeated game. Both players may repeat cooperation or defection strategies indefinitely. But they cannot repeat either of the other two strategies indefinitely one of which is a pure strategy Nash equilibrium.

Independent of the two strategies discussed if CTP decides to defect six times in every seven years indefinitely based on 2015 tax audit rate, the limiting average of payoffs is about \( \$117 \) thousand given that TRA will not audit in the six year period. Having succeeded in this strategy, CTP saves about \( \$140 \) thousand on average compared to tit-for-tat strategy. Raising audit rate by 5% in next period might lead CTP to defect four times in every five years indefinitely. In this case limiting average of payoffs would be \( \$125,533 \). This is about 8% increase in payoff which is a significant rate.

13. CONCLUSION

We calculated corporate tax payoffs using discounted sums in infinitely repeated pure strategy and mixed strategy games under NDTPL. We focused on four corporate tax payoff functions based on tax payment schedules, namely remorse exemption \((u_{ij1})\), tax completion after statutory period \((u_{ij2})\), settlement \((u_{ij3})\) and no settlement \((u_{ij4})\).

Regardless of payment type in pure strategy, cooperative outcomes sustained as Nash equilibria. For all payment types, mainly due to TRA’s audit cost, mixed strategy minmax payoff is less than pure strategy minmax payoff and less than mixed strategy payoff for each player which is in line with the theory.

Like in the infinitely repeated prisoner’s dilemma, we found that Grim-trigger cooperation for both players was an SPNE. Tit-for-tat was also an SPNE. Based on the discussions in the previous section, both strategies of defection are not appropriate for corporate tax payoffs. In practice, it is unlikely that TRA stays in the game due to the past low audit history. The limiting average of CTP tax payoffs from tit-for-tat strategy is substantially greater than that from CTP’s defect strategy in Grim-trigger. Therefore, Grim-trigger was not an appropriate strategy for TRA either.

In pure and mixed strategy equilibrium, TRA needs high audit rates in order to force CTP to point of indifference. Having this information and given low audit rate history, CTP will prefer evading tax indefinitely and request settlement when get caught for evading tax. In order to break the courage of CTP evading tax, TRA must increase the number of qualified tax inspectors and audit rates.

APPENDIX A

A.1. HIGHLIGHTS OF THE NEW DRAFT TAX PROCEDURE LAW

Highlights of the NDTPL are (a) there will be no delay in fines due to tax fraud; (b) there will be no reduction on actual tax liability; (c) tax rate will be decreased by 0.5 percent for honest CTPs paying their taxes regularly. Honest CTPs will pay 19.5% instead statutory 20%; (d) honest CTPs will be allowed to establish an e-business office on Twitter; (e) incentives will be provided to CTPs based on the degree of compliance to TPL; (f) those who violate tax privacy shall be imprisoned up to 3 years and adjudged to criminal fines for no less than 150 days.
A.2. TAX PAYMENT SCHEDULE WITH RESPECT TO FINE RATES

Cases corresponding to fine rates are: (a) voluntary submission of an e-tax return with REM for the base difference in which fine rate \( f = 0 \); (b) voluntary submission of an e-tax return with ASP for the base difference: \( f = 0.5 \); (c) underdeclaration and a base difference calculated from the books and records after an audit: \( f = 1 \); (d) partial or no calculation of tax liability from the books or records by tax audit committee after an ex-officio audit: \( f = 2 \); (e) an audit resulting with a tax fraud: \( f = 3 \).

REFERENCES


WEB REFERENCES
