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Restricted Optimal Control Problem for Stochastic Switching Linear Systems with Variable Delay

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ABSTRACT

An optimal control problem for stochastic switching systems with variable time delay on state and restriction at end point is investigated. Stochastic linear quadratic regulator (SLQR) problem for system governed by a set of stochastic linear differential equations with variable delay is examined. Necessary and sufficient condition of optimality by means of maximum principle and the transversality conditions at switching points are obtained. A design method of stochastic feedback control is proposed.
Keywords: Delayed Differential Equation, Restricted Optimization Problem, Stochastic Riccati Equations, Variable Delay

Değişken Gecikmeli Stokastik Doğrusal Geçiş Sistemleri İçin Kısıtlı Optimal Kontrol Problemi

ÖZ

Zamana göre değişken gecikmeli ve uç nokta kısıtlı stokastik geçiş sistemi için optimal kontrol problemi araştırılmıştır. Gecikmeli stokastik doğrusal diferansiyel denklemler dizisiyle ifade olunan sistem için doğrusal kuadratik regülatör (SDKR) problemi ele alınmıştır. Maksimum prensibine dayanarak optimallık için gerek ve yeter koşul, geçiş noktalarında karşıtlık koşulları elde edilmiştir. Optimal kontrol için geriye dönüş tasarım yöntemi önerilmiştir.
Anahtar Kelimeler: Değişken Gecikme, Gecikmeli Diferansiyel Denklem, Kısıtlı Optimizasyon Problemi, Stokastik Rikkati Denklemleri

INTRODUCTION

Systems with randomness have provided an interest for investigation of problems of physics, economics, finance and etc. [1,2]. Delayed differential equations are adequate models of processes that behavior depends not only on their present, but also on their previous states [3, 4]. Stochastic optimization problems for delayed differential equation are the study of dynamical systems subject to random perturbations and time lag. The research on control theory has developed considerably over last few years, inspired in particular by stochastic optimization problems emerging from life and nonlife insurance, mathematical finance, self-oscillating systems. [5-9].

The stochastic linear quadratic optimal control problem which deals with minimizing a performance index of a system governed by a set of stochastic differential equations is the most studied control problem.

The optimal control problem for linear systems was solved by Kalman [10]. Although the linear quadratic regulator (LQR) is one of the most studied control problem in the literature, there exist a lot of invariant and non invariant linear systems with still open optimal control problems. There has been a rich theory on LQ control, deterministic and stochastic alike (see [11-15]).

One of the elegant features of the LQ theory is the opportunity to give an explicit description for optimal control in a linear state feedback form in terms of Riccati equation. Riccati equations, associated with deterministic LQ problem or stochastic LQ problem with deterministic coefficients are deterministic backward ordinary differential equations [16, 17].

Bismut performed a detailed analysis for stochastic LQ control problem with random coefficients [18].

Switching systems that a peculiar class of non-invariant systems have a benefit to model the processes with continuous dynamic [19-25].

This article is concerned with optimal control problem of restricted stochastic linear switching systems with variable delay.

Next section formulates the notations, main problem and assumptions. Then, the necessary and sufficient condition of optimality for stochastic linear switching systems is proved. Finally, stochastic Riccati equations for description of optimal control problem are obtained.

Notations and Problem Formulation

In this section we fix notation and definition used throughout this paper. Let N be some positive constant, R^n denotes the n - dimensional real vector space, $|.|$ denotes the Euclidean norm and $\langle\cdot,\cdot\rangle$ denotes scalar product in R^n. E represents expectation; ' (the prime) denotes derivative; * is the matrix transposition operation; by a.c. we denote almost certainly.

Let $w^1(t), w^2(t),..., w^s(t)$ are independent Wiener processes that generate the filtrations $F_t^l = \bar{\sigma}(w^l(t), t_{l-1} \le t \le t_l)$, $l = 1,...,s$; (Ω, F^l, P) be a probability spaces with corresponding filtrations $\{F_t^l, t \in [t_{l-1}, t_l]\}$. By $L_{F^l}^2(a,b;R^n)$ we denote the space of all predictable processes $x(t,\omega) \equiv x(t)$ such that: $E\int_a^b |x(t,\omega)|^2 dt < +\infty$. $R^{m\times n}$ is the space of all linear transformations from R^m to R^n. Let $O_l \subset R^{n_l}$, $Q_l \subset R^{m_l}$ be open sets; $T = [0,T]$ be a finite interval and $0 = t_0 < t_1 < ... < t_s = T$. Unless specified otherwise we use the following notation: $t = (t_0, t_1, ..., t_s)$, $\mathbf{u} = (u^1, u^2, ..., u^s)$, $\mathbf{x} = (x^1, x^2, ..., x^s)$.

Assume that, $A^l, B^l, C^l, M^l, N^l, \Phi^l, K^l$ are deterministic and $n_l \times n_l, n_l \times n_l, n_l \times m_l, m_l \times m_l$, $n_l \times n_l, n_l \times 1$ dimensional scalar matrices, respectively. G, $N^l, l = 1,...,s$ are a positively semi-defined matrices; $N^1,...,N^s$ are a positively defined matrices.

Consider following linear controlled system with variable delay:

$$dx^l(t) = \left[A^l x^l(t) + B^l x^l(t - h(t)) + C^l u^l(t)\right]dt +$$
$$\left[D^l x^l(t) + L^l u^l(t)\right]dw^l(t), t \in (t_{l-1}, t_l], \ l = 1,...,s; \quad (1)$$

$$x^{l+1}(t) = \Phi^l x^l(t) + K^l \ , t \in \left[t_l - h(t_l), t_l\right], l = 1,...,s-1; \quad (2)$$

$$x^1(t) = K^0, \ t \in \left[t_0 - h(t_0), t_0\right]; \quad (3)$$

$$u^l(t) \in U_\partial^l \equiv \left\{u^l(t,\cdot) \in U^l \subset R^{m_l}\right\}, \quad (4)$$

where U^l, $l = 1,...,s$ are non-empty bounded sets. The elements of U_∂^l are called the admissible controls. Here $h(t) > 0, t \in [t_0, t_s]$ is continuously differentiable deterministic function, such that $\frac{dh(t)}{dt} < 1$.

The problem is concluded to find an optimal solution $(\mathbf{x},\mathbf{u}) = (x^1, x^2, ..., x^s, u^1, u^2, ..., u^s)$ and a switching sequence $\mathbf{t} = (t_1, t_2, ..., t_s)$ which minimize the cost functional:

$$J(u) = E\left\langle Gx^s(t_s), x^s(t_s)\right\rangle +$$
$$E\sum_{l=1}^{s}\int_{t_{l-1}}^{t_l}\left(\langle M^l x^l(t), x^l(t)\rangle + \langle N^l u^l(t), u^l(t)\rangle\right)dt \quad (5)$$

on the decisions of the system (1)-(3) among the set of admissible controls (4) at condition:

$$E\left\langle q^s(t_s), x^s(t_s)\right\rangle \in D \quad (6)$$

D is a closed convex set in R^{m_s}.

In sequel, some notations and definitions are given to facilitate the reading of paper.

Let $U = U^1 \times U^2 \times ... \times U^s$, and consider the sets: $A_i = T^{i+1} \times \prod_{j=1}^{i} O_j \times \prod_{j=1}^{i} O_j \times \prod_{j=1}^{i} U^j$ with the elements $\pi^i = (t_0, ..., t_i, x_{t_1}^1, ..., x_{t_i}^i, u^1, ..., u^i)$.

Definition 1. The set of functions $\left\{x^l(t) = x^l(t, \pi^l), \ t \in \left[t_{l-1} - h(t_{l-1}), t_l\right], l = 1,...,s\right\}$ is said to be a solution of the linear stochastic differential equation (1) corresponding to an element $\pi^s \in A_s$, if the function $x^l(t) \in O_l$ on the interval $[t_l - h(t_l), t_l]$ satisfies conditions (2),(3), it is absolutely continuous a.c. and satisfies the equation (1) almost everywhere while on the interval $[t_{l-1}, t_l]$.

Definition 2. The element $\pi^s \in A_s$ is said to be admissible if the pairs $(x^l(t), u^l(t))$, $t \in \left[t_{l-1} - h(t_{l-1}), t_l\right]$, $l = 1, \ldots, s$ are the solutions of switching system (1)-(4) and satisfies the condition (6) .

A_s^0 be the set of admissible elements.

Definition 3. The element $\tilde{\pi}^s \in A_s^0$, is said to be an optimal solution of problem (1)-(6) if there exist admissible controls $\tilde{u}_t^l, l = 1,...s$ and corresponding solutions $\tilde{x}_t^l, l = 1,...,s$ of system (1)-(4) with constraint (6), such that pairs $(\tilde{x}_t^l, \tilde{u}_t^l)$ minimize the functional (5).

Condition of Optimality

Next necessary and sufficient condition of optimality and explicit description of optimal control for SLQR problem (1)-(5) without restriction is obtained.

Theorem 1. Let there exist random processes $(\psi^l(t), \beta^l(t)) \in L_{F^l}^2(t_{l-1}, t_l; R^{n_l}) \times L_{F^l}^2(t_{l-1}, t_l; R^{n_l x n_l})$ that are the solutions of the following adjoint equations:

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$$\begin{cases} d\psi^l(t) = -\left[A^{l^*}\psi^l(t) + D^{l^*}\beta^l(t) + B^{l^*}\psi^l(r(t))r^{'}(t) - \right. \\ \qquad \left. M^l x^l(t)\right]dt + \beta^l(t)dw^l(t),\ t\in\left[t_{l-1}, t_l - h(t_l)\right), \\ d\psi^l(t) = -\left[A_t^{l^*}\psi^l(t) + D_t^{l^*}\beta^l(t) + B^{l+1^*}\psi^{l+1}(r(t))\Phi^l r^{'}(t)\right. \\ \qquad \left.-M^l x^l(t)\right]dt + \beta^l(t)dw^l(t),\ t\in\left[t_l - h(t_l), t_l\right), \\ \psi^l(t_l) = \Phi^{l*}\psi^{l+1}(t_l),\ l=1,\dots,s-1, \\ \psi^s(t_s) = -G^s x^s(t_s); \end{cases}$$

(7)

here $\tau = r(\tau)$ is a solution of the equation $\tau = t - h(t)$.

The element $\pi^s = (t_0,..,t_s, x_t^1,\dots,x_t^s, u^1,\dots,u^s)$ is an optimal solution of problem (1)-(5) only and only if:

a) the candidate of optimal controls $u^l(t)$ at $t\in\left[t_{l-1}, t_l\right]$ for each $l=1,\dots,s$ are defined by

$$N^{l*}u^l(t) = C^{l*}\psi^l(t) + L^{l*}\beta^l(t)\ ; \qquad (8)$$

b) for $l=1,\dots,s-1$ the following transversality conditions hold:

$$a_{l+1}\psi^l(t_l)\left[A^l x^l(t_l) + B^l x^l(t_l - h(t_l)) + C^l u^l(t_l)\right] - b_{l+1}\psi^{l+1}(t_l)\left[A^{l+1}x^{l+1}(r(t_l)) + B^{l+1}x^{l+1}(t_l) + C^{l+1}u^{l+1}(r(t_l))r^{'}(t_l)\right] = 0 \qquad (9)$$

where $a_1 = 0, a_1 = \dots a_s = b_1 = \dots b_{s-1} = 1, b_s = 0.$

To prove this theorem we adopt the similar method as in [23] and [25].

In the constrained case, we follow the approach proposed by Ekeland [26].

This approach consists in converting the given problem into the sequence of unconstrained problems.

Theorem 2. Let there exist random processes $(\psi^l(t), \beta^l(t)) \in L_{F^l}^2(t_{l-1}, t_l; R^{n_l}) \times L_{F^l}^2(t_{l-1}, t_l; R^{n_l x n_l})$ and a vector $\lambda = (\lambda_0^s, \lambda_1^s) \neq 0,\ \lambda_0^s \leq 0$ that are the solutions of the following adjoint equations:

$$\begin{cases} d\psi^l(t) = -\left[A^{l^*}\psi^l(t) + D^{l^*}\beta^l(t) + B^{l^*}\psi^l(r(t))r^{'}(t) - \right. \\ \qquad \left. M^l x^l(t)\right]dt + \beta^l(t)dw^l(t),\ t\in\left[t_{l-1}, t_l - h(t_l)\right), \\ d\psi^l(t) = -\left[A_t^{l^*}\psi^l(t) + D_t^{l^*}\beta^l(t) + B^{l+1^*}\psi^{l+1}(r(t))\Phi^l r^{'}(t)\right. \\ \qquad \left.-M^l x^l(t)\right]dt + \beta^l(t)dw^l(t),\ t\in\left[t_l - h(t_l), t_l\right), \\ \psi^l(t_l) = \Phi^{l*}\psi^{l+1}(t_l),\ l=1,\dots,s-1, \\ \psi^s(t_s) = -\lambda_0^s q^s(t_s) - \lambda_1^s G^s x^s(t_s); \end{cases} \qquad (10)$$

The element $\pi^s = (t_0,..,t_s, x_t^1,\dots,x_t^s, S^1,\dots,S^s, u^1,\dots,u^s)$ is an optimal solution of problem (1)-(6) only and only if:

a) the candidate optimal controls $u^l(t)$ are defined by (8);

b) for $l=1,\dots,s-1$ the transversality conditions (9) are hold.

c) following transversality condition at point t_s is satisfied:

$$q_t^s(t_s) + \psi^s(t_s)\left[A^s x^s(t_s) + B^s x^s(t_s - h(t_s)) + C^s u^s(t_s)\right] = 0; \qquad (11)$$

Here $q_t^s(t_s)$ represents the derivative of function $q^s(t)$ at point t_s.

Proof. For any natural j let's introduce the approximating functional:

$$I_j(\mathbf{u}) = \min_{(c,y)\in\varepsilon}\sqrt{\left|c - 1/j - ES(x,u,t)\right|^2 + \left|y - E\left\langle q^s(t_s), x^s(t_s)\right\rangle\right|^2}$$

here $\varepsilon = \left\{c : c \leq J^0, y\in D\right\}$,

$$S = \langle Gx^s(t_s), x^s(t_s)\rangle + \sum_{l=1}^{s}\int_{t_{l-1}}^{t_l}\left(\langle M^l x^l(t), x^l(t)\rangle + \langle N^l u^l(t), u^l(t)\rangle\right)dt$$

and J^0 is minimal value of the functional in the problem (1)-(5). Let $\mathbf{V} \equiv (V^1,\dots,V^r)$, here $V^k \equiv (U^k, d)$ be space of controls obtained by means of the following metric:

$d(\mathbf{u},\mathbf{v}) = (l\otimes P)\ \left\{(t,\omega)\in\left[t_0, t_s\right]\times\Omega : \mathbf{v}(\mathbf{t}) \neq \mathbf{u}(\mathbf{t})\right\}$.

According to Ekeland's variational principle, there are controls such as; $\mathbf{u}^j(t) : d(\mathbf{u}^j(t), \mathbf{u}(t)) \leq \sqrt{\varepsilon_j}$ and for $\forall\mathbf{u}\in\mathbf{V}$ the following is achieved:

$$I_j(\mathbf{u}^j) \leq I_j(\mathbf{u}) + \frac{1}{j}d(\mathbf{u}^j, \mathbf{u}).$$

Consequently, $(t_0, t_1,\dots,t_s, x_t^{1,j},\dots,x_t^{s,j}, u_t^{1,j},\dots,u_t^{s,j})$ is a solution of the next unconstrained control problem:

$$\begin{cases} J_j(\mathbf{u}) = I_j(\mathbf{u}^j) + E\sqrt{\varepsilon_j}\int_{t_0}^{t_s}\delta(\mathbf{u},\mathbf{u^j})dt \to \min \\ dx_t^{l,j} = \left[A^l x^{l,j}(t) + B^l x^{l,j}(t - h(t)) + C^l u^{l,j}(t)\right]dt + \\ \qquad \left[D^l x^{l,j}(t) + L^l u^{l,j}(t)\right]dw^l(t),\ t\in(t_{l-1}, t_l], \\ x_t^{1+1,j} = \Phi^l x^{l,j}(t) + K^l,\ t\in\left[t_l - h(t_l), t_l\right],\ l=1,\dots,s-1, \\ x_t^{1,j} = K^0,\ t\in\left[t_0 - h(t_0), t_0\right],\quad u_t^l \in U_\partial^l \end{cases}$$

Function $\delta(\mathbf{u},\mathbf{v})$ is the characteristic of $\mathbf{V}$.

Introduce the random processes $\left(\psi^{l,j}(t), \beta^{l,j}(t)\right)$ and non-zero vectors $(\lambda_0^{s,j}, \lambda_1^{s,j})$ that are the solutions of the system:

$$\begin{cases} d\psi^{l,j}(t) = -\left[A^{l^*}\psi^{l,j}(t)+D^{l^*}\beta^{l,j}(t)+B^{l^*}\psi^{l,j}(r(t))r^{'}(t)-M^{l}x^{l,j}(t)\right]dt \\ \qquad +\beta^{l,j}(t)dw^{l}(t), \quad t\in\left[t_{l-1},t_l-h(t_l)\right),\ l=1,...,s, \\ d\psi^{l,j}(t) = -\left[A_t^{l^*}\psi^{l,j}(t)+D_t^{l^*}\beta^{l,j}(t)+B^{l+1^*}\psi^{l+1,j}(r(t))r^{'}(t)\Phi^{l}-M^{l}x^{l,j}(t)\right]dt \\ +\beta^{l,j}(t)dw^{l}(t), \quad t\in\left[t_l-h(t_l),t_l\right),\ l=1,...,s, \\ \psi^{l,j}(t_l)=\Phi^{l^*}\psi^{l+1,j}(t_l),\ l=1,...,s-1, \\ \psi^{s,j}(t_s) = -\lambda_0^{s,j}q^{s}(t_s)-\lambda_1^{s,j}G^{s}x^{s,j}(t_s); \qquad (13) \end{cases}$$

(11)

Here $(\lambda_0^{s,j},\lambda_1^{s,j})$ is defined as:

$$(\lambda_0^{s,j},\lambda_1^{s,j}) = \frac{\left(-y+Eq^{s}x^{s,j}(t_s),-c+\varepsilon_j+ES(x,u,t)\right)}{J_j^0}$$

(12)

where

$$J_j^0=\sqrt{\left|c-1/j-ES\left(x^j,u^j,t\right)\right|^2+\left|y-E\left\langle q^s(t_s),x^{s,j}(t_s)\right\rangle\right|^2}$$

Based to the Theorem 1 there follow that $u^{l,j}(t)$ is optimal control only and only if :

$$N^{l^*}u^{l,j}(t)=C^{l^*}\psi^{l,j}(t)+L^{l^*}\beta^{l,j}(t), t\in[t_{l-1},t_l], l=1,...,s. \qquad (13)$$

According to (9), the following transversality conditions hold for $l=1,...,s-1$:

$$a_l\psi^{l,j}(t_l)\left[A^{l}x^{l,j}(t_l)+B^{l}x^{l,j}(t_l-h(t_l))+C^{l}u^{l,j}(t_l)\right]=$$
$$b_l\psi^{l+1,j}(t_l)\left[A^{l+1}x^{l+1,j}(t_l)+B^{l+1}x^{l+1,j}(t_l-h(t_l))+C^{l+1}u^{l+1,j}(t_l)\right]$$
$$q_t^{s}(t_l)=-\psi^{s,j}(t_s)\left[A^{s}x^{s,j}(t_s)+B^{s}x^{s,j}(t_s-h(t_s))+C^{s}u^{s,j}(t_s)\right] \qquad (14).$$

Clearly, by (12) we obtain that $\left|\lambda_0^{s,j}\right|^2+\left\langle\lambda_1^{s,j},\lambda_1^{s\,j}\right\rangle=1$, so there exits subsequence of $(\lambda_0^{s},\lambda_1^{s})$ such that $(\lambda_0^{s,j},\lambda_1^{s,j})\to(\lambda_0^{s},\lambda_1^{s})$ if $j\to\infty$.

Since $I_j(\mathbf{u})$ is a convex function that differentiable at points $\left(ES^j(x,u,t),E\left\langle q^s(t_s),x^{s,j}(t_s)\right\rangle\right)$, we get $\lambda_1^s\le 0$ and λ_0^s is a normal to the set D at point $E\left\langle q^s(t_s),x^s(t_s)\right\rangle$.

Furthermore, from

$$\begin{cases}\psi^{l,j}(t_l)=\Phi^{l^*}\psi^{l+1,j}(t_l),\ l=1,...,s-1,\\ \psi^{s,j}(t_s)=-\lambda_0^{s,j}q^s(t_s)-\lambda_1^{s,j}Gx^{s,j}(t_s)\end{cases}$$

it follow that

$$\begin{cases}\psi^{l}(t_l)=\Phi^{l^*}\psi^{l+1}(t_l),\ l=1,...,s-1,\\ \psi^{s}(t_s)=-\lambda_0^{s}q^s(t_s)-\lambda_1^{s}Gx^{s}(t_s).\end{cases}$$

Hence, the weak limits of the sequences $\psi_t^{l,j},\beta_t^{l,j}$ to ψ_t^{l},β_t^{l} are provided. Finally, the affirmations of a), b) and (12) imply from the (13), (14) by taking the limits.

Delayed Riccati Equations

In the theory it is natural to connect the LQ problem with Riccati equation for the possible feedback design. To establish this fact we use the following linear relation:

$$\psi^{l}(t)=-p^{l}(t)x^{l}(t)\ ,\ l=1,...,s,\text{, a.c.} \qquad (15)$$

According to formula Ito it is obtained:

$$-d\psi^{l}(t)=dp^{l}(t)x^{l}(t)+p^{l}(t)dx^{l}(t)+\gamma^{l}(t)D^{l}x^{l}(t)+$$
$$\gamma^{l}(t)L^{l}x^{l}(t)dt=dp^{l}(t)x^{l}(t)+$$
$$p_t^{l}\left[A^{l}x^{l}(t)+B^{l}x^{l}(t-h(t))+C^{l}(t)u^{l}(t)\right]dt+$$
$$\gamma^{l}(t)\left[L^{l}u^{l}(t)+D^{l}x^{l}(t)\right]dt+p^{l}(t)\left[D^{l}x^{l}(t)+L^{l}u^{l}(t)\right]dw^{l}(t).$$

(16)

Using (15) in the (16) for each $l=1,...,s$ we have:

$$\int_{t_{l-1}}^{t_l-h(t_l)}\left[A^{l^*}\psi^{l}(t)+D^{l^*}\beta^{l}(t)+B^{l^*}\psi^{l}(r(t))r^{'}(t)-M^{l}x^{l}(t)\right]\chi_l dt+$$
$$\int_{t_l-h(t_l)}^{t_l}\left[A^{l^*}\psi^{l}(t)+D^{l^*}\beta^{l}(t)+B^{l+1^*}\psi^{l+1}(r(t))r^{'}(t)\Phi^{l}-M^{l}x^{l}(t)\right]$$
$$(1-\chi_l)dt-\int_{t_{l-1}}^{t_l}\beta^{l}(t)dw^{l}(t)=$$
$$\int_{t_{l-1}}^{t_l}\left[p^{l}(t)x^{l}(t)+p^{l}(t)A^{l}x^{l}(t)+p^{l}(t)B^{l}x^{l}(t-h(t))+\right.$$
$$\left.p^{l}(t)C^{l}u^{l}(t)+\gamma^{l}(t)L^{l}u^{l}(t)+\gamma^{l}(t)D^{l}x^{l}(t)\right]dt$$
$$+\int_{t_{l-1}}^{t_l}\left[\gamma^{l}(t)x^{l}(t)+p^{l}(t)D^{l}x^{l}(t)+p^{l}(t)L^{l}u^{l}(t)\right]dw^{l}(t),$$

(17)

here χ_l is characteristic function of interval $\left[t_{l-1},t_l-h(t_l)\right]$ and random processes $\beta^{l}(t)$ we are having next form at $t\in\left[t_{l-1},t_l\right]$ for each l:

$$\beta^{l}(t)=-\left[\gamma^{l}(t)x^{l}(t)+p^{l}(t)D^{l}x^{l}(t)+p^{l}(t)L^{l}u^{l}(t)\right] \qquad (18)$$

By means of simple transformations taking into account (18) the expression (17) can be rewritten as

$$-\left[p^{l}(t)+p^{l}(t)A^{l}+A^{l*}p^{l}(t)+B^{l*}p^{l}(t)+D^{l*}\gamma^{l}(t)+M^{l*}\right]x^{l}(t)=$$
$$\left[D^{l*}\beta^{l}(t)+p^{l}(t)C^{l}u^{l}(t)+\gamma^{l}(t)L^{l}u^{l}(t)+p^{l}(t)B^{l}x^{l}(t-h(t))\right].$$

Finally, we obtain the stochastic analogue of the Riccati equations that determine the pairs $\left(p^l(t),\gamma^l(t)\right)$:

$$dp^l(t)=-\Big[p^l(t)A^l+A^{l^*}p^l(t)+\gamma^l(t)D^l+D^{l^*}\gamma^l(t)+D_t^{l^*}p^l(t)D^l+$$
$$M^l-\Gamma^l(t)\left(N_t+L^{l^*}p^l(t)L^l\right)^{-1}\Gamma^{l*}(t)+p^l(r(t))r^{'}(t)B^l+$$
$$B^{l^*}p^l(r(t))r^{'}(t)-\Gamma^l(r(t))\left(N^l+L^{l^*}p^l(r(t))L^l\right)^{-1}\Gamma^{l*}(r(t))r^{'}(t)\Big]dt$$
$$+\gamma^l(t)dw^l(t),\ t\in[t_{l-1},t_l-h(t_l));$$
$$dp^l(t)=-\Big[p^l(t)A^l+A^{l^*}p^l(t)+\gamma^l(t)D^l+D^{l^*}\gamma^l(t)+D_t^{l^*}p^l(t)D^l+$$
$$M^l-\Gamma^l(t)\left(N_t+L^{l^*}p^l(t)L^l\right)^{-1}\Gamma^{l*}(t)]dt$$
$$+\gamma^l(t)dw^l(t),\ t\in[t_l-h(t_l),t_l);$$

here $\Gamma^l(t)=\left(p^l(t)C^l+\gamma^l(t)L^l+D^{l^*}p^l(t)L^l\right)$.

At the end, considering (15) and (18) in expression (8), the optimal control in the intervals $t\in[t_{l-1},t_l)$ can be explicitly defined as:

$$\left(N^l+L^{l^*}p^l(t)L^l\right)u^l(t)=-\Gamma^{l*}(t)x^l(t),$$
$$dp^l(t)=-\Big[p^l(t)A^l+A^{l^*}p^l(t)+\gamma^l(t)D^l+D^{l^*}\gamma^l(t)+D_t^{l^*}p^l(t)D^l$$
$$-M^l-\Gamma^l(t)\left(N_t+L^{l^*}p^l(t)L^l\right)^{-1}\Gamma^{l*}(t)\Big]dt+\gamma^l(t)dw^l(t)$$

in $[t_l-h(t_l),t_l]$.

CONCLUSION

As it is well known LQR and SQLR problems widely used to investigate the optimization questions arising for linear models. Also these problems have had a profound impact on decision of non-linear control problems encountered in economics, engineering, chemical, financial and other applications by approximating original ones to corresponding LQR problems.

This study deals with delayed SQLR problems under restrictions. An explicit solution to the SLQR problem for stochastic switching systems with variable time delay on state is obtained. Finally, SLQR controller is constructed by the solution of stochastic delayed Riccati differential equation. The condition of optimality developed in this manuscript can be viewed as stochastic analogues of results [9, 10, 20]. The SLQR problem considered in this manuscript is a natural improving of the problem given in [25].

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