Mineral resource estimation using a combination of drilling and IP-Rs data using statistical and cokriging methods

Kamran MOSTAFAEI* and Hamidreza RAMAZI*†

*Faculty of Mining and Metallurgical Engineering, Amirkabir University of Technology (Tehran Polytechnic), Iran.

ABSTRACT
The aim of this research is the mineral resource estimation using a combination of drilling and IP-Rs data. Therefore the approach of this paper is to study the correlation of induced polarization (IP) and Electrical resistivity (Rs) data with drilling data in order to grade estimation and mineral resource estimation. Reducing the boreholes number and optimization of the boreholes location is another aim of this research. The Abassabad copper mine located in Miami-Sabzevar mineralization belt northeast Iran was chosen as a case study. Within the borehole locations, geophysical profiles were designed and surveyed. After IP-Rs data inversion, 2D sections were prepared. The 3D block models of IP-Rs were constructed by geostatistical methods. The correlation between IP-Rs and drilling data were examined by statistical and geostatistical methods using regression, multivariate regression analysis, and cokriging. Based on the mentioned methods copper grade was estimated and the 3D block models of Cu grade were constructed. Obtained models were checked and compared with real Cu model compiled according to drilling data which was done after geophysical measurements. Results showed that the regression between IP data and Cu grade was more appropriate with least error. Rs data are not suitable for Cu estimation, due to changing intervals which led to increasing estimation error. Based on the suggestions of this paper, we could reduce the number of boreholes to 30% of the initial number and optimize the boreholes locations.

1. Introduction

Ore reserve modelling and estimation play crucial part in mineral exploration and exploration planning and require an extensive database that make it a costly and time-consuming process. Mineral exploration is a complex process which is carried out through integration of different methods. Building a 3D model of a deposit is the main goal of mineral exploration. The main issue is to acquire more information in less time and by lower costs. Grade estimation is the most important phase which can have a major effect on mining feasibility and its future management (Tahmasebi and Hezarkhani, 2012). Ore modeling and reserve estimation was done according to exploration studies results, such as boreholes and exploratory drifts, which are located in a grid. (Wang Q. et al., 2011).

There are numerous researches regarding ore modeling and grade estimation including ore reserve estimation by fuzzy modeling based on the spatial variability (Tutmez et al., 2007); ore reserve estimation using fractal methods in China (Wang Q. et al., 2010) using Wavelet Neural Network (WNN) and Artificial Neural Network (ANN) methods for mineral deposit evaluation (Li et al., 2010); grade estimation and reserve evaluation for Iron ore deposit in Iran (Shademan et al., 2013); grade estimation and modeling using support vector machine methods (Li...
et al., 2013), combination of geostatistical and fractal methods in order to assessment of mineral resource in the Tongshan porphyry copper deposit (Wang G. et al., 2013) and management of resources and reserve in Brazil (Seccatore et al., 2014). It is noted that grade estimation and ore reserve estimation of these studies were based on drilling results.

Geophysical methods especially IP-Rs methods are widely used in earth sciences including mineral exploration, engineering geology, environmental studies and etc. We can be mentioned some case as follow; Gold-silver deposit exploration by IP in Russia (Gurin et al., 2015), Uranium mineralization detection in India (Biswas and Sharma, 2016), Bitumen exploration in Iran using IP-Rs (Mashhadi et al., 2017), detection deeply buried cave in Spain using combination of IP-Rs and gravity (Martinez-Moreno et al., 2013), sinkhole investigation in urban area using ERT and GPR (Sevil et al., 2017), investigation of coal washing waste pile in Iran by geoelectrical methods (Jodeiri et al., 2014), investigation of landfill leaching plume using 2D and 3D ERT (Maurya et al., 2017).

In mineral exploration, geophysical surveys are predominantly carried out for anomaly separation and delineation of geological structures. However, there are some studies about grade and reserve estimation by geophysical methods including copper grade estimation in blast holes using prompt gamma neutron activation analysis (PGNAA) in Chuquicamata copper mine in Chile (Charbucinski et al., 2003), investigation of organic pollutions effect on IP-Rs measurements in Aveiro, Portugal (Martinho and Almeida, 2006), ore reserve estimation by VES and chemical analyses (Ehinola et al., 2009), correlation between geoelectrical data and aquifer parameters in evaluation of ground water potential (Batte et al., 2010), estimation of Ni grade using crosshole seismic velocity tomography in Canada (Perozzi et al., 2012), coal quality estimation using borehole geophysical data (Webber et al., 2013), reserve estimation of limestone and sand using geoelectrical data (Ushie et al., 2014) and predicting the pyrite oxidation and transport process in coal waste pile using resistivity methods in Iran (Jodeiri et al., 2016). At large, these studies include three subjects: I) estimation of hydraulic parameters of aquifer and hydrogeological parameters by geoelectrical data; II) estimation of grade and chemical parameters using well logging geophysical data and III) evaluation of lithology and dynamic parameters using geophysical methods that used in geotechnical investigations.

The attempt of this research was to combine geophysical induced polarization (IP) and resistivity (Rs) and drilling data to enhance copper ore modeling and grade estimation. In contrast of drilling, IP-Rs surveying is fast, continuous and cheap so it can cover more areas. Therefore combination of IP-Rs and drilling, can save time and cost in exploration. Therefore, in the first stage of this study, IP-Rs surveys were carried out in the available borehole locations and then inversion and modeling of IP-Rs data was conducted. After that the relationship between grade and IP-Rs has been checked and established in the borehole locations using statistical methods including regression and multivariate regression analyses (MRV) and cokriging. Finally, the 3D block model of the deposit was constructed that used for grade prediction and ore reserve calculation in the all of the study area. Studying the spatial relationships of IP and Cu on one hand and applying the spatial regression to predicting the Cu grade based on the other regionalized variable is a new method which has been assessed in one case study.

2. Methods

2.1. Induced Polarization and Resistivity (IP-Rs)

In mineral exploration, geophysical methods are used for measurement of physical features of bodies or rocks, also especially, for identification of differences between studied targets. Geophysical exploration can be used to detection the mineralization properties by measuring physical properties directly (Gadallah and Fisher, 2009). Recently, application of geophysical exploration is increasing due to optimization in cost and time (Mostafaie and Ramazi, 2015). The base of geophysical methods is the identifying the contrasts of physical properties in materials. (Telford et al., 1990). Geophysical method(s) selection for a mineral deposit exploration depended on physical properties of mineral target and its accompanied rocks geological setting, and also its topography. Integrated geophysical methods are commonly used in mineral exploration to obtain qualified results and more
certain results (Mandal et al., 2013; Biswas et al., 2014; Mandal et al., 2015). Electrical resistivity (Rs) and induced polarization (IP) are the most commonly used and the oldest subset of geophysical exploration (White et al., 2003; Dahlin and Loke, 2015). The measured parameter in geoelectrical surveys is apparent resistivity that shows by Rs (Loke and Dahlin, 2002). The base of IP method is the study of secondary electric fields that generated in ground by electric currents. IP observation are performed in time or frequency domains. In the time domain, the decay voltage is measured as a function of time while in the frequency domain, apparent resistivity p, is measured at two or more frequencies, generally below 10 Hz. The measured parameter in IP survey is apparent chargeability (Dahlin et al., 2002).

There are many arrays in IP-Rs surveying, but dipole - dipole, pole - dipole, Wenner and Schlumberger arrays are the most commonly used. Dipole - dipole array that is more conventional in profiling is sensitive to horizontal resistivity variations but very low sensitivity to vertical variations. It also has low depth of investigation compared to other conventional arrays like Wenner and Schlumberger. The other disadvantage of dipole - dipole array is that its pseudo-sections could be very different from the real geological structures (Loke, 2015). Wenner and Schlumberger arrays cannot be used for profiling. Although the conventional arrays (e.g. Dipole-Dipole, pole- Dipole, Wenner, and Schlumberger) have been used in many mineral exploration studies successfully, in some topographical and geological conditions especially in thin and high dip-angle mineralized vines, these arrays may not lead to satisfying results (Mostafaie and Ramazi, 2015).

Regarding the mentioned factors and problems, Ramazi and Mostafaei (2013) tried to design a new array, which lacks some of the mentioned problems, provides a good image of the subsurface, and can be used practically in the field, especially for one channel measurement systems. The designed array is named CRSP that is the abbreviation of (Combined Resistivity Sounding and Profiling). CRSP is a combination of sounding and profiling which can lead to good results in mentioned geological conditions (Ramazi and Mostafaei, 2013). This array (CRSP array) has successfully been applied in many exploration and/or site investigation projects using IP-Rs studies. For more information on CRSP array please refer to Ramazi and Mostafaei, 2013; Ramazi and Jalali, 2014; Mostafaie and Ramazi, 2015; Amini and Ramazi, 2016a, b.

2.2. Regression

Regression is a subset of statistical methods that used to estimate the relationship between variables. Regression has several techniques for modeling and analyzing variables that focus on identifying the relationship between a dependent variable and one or more independent variables (Howarth, 2001). Regression techniques are widely used for prediction, and also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships (Armstrong, 2012).

According to data and research goal, there are several types of regression such as linear, logistic, polynomial, ridge and etc., (Faul et al., 2009). Linear regression is the simplest type of the regressions where there are two correlated random variables; X and Y. The outcomes generate a cloud of data in the plane X-Y and we want to determine the best affine function $Y=aX+b$ that fits the observations (Howarth, 2001). In most cases, including this study, linear regression dose not satisfactory results so we should use the nonlinear regression that include several methods. The polynomial models can be used in those situations where the relationship between variables is curvilinear. Sometimes a nonlinear relationship in a small range of explanatory variable can also be modeled by polynomials (Helsel and Hirsch, 2002). This model depends on the number of variables. For one variable the polynomial model is given by eq.1.

$$Y=B_0+B_1 x_1+B_2 x_2 +...+B_n x^n$$  \hspace{1cm} (eq.1)

Where

$Y$: dependent variable, $x$: independent variable, $B_0$ to $B_n$: constant number

2.3. Multivariate Regression Analyses (MRA)

Functions obtained from regression analyses are usually used to describe the relationship between response and predictor variables (Chiou et al., 2016). Multivariate regression analysis is a method with one
dependent and many independent variables (Braglia et al., 2012; Zhang and Goh, 2016). In this method, it is assumed that a dependent variable \( Y \) is expressed as the function of independent variable \( X_i \):

\[ Y = f(X_i) \]

The regression type depends on the type of obtained function so that if the function was linear the regression is called linear and if the function was non-linear the regression is called non-linear (Granian et al., 2015).

The general form of the model is as follows:

\[ y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n + \epsilon \quad \text{(eq.2)} \]

That in this function \( y \) is dependent variable, \( x_1 \) to \( x_n \) are independent variables and \( \alpha_0 \) to \( \alpha_n \) are regression coefficient. In linear regression analysis, the regression coefficients are calculated by least square method. In linear regression analysis, correlation coefficient \( R^2 \) can be obtained from the following equation:

\[ R^2 = \frac{\sum (\hat{Y}_i - \bar{Y})^2 - \sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y})^2} = 1 - \frac{\sum (Y_i - \hat{Y}_i)^2}{\sum (Y_i - \bar{Y})^2} \]

In this equation \( \hat{Y}_i \) is calculated value of \( i \)th sample of dependent variable, \( \bar{Y} \) is mean of the variable and \( Y_i \) is the value of \( i \)th sample of the dependent variable (Braglia et al., 2012). If \( R^2 \) is close to 1, it means the result is desirable if \( R^2 \) is closed to zero this means that dependent and independent variables have not correlation (Mogaji, 2016).

Multivariate regression has been shown to suitable model for various earth sciences studies. Over the past few years, multivariate regression analysis was used widely in different earth sciences for predicting various objectives (e.g., Noori et al., 2010; Khanlari et al, 2012; Mokhtari, 2014; Habibi et al., 2014; Granian et al., 2015). Therefore, multivariate linear regression has been used to obtain the relationship between IP, Rs and copper grade in the Abassabad copper mine.

2.4. Cokriging

Geostatistics aims at providing quantitative descriptions of natural variables distribute in space or in time and space (Chiles and Delfiner, 2012). Geostatistics is the application of statistical estimation techniques considering the spatial correlation between data. Spatial correlation data, including distance and direction, can be expressed in mathematical form, considering the spatial structure. Spatial structure is studied by means of a variogram in geostatistics (Mostafaie et al., 2014). Geostatistical methods was developed for estimation of the regionalized variables such as grade in ore body at a known location. Regionalized variables are variables of a phenomenon defined in space (and/or time) that possesses a certain structure (Kumar et al., 2007). Kriging is an optimal interpolation based on regression against observed \( z \) values of surrounding data points, weighted according to spatial covariance values. Cokriging is kriging using information from one or more correlated secondary variables, or multivariate kriging in general (Bohling, 2005). The cokriging procedure is a natural extension of kriging when a multivariate variogram or covariance model and multivariate data are available (Wackernagel, 2003). Cokriging and kriging are subset of the geostatistical methods used for estimation and interpolation. These methods are generalized form of univariate and multivariate linear regression for estimation at a point, an area or within a volume. Cokriging methods are used to take advantage of the covariance between two or more regionalized variables that are related, and are appropriate when the main attribute of interest is sparse, but related secondary information is abundant (Deutsch and Journel, 1998).

The information available on a natural phenomenon is rarely limited to the values assumed by a single variable over set of sample points. In the most real studies (especially earth science) involve more than one variable, so we have to use the multivariate generalization of kriging which is named cokriging (Chiles and Delfiner, 2012). Using auxiliary variables we can improve the precision of a main variable. auxiliary variables usually are cheap in measuring so we can reduce the number of observations for the main variable that it is expensive and needed for optimizing the interpolations (Knotters, et al., 1995). A variable of interest is cokriged at a specific location from data about itself and about auxiliary variable in the neighborhood. The data set may not cover all data variables at all sample locations. Ordinary cokriging requires at least on data value about the variable of interest, while simple cokriging, relying on its knowledge of the mean, can be performed with data solely about the auxiliary variables (Wackernagal,
As mentioned before, cokriging method consisted of one primary and one secondary variable. Moreover, spatial structure should be studied in any geostatistical method. So in the cokriging analyses, the cross semi-variogram (or cross-variogram) should be determined in prior (Hooshmand, et al., 2011). There are some method for coregionalization model investigation such as: linear model of coregionalization (LMC); Markov-type model; intrinsic linear model (Madani and Emery, 2018). The linear model of coregionalization (LMC) is the one of the approach to simultaneously model direct and cross variogram in a multivariate setting. An LMC is suitable in cokriging and cosimulation (Leuangthong et al., 2008, Goulard and Voltz, 1992).

The measurements available for different variables in given domain may be located either at the same sample points or at different points for each variables. Complete heterotopy: the variables have been measured on different sets of sample points and have no sample locations in common. Partial heterotopy: some variables share some sample locations. Isotopy: data is available at all sampling points (Wackernagal, 2003). The sampling is partially heterotopic in this study. By considering the data located in a neighborhood of the target location and data distribution situation, the search strategies to select neighboring data will be selected. There are some search strategies including: collected neighborhood, multi-collected neighborhood, full neighborhood (Chiles and Delfiner, 2012, Madani and Emery, 2018). In collected cokriging only retained secondary data are the ones available at the target location (Madani and Emery, 2018). The collected cokriging and full cokriging have been used in this study.

For more information about Cokriging in ore modeling and grade estimation refer to Knotters et al., 1995; Boezio et al., 2011 and Xu et al., 2015.

3. Case Study-Abassabad Copper Mine

The Abassabad cooper mine is an active mine that is located 120 km East of Shahrud, Semnan Province, Northeastern Iran. This mine is located 10 km north of Abassabad village, named Abassabad copper mine. The location map are presented in figure 1.

From the geological point of view, Abassabad is a part of a wide mineralization belt named Miami-Sabzevar copper belt. There are many lithology units in all of the Miami-Sabzevar copper belt including 2 major geological units: igneous rocks (porphyritic andesite and trachyandesite) and sedimentary rocks (limestone with marl). Mineralization has occurred in the contact of andesite and limestone. The geology map of Abassabad was presented in figure 2. Based on the geology map there are some lithology units include porphyritic trachyandesite, conglomerate,
limestone, sandstone and shale. Quaternary alluvium occur in some locations. Sedimentary rocks consist of limestone, and in some parts clay minerals were converted to marl. Mineralogy studies shows that limestone is a geochemical barrier to settle the copper content. Limestone caused deposition of copper minerals so mineralization has occurred in the contact of andesite and limestone. According various studies such as; to mineralogy, petrology, alteration and economic geology, this deposit is Manto-type copper deposit (Salehi et al., 2016).

4. Results

In this research, the integration of IP-Rs data and grade data for construction of 3D block model and resource estimation was investigated. For this purpose, the Abassabad copper mine was selected which is an active copper mine in the Semnan Province, Northeastern Iran. In the first step, geophysical surveying was performed in the exploratory boreholes. Then, inversion of obtained data carried out using Res2Dinv software package and 2D sections were prepared. Based on the geostatistical methods, 3D block model of IP-Rs were prepared in the study area. In the second step, the relationship of IP-Rs data and the copper grade was checked out. Based on correlation between IP and Cu grade obtained by regression, Cu grade was estimated in all of the study area and the 3D model of Cu grade was prepared. Then the relationship of IP-Rs data and Cu grade were calculated by multivariate regression analyses (MRA).

Based on the MRA results, Cu grade was estimated and the 3D model of Cu grade was obtained. Based on the obtained model, 7 boreholes were proposed. The accuracy of grade estimation and prepared models was checked out by new exploratory boreholes. Then prepared models -using regression and MRA methods- were reviewed and remodeled by adding new drilling data. For more accuracy and comparison, cokriging was used. Relationship of IP values and Cu grade was examined and Cu grade was estimated and then, 3D model of Cu was prepared using cokriging. Finally estimated Cu grade and obtained Cu models were compared. Prepared models based on the IP-Rs data and Cu grade were compared with real block model of Cu grade.

4.1. Geophysical Results

In the first stage, location of profiles were detected then data surveying was done. Data inversion was done by Res2Dinv and the 2D imaging of profiles is prepared. The 3D model of data was prepared based on geostatistical methods. Also for the determination of the IP-Rs data thresholds, statistical methods and fractal methods were used. The result of 3D modeling was checked out by geostatistical methods and drilling results.

4.1.1. Designing and Surveying Data

Based on the exploratory borehole plan, the IP-Rs profiles were designed. Geophysical profile position was chosen in a way that it covers most of exploration boreholes (Figure 3). According to the mineralization
type, mine situation and surveying conditions, CRSP (combined resistivity sounding and profiling) array was selected for this investigation. CRSP array is a combined array which can lead to useful results in the various topographical and geological conditions (Ramazi and Mostafaie, 2013). Therefore, IP-Rs data have been surveyed by using CRSP array. Generally, 4 profiles were designed and surveyed. Three profiles P1, P2 and P3 are parallel. P1 and P2 were surveyed along borehole profiles with 10 meters electrode spacing. P3 and P4 were surveyed for the evaluation of IP-Rs results, therefore along these profiles there are no exploration boreholes. P3 was located 50 meters south of P1 and P2, and is also parallel to them. The electrode spacing of P3 is 20 meters. P4 is located 500 meters from the western part of the others by 10 m electrode spacing. Finally, 2000 points were read in the length of all profiles. In each point, resistivity and induced polarization in time domain have been recorded. Also the drilling data of 20 boreholes was available.

Data acquisition has been carried out according to designed plan field in 09/01/2017 to 09/22/2017. In order to check the data quality, several measurements were randomly repeated in the field. The surveyed data was revised, the data accuracy was checked out, and then data processing was done.

4.1.2. Inversion Results

As mentioned earlier in this paper, IP-Rs data inversion was done in the first stage resulted in 2D IP-Rs imaging. The resistivity and IP data sets were inverted using the RES2DINV software. To prepare IP-Rs sections, the resistivity and IP data sets were inversed by the Newton and Gauss–Newton methods, from the RES2DINV software package (Loke and Dahlin, 2002). In the inversion process number of iterations were 5. The best iteration was selected based on the RMS error and geological situations. The RMS error level is between 3.2% to 5.5% for IP data, and RMS error is between 8% to 12.5% for resistivity data. The inversion results determined as IP and resistivity sections shown by the surfer software. Inversion results and compiled sections are presented in figure 4 for profiles 1 to 4. Anomaly values were defined based on the fractal methods. Due to the data, the “concentration–area” method (Ferdows and Ramazi, 2015) has been used in this research for separation of the anomalous value from the background. The obtained threshold correlated with drilling results and the obtained threshold was modified. The threshold of the inversed IP data is equal to 25 mV/V and the threshold of inversed Rs data is 250 Ωm. According to the obtained results of the geophysical study and those correlated with the drilling data, it may be say that resistivity and IP
section could detect mineralization zones so well. In the profile locations, the mineralization was traced with acceptable accuracy by a high value of resistivity and IP. The efficiency of IP-Rs in the Abassabad copper mine is high and this investigation reduced and optimized the drilling operation efficiently. For further investigation and optimization of the exploration boreholes, we need a 3D model of the study area. By a 3D model, we can investigate the area between the profiles and also all of the study area.

4.1.3. 3D Modeling

After inversion and preparation of 2D sections presented in figure 4, the 3D models of IP-Rs data were obtained based on the geostatistical methods. In this study, data of P1, P2 and P3 were used for modeling,
because these profiles were parallel and the distance between them was less. As mentioned, P4 is located 500 meters from the western part of the other profiles, so we cannot use it in modeling. In geostatistics, the variables with spatial structure are investigated. In the other word, spatial structure is essential for using geostatistical methods. The variogram is a fundamental tool in geostatistics for investigating spatial structure because it provides critical parameters for various Kriging estimators. Accuracy of the proposed parameters from the variogram are of crucial importance and can have significant positive or negative influence on the estimated blocks (Mostafaie et al., 2014). In order to study the spatial structure, the data were reviewed and variography was carried out. According to the above factors and applying the related software such as (SGeMS) (Bohling, 2007) experimental variogram for data were calculated and presented scientifically. Variogram for various parameters such as different azimuth and dip were calculated. The appropriate theoretical models based on the least square differences were fitted to the variogram (Figures 5 to 7).

Variogram models and parameters for maximum range; median range and minimum range that are perpendicular each other and essential for modeling are presented in table 1 briefly. The angels of anisotropy were obtained for X, Y and Z respectively, -30, 0 and 120.

In the next step, 3D modeling was done based on the obtained parameters of the variography. Datamine

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**Figure 5-** Variogram model for minimum range of data (azth: 330, dip: 30).

**Figure 6-** Variogram model for median range of data (azth: 150, dip: 60).
Studio3 software package was used to prepare 3D models. The input data of Datamine Studio3 software were two-dimensional modeling results obtained from inversion. As we know the output of inversion including X, Y, Z, IP-Rs for each point that were prepared to input dataset as continuous data according to the setting of Datamine database. The input data were composited and the composite length was selected 5 meters. Then, the 3D block models of IP-Rs were prepared by Datamine Studio software. The two models include induced polarization (IP) and Resistivity (Rs) are presented as a 3D block model in figures 8 and 9, respectively. It should be noted that the block size were identical and equal to 5 meters in the whole of the model. In these models, IP-Rs parameter distribution has been shown with high accuracy and the mineralization zone is marked in different directions. Maximum value of the IP is 45 mv/v and the maximum value of resistivity is 850 Ωm. The threshold of IP is 25 mv/v and the Rs threshold is 250 Ωm. IP-Rs models show that high values of IP-Rs are the anomalous values. In the other word, anomaly value is the location with high value of the IP-Rs. Based on the 2D section and the 3D model of geophysical results, 7 boreholes were proposed and drilled (Figure 3). Drilling data confirmed the geophysical results. In the drilling planning of this mine, boreholes were drilled at a distance of 30 meters. By geophysical modelling the mineralization zone and it’s characterizes including; depth, thickness, continuity were determined. Geophysical modelling showed that mineralization zone has an acceptable continuity. So the boreholes are not needed at a distance of 30 meters, and using the boreholes at intervals more than 60 meters we can obtained required data to mineralization identify. Therefore using geophysical results we were able to reduce the borehole number significantly

According to drilling results, the database includes boreholes and geophysical models was constructed. In order to better correlation the drilling and IP-Rs data combined and composited. After data composite, dataset was selected to correlation studying (Table 2).

### 4.2. Regression Results

As mentioned before, one of the main goal of this research is to estimate Cu grade based on the IP-Rs data in order to use regression method. Firstly,
primary statistical analysis of data was performed. In regression analysis, the relationship among variables is examined. For this purpose, correlation between IP and Cu grade were checked out. So, IP was assumed as an independent variable and Cu assumed as dependent. Therefore there is a function; \( Y = f(X) \) that \( X = \text{IP} \) and \( Y = \text{Cu} \).

After checking the correlation between IP and Cu, the best equation was selected and the obtained result is presented in figure 10. As mentioned previously, there are several type of regression. So the polynomial regression type is selected as the best type of regression and the obtained equation (eq.3) is as fallow;

\[
Y = 0.0043x^2 - 0.13x + 1.002
\]

where; \( X = \text{IP} \), \( Y = \text{Cu} \) eq.3

Table 2- The statistical description of used data in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>No. of samples</th>
<th>Max.</th>
<th>Ave.</th>
<th>Min.</th>
<th>Std. dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Induce polarization</td>
<td>IP</td>
<td>106</td>
<td>46.45</td>
<td>25.75</td>
<td>11.370521</td>
<td>7.91</td>
</tr>
<tr>
<td>Resistivity</td>
<td>Rs</td>
<td>106</td>
<td>735.1</td>
<td>362.7</td>
<td>81.585678</td>
<td>179.5</td>
</tr>
<tr>
<td>Cu grade</td>
<td>Cu</td>
<td>106</td>
<td>5.4</td>
<td>0.74</td>
<td>0.002</td>
<td>1.08</td>
</tr>
</tbody>
</table>

The R-sq of equation 1 is about 75%, and the S is about 4.9% in this analyze that it is acceptable. It is notable that S is the standard deviation of the distance between the data values and fitted values.

Based on the IP, copper reserve was estimated in all of the study area according to equation 1 and the 3D model of estimated copper was constructed (Figure 11). In this copper mine, Cut of grade was considered as 0.1%, so the model consists of two group; values less than 0.1% and values more than 0.1% in the presented models. The dimensions of the block model are selected 5*5*5 meters. Based on the block model, estimated Cu reserve is about 2.11 million ton (Table 3).

The correlation between Rs and Cu grade was checked out. The correlation coefficient between Rs
and Cu grade is less than 50% that is not a satisfactory value. Therefore its results were not used in copper estimation.

4.3. Multivariate Regression Analyses (MRA) Result

In the previous part, we checked out the correlation between IP and Cu grade, and now we want to investigate the correlation between IP-Rs and Cu grade simultaneously. In the other word, we want to check the IP-Rs influence on Cu estimation together. So we used the Multivariate Regression Analyses (MRA), because as mentioned, MRA was used for examination of one dependent and more independent variables relation. In this paper, IP-Rs values were considered as independent variables and Cu grade were considered as dependent variable in order to predict the Cu grade by IP-Rs parameters using Minitab software. As mentioned in the previous parts-2.2 regression- there are several type of regression, also in the multivariate regression analyses, there are several type of regression. After the studying these types polynomial type is selected. Then the best equation with high R-Sq. was selected based on trial and error method. The obtained equation is presented as eq.4.

Table 3- Result summaries of estimated Cu using regression.

<table>
<thead>
<tr>
<th>Min (%)</th>
<th>Max (%)</th>
<th>Cutoff grade (%)</th>
<th>Mean (%)</th>
<th>Block size(m)</th>
<th>Estimated ore(million ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>4.2</td>
<td>0.1</td>
<td>0.78</td>
<td>5x5x5</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Figure 10- Diagram of Cu grade versus IP.

Figure 11- 3D model of the estimated Cu based on the regression results.
Cu (%) = 5587 + 6.80 IP - 0.120 Rs - 6002 (Rs)\(^{0.1}\) - 0.143 (IP)\(^2\) - 51.0 Ln (IP) + 620 Ln (Rs) + 0.00130 (IP)\(^3\) + 288 (Rs)\(^{0.3}\)  
(eq.4)

Correlation Coefficient = 67.3%

Based on the eq.2 the Cu grade in the study area was estimated and the Cu 3D block model was prepared (Figure 12). The Cu reserve was estimated about 2.46 million ton (Table 4).

We mentioned cokriging method as consisted of one primary and one secondary variable. The primary statistical analyses shoed that correlation between IP and Cu is more than correlation between Rs and Cu, also the Rs data variation range is more than IP which causes an increase in estimation error level. Therefore Rs data were not used in cokriging estimation. In this paper, primary variable was Cu grade and secondary variable was IP data. Due to abundance of IP data, we attempted to estimate Cu grade using IP data. As mentioned in cokriging cross-variogram is necessary, so variography was done based on the linear model of coregionalization (LMC). At first, variography of IP and Cu grade was performed. Subsequently, cross-variogram of IP and Cu grade was calculated and required parameters were obtained (Figures 13 to 15).

![Figure 12- 3D model of estimated Cu based on the MRA results.](image1.png)

**Table 4- Results summaries of estimated Cu using MLR.**

<table>
<thead>
<tr>
<th>Min (%)</th>
<th>Max (%)</th>
<th>Cutoff grade (%)</th>
<th>Mean (%)</th>
<th>Block size(m)</th>
<th>Estimated ore(million ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>0.1</td>
<td>1.43</td>
<td>5x5x5</td>
<td>2.46</td>
</tr>
</tbody>
</table>

![Figure 13- Variogram model for IP data.](image2.png)
The standardized variograms and their parameters presented in (Table 5).

Then according to the variogram parameters, Cu grade was estimated and the 3D block model of Cu grade was constructed by SGeMS software (Figure 16) and the results presented in table 6. In the estimation maximum and minimum of data equal to 3 and 15 respectively and also the search radius considered 60 meters in the cokriging estimation process. Ordinary type of cokriging (OK) was used for estimation. The secondary data (IP data) covers

Table 5- The summaries of used parameters in cokriging based on the standardized variograms.

<table>
<thead>
<tr>
<th>Experimental semivariogram</th>
<th>Fitted model</th>
<th>sill</th>
<th>Range(m)</th>
<th>Nugget effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP</td>
<td>Spherical</td>
<td>1</td>
<td>150</td>
<td>0.18</td>
</tr>
<tr>
<td>Cu</td>
<td>Spherical</td>
<td>1</td>
<td>65</td>
<td>0.35</td>
</tr>
<tr>
<td>Cross</td>
<td>Spherical</td>
<td>1</td>
<td>185</td>
<td>2.6</td>
</tr>
</tbody>
</table>
the whole domain because we want to estimate the Cu grade based on the 3D block model of IP data (Figure 8 in 4.1.3). Given that the secondary data is more than primary data, so we used the full cokriging and also collected cokriging with Markov Model 2 (MM2). The results of full cokriging that it was used all data are better and acceptable, so we presented the full cokriging results in this paper. For more details see in section 4.2. Cokriging and presented explanations in (Wackernagal, 2003; Chiles and Delfiner, 2012; Madani and Emery, 2018).

5. Discussion

The Miami-Sabzevar copper belt is a wide mineralization zone that it is currently under reconnaissance and exploration. This vast areas of this belt entails high cost and time-consuming exploration activity. Thus the main goal of this research was to devise an exploration plan to overcome this hurdle. Geophysical methods are among the cheapest and fastest so we decided to use geophysical methods combined with drilling results in ore modeling. For this purpose, the active mine of Madan Bozorg in the Miami-Sabzevar belt was selected as a case study.

At first, the application of the IP-Rs methods were prepared. IP-Rs results were compared with drilling results. Then in the locations that there is no boreholes, 2 profiles were designed and surveyed. Also these data were inverted and sections were prepared (see 4.1.2). In the next stage, 3D models of IP-Rs data were prepared by geostatistical methods and the correlation between Cu grade and IP-Rs was examined. To checking obtained results, 7 exploratory boreholes were proposed, and drilled. Drilling results showed that the geophysical method could detect anomalous region for Cu mineralization, significantly. Moreover, the correlation between Cu grade and IP-Rs is appropriate and applicable (see 4.1.2).

Based on the drilling, the database includes 20 boreholes (available boreholes and drilled boreholes after geophysical modeling) was constructed. The correlation between Cu grade and IP-Rs were revised and investigated. For this purpose regression, MRA and cokriging were used and according to the obtained correlation, Cu grade was estimated in the study area. The 3D block model of the estimated Cu grade was constructed (see; 4.2, 4.3, 4.4).

As mentioned before the case study is an active copper mine, so the extraction block model based on the measured copper grade is available. The mineral reserve estimation results according to the measured copper grade presented briefly (Table 7). The obtained results showed that the estimated copper is about 1.95 million ton with 0.71% mean.

<table>
<thead>
<tr>
<th>Min (%)</th>
<th>Max (%)</th>
<th>Cutoff grade (%)</th>
<th>Mean (%)</th>
<th>Block size (m)</th>
<th>Estimated ore (million ton)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.44</td>
<td>0.1</td>
<td>0.92</td>
<td>5x5x5</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Figure 16- The 3D block model of Cu estimation using cokriging.
The obtained models based on the combination of IP-Rs and drilling data were compared with actual Cu models, according to this we can say the obtained results are acceptable. To better comparison, we calculated the estimation error of obtained models over the actual model (Table 8).

Table 8 - The error level of estimation methods over actual model.

<table>
<thead>
<tr>
<th>No.</th>
<th>Method</th>
<th>Estimation error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Regression</td>
<td>8.2%</td>
</tr>
<tr>
<td>2</td>
<td>MLA</td>
<td>26.1%</td>
</tr>
<tr>
<td>3</td>
<td>Cokriging</td>
<td>5.1%</td>
</tr>
</tbody>
</table>

Results showed that the model constructed by the cokriging method is close to reality with minimum error. Beside cokriging method, the results of the regression method (correlation between IP and Cu) is better. Due to the smoothing effect of the cokriging, the estimated copper is less than the actual result. The results of MRA methods are of most error due to consideration of the simultaneous effect of IP-Rs values. The Rs data variation range is more than of IP data, so estimation of Cu grade entails more error rather than regression and cokriging methods.

Conclusion

IP-Rs methods have been used successfully in the copper detection in the Abbasabad copper mine with highest efficiency. Through geostatistical methods, we were able to construct 3D models for IP-Rs data that surveyed in 2D which were confirmed by drilling results.

The number of boreholes could be decreased from 20 to 7 in two places with acceptable results by geophysical modeling that led to significant cost saving.

Correlation between Cu grade and IP-Rs were examined and calculated using regression, MRA and cokriging methods and based on them, Cu grade was estimated and the 3D block model of Cu was constructed.

Results proved that Cu estimation using IP data is of better quality than Rs data, because of high variation of Rs data that increases error rate. However, Rs data can be used for geological prediction.

Results of this paper shows that the location and number of additional boreholes can be optimized by the combination of geophysical data and drilling results. We could reduce the number of boreholes and the cost of the exploration operation significantly. Based on these results, we devised exploration plan for the other areas of the Miami-Sabzevar mineralization belt.

References


Biswa, A., Sharma, S. P. 2016. Integrated geophysical studies to elicit the structure associated with Uranium mineralization around South Purulia Shear Zone, India: A Review. Ore Geology Reviews, 72, 1307-1326.


Jodeiri, B., Faramarz, S., Ardejani, D., Ramazi, H., Moradzadeh, A. 2016. Predicting pyrite oxidation
and multi-component reactive transport processes from an abandoned coal waste pile by comparing 2D numerical modeling and 3D geoelectrical inversion, International Journal of Coal Geology, 164: 13-24


Leuangthong, O., Daniel Khan, K., Deutsch, C.V. 2008. Solved problem in geostatistics, Chapter 9: Multiple Variable.


Loke, M.H., 2015, Tutorial: 2-D and 3-D electrical imaging surveys: (Revision date: 17th October 2015), (www.geotomosoft.com)


