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Research Article

Natural-Frequency Uncertainty of Cracked Microbeams with Random Crack Flexibility

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Abstract: Uncertainty in crack flexibility can significantly affect the vibration characteristics of cracked microbeams, particularly in microscale structures. This study investigates the stochastic free vibration behavior of cracked microbeams with random crack flexibility within the framework of modified couple stress theory. The crack is modeled as a torsional spring; its position defines the crack location, while its rotational flexibility characterizes the crack severity. In the stochastic formulation, the crack location is kept deterministic, and the crack flexibility is modeled as a random variable. Natural frequencies are obtained for simply supported and clamped-clamped microbeams using a numerical procedure based on the characteristic equation. Deterministic frequency maps are first generated in the crack parameter plane to identify regions where the modal response is most sensitive to crack location and crack flexibility. Monte Carlo simulation is then used to propagate the uncertainty in crack flexibility, and the resulting samples of the first mode natural frequency are evaluated using the mean, standard deviation, coefficient of variation, and percentile intervals. The results show that the coefficient of variation increases as the crack location moves toward the midspan, and the simply supported microbeam exhibits larger uncertainty than the clamped-clamped microbeam. A sensitivity analysis of the uncertainty level further confirms that greater dispersion in crack flexibility leads to greater uncertainty in the natural frequency.

Keywords: *Crack flexibility uncertainty, cracked microbeams, natural frequency, modified couple stress theory, stochastic free vibration*

Rastgele Çatlak Esnekliğine Sahip Çatlaklı Mikro Kirişlerin Doğal Frekans Belirsizliği

Öz. Çatlak esnekliğindeki belirsizlik, özellikle mikro ölçekli yapılarda, titreşim karakteristiklerini önemli ölçüde etkileyen bir unsurdur. Bu çalışmada, rastgele çatlak esnekliğine sahip çatlaklı mikro kirişlerin stokastik serbest titreşim davranışı, değiştirilmiş çift gerilme teorisi çerçevesinde incelenmektedir. Çatlak, burulma yayı olarak modellenmiştir; yayın konumu çatlağın yerini tanımlarken, yay direngenliği, çatlak şiddetini karakterize etmektedir. Stokastik formülasyonda çatlak konumu sabit alınmış, çatlak esnekliği ise rastgele değişken olarak modellenmiştir. Basit mesnetli ve ankastre-ankastre mikro kirişler için doğal frekanslar, karakteristik denkleme dayalı sayısal bir yöntemle elde edilmiştir. Öncelikle, modal yanıtın çatlak konumu ve çatlak esnekliğine en duyarlı olduğu bölgeleri belirlemek amacıyla çatlak parametre düzleminde deterministik frekans haritaları oluşturulmuştur. Daha sonra, çatlak esnekliğindeki belirsizliği yaymak için Monte Carlo simülasyonu kullanılmış ve elde edilen birinci mod frekans örnekleri ortalama, standart sapma, değişim katsayısı ve yüzdelik aralıklar kullanılarak değerlendirilmiştir. Sonuçlar, değişim katsayısının, çatlak konumu kiriş ortasına yaklaştıkça arttığını ve basit mesnetli mikro kirişte, ankastre-ankastre mikro kirişe göre daha yüksek belirsizliğin ortaya çıktığını

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göstermektedir. Belirsizlik seviyesi duyarlılık analizi sonuçları ise, çatlak esnekliğindeki daha geniş dağılımın daha güçlü frekans belirsizliğine yol açtığını doğrulamaktadır.

Anahtar kelimeler: *Çatlak esnekliği belirsizliği, çatlaklı mikro kirişler, doğal frekans, modifiye çift gerilme teorisi, stokastik serbest titreşim.*

1. Introduction

Microbeam-type structures are fundamental components of many modern microscale devices, including MEMS, sensors, actuators, and resonators. Their dynamic behavior plays an important role in device performance, stability, and reliability. When structural dimensions decrease to the microscale, classical continuum models may become insufficient because size effects become significant. To capture these effects, the modified couple stress theory proposed by Yang et al. [1] has been widely adopted. By introducing a material length-scale parameter, this theory provides a size-dependent representation of the stiffness and deformation characteristics of microstructures. The applicability of the modified couple stress theory to beam-like microsystems has been demonstrated in many studies [2–18].

Cracks are common defects in micro- and nanoscale structural elements and may significantly alter their static and dynamic behavior. Their influence has therefore been examined in several deterministic studies. Loya et al. [19] analyzed cracked nanobeams and reported significant size-dependent changes in the natural frequencies. Tadi Beni et al. [20] showed that natural frequencies decrease with increasing crack severity. Ha and Lien [21] used an overall additional flexibility matrix to include crack characteristics and investigated the effects of material properties, foundation parameters, length scale, damping, and crack parameters on functionally graded microbeams. Further studies [22–27] demonstrated that crack location, crack severity, and boundary conditions play important roles in the vibration response of cracked microbeams.

Nonlinear vibrations of cracked microbeams have also been investigated within the framework of the modified couple stress theory [28, 29]. These studies showed that when the crack is modeled by a torsional spring, increasing crack flexibility reduces both linear and nonlinear frequencies, even in the presence of additional effects such as geometric nonlinearity, tip mass, and magnetic loading. These findings further confirm the important roles of crack location and crack flexibility in the vibration behavior of cracked microbeams.

Recent researches have also addressed uncertainty in nanobeam dynamics. The stochastic nonlinear response of a nanobeam resting on a viscoelastic foundation and subjected to random external excitation was investigated with particular emphasis on material nonlinearity, damping, and response uncertainty bounds [30]. The free vibration of nanobeams with random non-ideal supports was analyzed within a probabilistic framework [31]. Moreover, mechanical behaviors of functionally graded nanobeams under uncertainty in material properties and random loading were investigated [32]. Despite these contributions, the influence of uncertainty in crack flexibility on the vibration characteristics of size-dependent cracked microbeams has received limited attention. In practical applications, crack flexibility may vary because of manufacturing imperfections, damage evolution, and uncertainty in estimating crack severity. Consequently, deterministic analysis alone may not adequately characterize the modal response, and a stochastic framework is required to quantify the propagation of crack-severity uncertainty into the natural frequencies.

Previous studies examined the deterministic vibration characteristics of cracked microbeams [28, 29] while other investigations considered random excitation, uncertain supports or material properties in micro and nanoscale beam systems [30–32]. The present study therefore investigates the stochastic free vibration behavior of cracked microbeams with uncertain crack flexibility within the framework of the modified couple stress theory. The crack is modeled as a torsional spring, whose position defines the normalized crack location and whose rotational flexibility characterizes the crack severity. The analysis is conditional on known crack locations, while the crack flexibility is modeled as a random variable. Although crack location may also be uncertain in practice, it is assumed to be known in the present study to isolate the effect of crack-flexibility uncertainty. The consideration of random crack location is reserved for future work.

The principal contribution of this study is the combined use of deterministic frequency maps and Monte Carlo simulation. The deterministic maps identify location-dependent sensitivity regions, while Monte Carlo simulation propagates uncertainty in crack flexibility to the natural frequencies. The resulting frequency samples are characterized using the mean, standard deviation, coefficient of variation, percentile intervals, and convergence analysis. Uncertainty amplification is also compared under simply supported and clamped-clamped boundary conditions. This combined deterministic–stochastic approach provides probabilistic frequency bounds and sensitivity information that cannot be obtained from deterministic analysis alone.

2. Stochastic Modeling of Cracked Microbeams

The cracked microbeam model considered in this study is based on the modified couple stress theory and follows the deterministic formulation presented in [28]. The schematic representation of the cracked microbeam is shown in Fig. 1. The beam is described using the Euler–Bernoulli beam model, while the crack is modeled as a torsional spring. The normalized rotational flexibility of the torsional spring is denoted by K_t. This parameter therefore represents the crack flexibility. Accordingly, a larger value of K_t corresponds to a more severe crack. Thus, the cracked microbeam is represented as two beam segments connected through a rotational spring at the crack position.

All length quantities used in the stochastic model are nondimensionalized with respect to the beam length L. Therefore, the normalized crack coordinate is defined as $l_c = x_c/L$, where x_c is the dimensional crack position. The physically admissible interval is $0 < l_c < 1$.

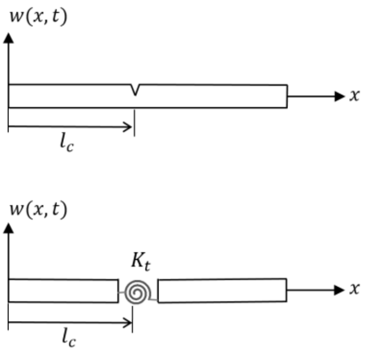

***Fig. 1.** Schematic figure of the cracked microbeam modeled by a torsional spring.*

The dimensionless linear governing equations are:

$$(1+\gamma)\, w_1{}^{(iv)}(x,t) + \ddot{w}_1(x,t) - \alpha_1 w_1{}''(x,t) = 0 \qquad 0 < x < l_c \tag{1}$$

$$(1+\gamma)\, w_2{}^{(iv)}(x,t) + \ddot{w}_2(x,t) - \alpha_1 w_2{}''(x,t) = 0 \qquad l_c < x < 1 \tag{2}$$

where $w_1(x, t)$ and $w_2(x, t)$ denote the transverse displacements of two segments of the microbeam. α_1 is the dimensionless axial-loading parameter. γ is the micro size-effect parameter which includes the material length scale parameter (l) of the modified couple stress theory [2]. Assuming harmonic motion,

$$w_i(x,t) = Y_i(x)e^{i\omega t} \qquad i = 1,2 \tag{3}$$

where $Y_i(x)$ is the mode shape function of two segments of the microbeam and ω is the frequency of vibration. The spatial vibration functions satisfy

$$(1+\gamma)\, Y_i^{(4)}(x) - \omega^2 Y_i(x) - \alpha_1 Y_i''(x) = 0 \qquad i = 1,2. \tag{4}$$

The crack is incorporated through compatibility conditions at $x = l_c$, together with the end conditions corresponding to simply supported and clamped-clamped beams. This leads to a homogeneous linear algebraic system, from which natural frequencies are obtained through the characteristic equation.

$$\det \mathbf{M}(\omega, K_t, l_c) = 0 \tag{5}$$

where $\mathbf{M}$ is the system matrix assembled from the boundary and crack compatibility conditions.

The boundary conditions considered for the simply supported and clamped-clamped microbeams and the compatibility conditions are summarized in Table 1.

For each specified crack location and crack severity, the modal roots are computed numerically from (5). In the present study, the roots are first localized by a uniform grid-based search over the prescribed frequency interval and then refined by a golden-section procedure applied to the local minima of the smallest singular value of the system matrix. The deterministic natural frequencies obtained in this way provide the basis of the stochastic analysis.

Table 1 Boundary and compatibility conditions for simply supported and clamped-clamped cracked microbeams.

simply supported	left end: $Y_1(0) = 0$ $Y_1{}''(0) = 0$	right end: $Y_1(1) = 0$ $Y_1{}''(1) = 0$
clamped-clamped	left end: $Y_1(0) = 0$ $Y_1{}'(0) = 0$	right end: $Y_2(1) = 0$ $Y_2{}'(1) = 0$
Compatibility conditions at crack location	$Y_1(l_c) = Y_2(l_c)$ $Y_2'(l_c) - Y_1'(l_c) - K_t Y_1{}''(l_c) = 0$ $Y_1{}''(l_c) = Y_2{}''(l_c)$ $(1+\gamma)\, Y_1'''(l_c) - \alpha_1\, Y_1'(l_c)$ $-(1+\gamma) Y_2'''(l_c) + \alpha_1 Y_2'(l_c) = 0$	

In this study, the crack location is kept deterministic, while the crack flexibility K_t is modeled as random in order to represent the crack-severity uncertainty.

The crack flexibility K_t is modelled using a lognormal distribution because it guarantees positive realizations and provides a simple right-skewed prior for damage-related flexibility. Let

$$K_t \sim \text{Lognormal}(\mu, \sigma^2) \tag{6}$$

where μ and σ are the mean and standard deviation of $\ln K_t$, respectively. The corresponding probability density function is

$$f_{K_t}(k) = \frac{1}{k\sigma\sqrt{2\pi}} \exp\left[-\frac{(\ln k - \mu)^2}{2\sigma^2}\right], \quad k > 0. \tag{7}$$

In the numerical simulations, the generated samples are restricted to the bounded interval

$$0 < K_t < 2 \tag{8}$$

The upper bound is introduced to keep the random samples within the deterministic parameter domain considered in the frequency maps and to avoid unrealistically severe cracks outside the intended range of the torsional-spring crack model. Values outside this interval are rejected and resampled.

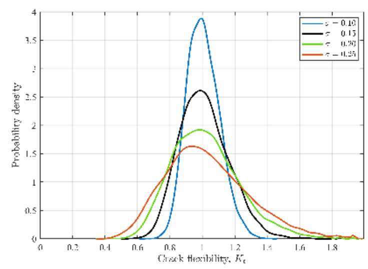

Fig. 2. *Probability density function of the truncated lognormal random crack flexibility parameter K_t with μ=0 and varying σ values.*

Figure 2 illustrates the probability density functions of the truncated lognormal crack flexibility parameter K_t for different values of σ, while the mean parameter is fixed at μ=0. The parameter σ represents the level of uncertainty; increasing σ results in a wider distribution and greater variability in the crack flexibility parameter.

The stochastic analysis is evaluated by Monte Carlo simulation. For each boundary condition, $\mathrm{N} = 8{,}000$ realizations are generated. For the *j-th* realization, a random sample $K_t^{(j)}$ is generated and substituted into the deterministic characteristic equation,

$$\det\, \mathbf{M}(\omega, K_t^{(j)}) = 0. \qquad (9)$$

The corresponding natural frequency is then computed numerically using the same root-search procedure described above. Repeating this procedure for all Monte Carlo realizations yields the frequency sample set

$$\left\{\omega_n^{(j)}\right\}_{j=1}^{N} \qquad n = 1, 2, 3, 4. \qquad (10)$$

In the stochastic results presented in this study, the analysis focuses on the first dimensionless natural frequency, and the resulting samples are post-processed using statistical measures such as the mean, standard deviation, coefficient of variation, and selected percentiles.

3. Numerical Results

Before performing the stochastic analysis, deterministic frequency maps were generated in the (l_c, K_t) plane for both simply supported and clamped-clamped cracked microbeams. These maps were used to examine how the natural frequencies vary with crack location and crack flexibility, and to identify the regions where the modal response is most sensitive to the crack parameters. In this deterministic stage, both l_c and K_t were varied over prescribed ranges, and the first four dimensionless natural frequencies were obtained from the characteristic equation. For each specified crack location and crack flexibility value, the modal roots were initially identified through a uniform grid-based search and subsequently refined using a golden-section procedure applied to the local minima of the smallest singular value of the system matrix. The case $K_t = 0$ was included as an intact-beam reference. After this deterministic sensitivity assessment, the stochastic analysis was performed by fixing the crack location and modeling K_t as a random variable.

Figures 3 and 4 present the deterministic distributions of the first four dimensionless natural frequencies in the crack-parameter plane (l_c, K_t) for simply supported and clamped-clamped microbeams, respectively. The results show that the influence of crack flexibility depends strongly on the crack location and vibration mode. Regions associated with large modal curvature and bending moment exhibit more pronounced frequency variation, whereas weak-curvature regions show limited sensitivity to crack flexibility. For higher modes, several sensitive regions appear along the beam due to the nodal points of the corresponding mode shapes. The clamped-clamped microbeam exhibits higher frequency levels than the simply supported microbeam because of its stronger boundary constraint. However, the same mode-dependent sensitivity mechanism is observed. These deterministic maps provide the physical basis for the stochastic analysis by indicating where uncertainty in K_t is expected to produce greater or smaller frequency scatter.

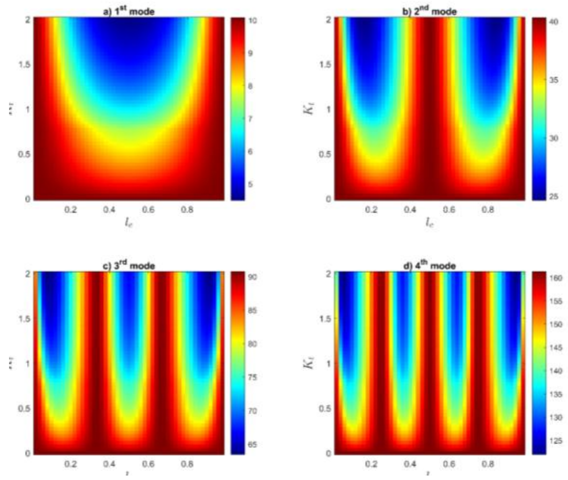

Fig. 3*. Deterministic distribution of the first four dimensionless natural frequencies in the crack-parameter plane (l_c, K_t) for the simply supported microbeam. In each panel, the color bar represents the value of the corresponding dimensionless natural frequency, ω_n.*

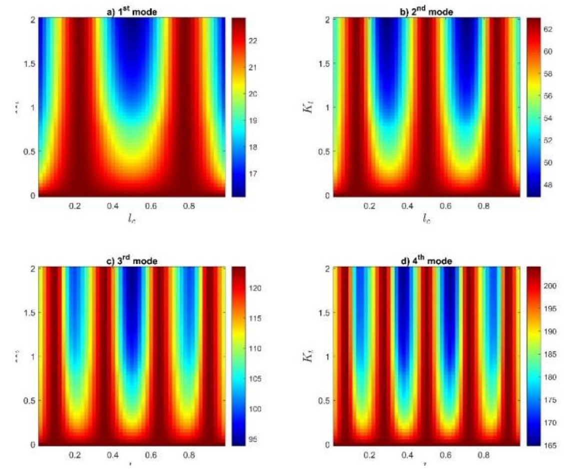

Fig. 4. *Deterministic distribution of the first four dimensionless natural frequencies in the crack-parameter plane (l_c, K_t) for the clamped-clamped microbeam. In each panel, the color bar represents the value of the corresponding dimensionless natural frequency, ω_n.*

Following the deterministic assessment, the stochastic analysis was carried out by fixing the crack location and modeling the crack flexibility K_t as a random variable. The analysis focused on the first dimensionless natural frequency, since the first mode is of primary importance in many microbeam resonator applications. Four deterministic values of the normalized crack location were considered: l_c =0.2, 0.3, 0.4, and 0.5. For each crack location, K_t was modeled as a truncated lognormal random variable with $\mu = 0$ and $\sigma = 0.15$ and $0 < K_t < 2$.

Both simply supported and clamped-clamped boundary conditions were examined.

For each Monte Carlo realization, a random value of K_t was generated and directly substituted into the characteristic equation. The corresponding first dimensionless natural frequency was then obtained using a grid-based search followed by golden-section refinement. Based on the convergence study, N=8000 Monte Carlo realizations were adopted for the final stochastic simulations. The resulting frequency samples were evaluated in terms of the mean, standard deviation, coefficient of variation, and the 5th and 95th percentiles.

Figure 5 shows the coefficient of variation with respect to the crack location for the first dimensionless natural frequency. For both boundary conditions, the coefficient of variation increases as the crack location moves from $l_c = 0.2$ to $l_c = 0.5$. This indicates that the uncertainty in crack flexibility propagates more strongly to the first natural frequency when the crack is located closer to a dynamically sensitive region of the first mode. The simply supported microbeam exhibits a noticeably larger coefficient of variation than the clamped-clamped microbeam for all crack locations. This shows that the first-mode response of the simply supported beam is more sensitive to random crack flexibility, whereas the clamped-clamped boundary condition reduces uncertainty propagation because of its stronger boundary constraint.

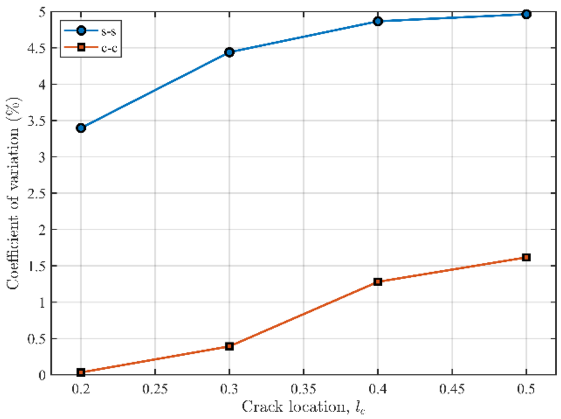

***Fig. 5**. Coefficient of variation with respect to the crack location for the first dimensionless natural frequency.*

Figure 6 presents the mean first dimensionless natural frequency together with the 5%-95% uncertainty interval. The mean frequency decreases as the crack location moves toward $l_c = 0.5$. This trend shows that the crack becomes more influential when it is positioned closer to the midspan region. The clamped-clamped microbeam has higher natural frequencies than the simply supported microbeam, as expected, because of its larger boundary stiffness. The shaded uncertainty intervals show that the frequency dispersion becomes more visible as the crack location approaches $l_c = 0.5$, especially for the simply supported case. This result confirms that the effect of random crack flexibility is not uniform but depends strongly on both crack location and boundary condition.

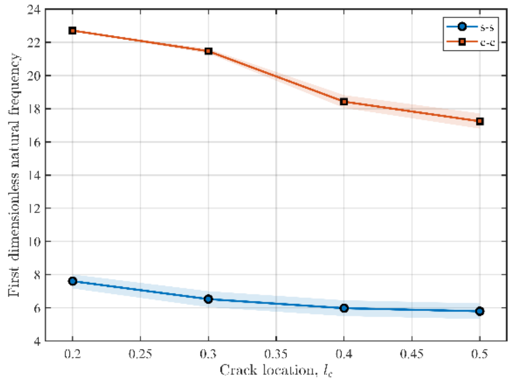

***Fig. 6.** Mean first dimensionless natural frequency with 5th – 95th percentile interval for different deterministic crack locations. The shaded regions indicate the percentile interval obtained from 8000 Monte Carlo realizations of the random crack flexibility K_t.*

Tables 2 and 3 summarize the statistical characteristics of the first dimensionless natural frequency for simply supported and clamped-clamped cracked microbeams, respectively. For both boundary conditions, the mean frequency decreases as the crack location moves from $l_c = 0.2$ toward $l_c = 0.5$. This result shows that the crack becomes more influential when it is located closer to the dynamically sensitive region of the first mode. In the simply supported case, the coefficient of variation increases from 3.397% at $l_c = 0.2$ to 4.962% at $l_c = 0.5$. This shows that the uncertainty induced by random crack flexibility becomes more pronounced as the crack approaches the midspan. A similar trend is observed for the clamped-clamped case; however, the uncertainty level is considerably smaller. The coefficient of variation increases from 0.033% at $l_c = 0.2$ to 1.616% at $l_c = 0.5$. This comparison indicates that the first natural frequency of the simply supported microbeam is more sensitive to crack-flexibility uncertainty. In contrast, the stronger boundary constraint of the clamped-clamped beam reduces the propagation of uncertainty, especially when the crack is located near $l_c = 0.2$.

Table 2 Statistical characteristics of the first dimensionless natural frequency of the simply supported cracked microbeam under random crack flexibility. P5 and P95 denote the 5th and 95th percentiles, respectively.

l_c	Mean	Std. dev.	CV (%)	P5	P95
0.2	7.609	0.2585	3.397	7.172	8.016
0.3	6.529	0.2898	4.439	6.044	7.007
0.4	5.982	0.2910	4.865	5.508	6.457
0.5	5.799	0.2877	4.962	5.333	6.275

Table 3 Statistical characteristics of the first dimensionless natural frequency of the clamped-clamped cracked microbeam under random crack

flexibility. P5 and P95 denote the 5th and 95th percentiles, respectively.

l_c	Mean	Std. dev.	CV (%)	P5	P95
0.2	22.706	0.0076	0.033	22.693	22.718
0.3	21.465	0.0842	0.392	21.328	21.608
0.4	18.423	0.2358	1.280	18.048	18.817
0.5	17.242	0.2787	1.616	16.801	17.713

Figures 7 and 8 show the Monte Carlo convergence of the stochastic simulations. The relative changes in the mean first dimensionless natural frequency and the coefficient of variation are evaluated with respect to the N=8000 result. In Fig. 7, the relative change in the mean frequency decreases rapidly as the number of Monte Carlo samples increases. Figure 8 shows a similar convergence behavior for the coefficient of variation. These results indicate that the statistical quantities are already nearly converged at N=4000. Therefore, N=8000 was selected for the final simulations to obtain conservative and statistically reliable estimates.

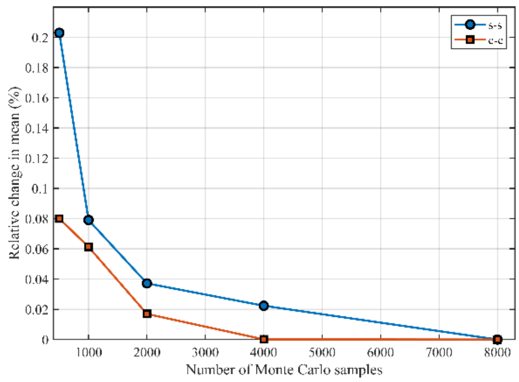

***Fig.** 7 Convergence of the mean first dimensionless natural frequency with respect to the number of Monte Carlo samples. The relative change is calculated with the N=8000 result as the reference.*

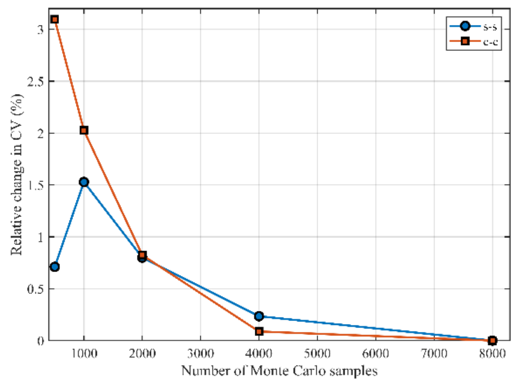

***Fig.** 8 Convergence of the coefficient of variation of the first dimensionless natural frequency with respect to the number of Monte Carlo samples. The relative change is calculated with the N=8000 result as the reference.*

Figure 9 shows the effect of uncertainty level σ on the first dimensionless natural frequency. The crack location is fixed at $l_c = 0.5$, since this location produced the largest first-mode uncertainty in the previous analysis. For both boundary conditions, the coefficient of variation increases as σ increases. This result indicates that a larger dispersion in the random crack flexibility K_t leads to stronger uncertainty propagation in the natural-frequency response. The simply supported microbeam exhibits a larger coefficient of variation than the clamped-clamped microbeam for all considered values of σ. This confirms that the simply supported boundary condition is more sensitive to crack flexibility uncertainty. The selected value $\sigma = 0.15$ can therefore be interpreted as a moderate uncertainty level within the examined range.

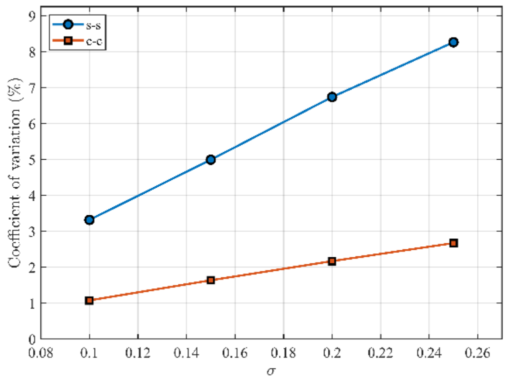

***Fig.** 9 Sensitivity of the coefficient of variation of the first dimensionless natural frequency to the lognormal spread parameter σ. The crack location is fixed at $l_c = 0.5$, and (N=8000) Monte Carlo realizations are used for each value of σ.*

4. Concluding Remarks

This study investigated the natural-frequency uncertainty of cracked microbeams with random crack flexibility within the framework of the modified couple stress theory. The crack was modeled as a torsional spring, and its normalized rotational flexibility was modeled as a random variable to represent uncertainty in crack severity. The crack location was kept deterministic, and both simply supported and clamped-clamped boundary conditions were considered.

Before the stochastic analysis, deterministic frequency maps were generated in the (l_c, K_t) plane. These maps showed that the influence of crack flexibility depends strongly on the crack location, vibration mode, and boundary condition. This deterministic assessment provided the physical basis for interpreting the stochastic results.

The stochastic analysis was performed using Monte Carlo simulation. For each realization, a random value of K_t was generated from a truncated lognormal distribution and directly substituted into the characteristic equation. The results showed that crack flexibility uncertainty does not propagate uniformly to the first natural frequency. Instead, the frequency uncertainty increases as the crack location moves toward the midspan region. The simply supported microbeam is more sensitive to crack-flexibility uncertainty, whereas the stronger boundary constraint of the clamped-clamped beam reduces uncertainty propagation.

The sensitivity analysis of the uncertainty level showed that larger dispersion in K_t leads to greater natural-frequency uncertainty. Therefore, the assumed uncertainty level of the crack flexibility has a direct influence on the predicted frequency scatter.

The present results demonstrate that random crack flexibility can produce significant and location-dependent uncertainty in the natural frequencies of cracked microbeams. Because the current analysis is conditional on fixed crack locations, a direct extension is to model crack location and crack flexibility jointly as random variables, including possible statistical dependence, and to examine their combined effect on the frequency distributions. Further work may also address multiple cracks, experimentally calibrated probability distributions, and Bayesian updating of crack parameters from measured frequency data.

Author Contribution

Data curation – Serpil Yılmaz Kutluay (SYK); Formal analysis - SYK; investigation – SYK, Duygu Atcı (DA); Literature review – SYK, DA; Writing – SYK, DA; Review and editing – SYK, DA.

Declaration of Competing Interest

The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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