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ENERGY STORAGE and RENEWABLE ENERGY: AN ECONOMIC APPROACH

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ÖZET

Makalede, kesintili (veya değişken) yenilenebilir enerji üretimi, fosil enerji üretimi ve enerji depolama teknolojisinin var olduğu bir iktisadi modelde enerji depolamanın refahı artırdığı koşullar ve iktisadi-enerji değişkenlerin uzun dönem eğilimleri incelenmektedir. Sonuçlar iki başlık altında toplanmaktadır. Birincisi, marjinal fayda eğrisinin konveksitesi, marjinal maliyet eğrisinin konveksitesi ve yenilenebilir enerji üretimindeki dalgalanmaların derecesi enerji depolamayı artıran faktörlerdir. İkincisi, yenilenebilir enerji üretiminin düşük seviyelerde olduğu durumlarda doğrusal bir maliyet eğrisinin varlığı enerji depolamayı refah artırıcı olmaktan çıkarmaktadır. Enerji depolamanın enerji üretim kararları üzerindeki etkisini gösterdiğinden, mevcut çalışma, ileriki iktisadi-enerji modellerde enerji arzının enerji depolamayı da içine alacak şekilde modellenmesinin gerekliliğini orataya koymaktadır.

ABSTRACT

I consider an economy with fossil fuels, intermittent renewable energy, and energy storage, identify the conditions under which energy storage is optimal, and analyze the long-run tendencies of the economy-energy variables. The findings are twofold. First, the amount of energy stored in the economy is highly dependent on the shape of the demand and supply schedules. In particular, energy storage is fostered by the convexity of the marginal utility, convexity of the marginal cost function for fossil fuel energy, and the degree of volatility in renewable energy. Second, considering a low level of renewable energy capacity, storing energy is not welfare improving when the unit cost of providing fossil fuel energy is constant. By showing the influence that energy storage can have on energy generation decisions, I believe that the current work can be influential in a more generous treatment of energy supply in future energy-economy models.

Keywords: *Energy storage; Fossil fuel energy; Renewable energy; Precautionary savings; Collocation method; Monte Carlo simulations*
JEL codes: Q21, Q41, Q42, Q47, C61, C63, G31

1. INTRODUCTION

The average cost of onshore (land-based) wind energy decreased by 35% between 2008 and 2015. The figure was even more dramatic for solar photovoltaics. Accordingly, the average cost of solar PV decreased by 80% (Mueller et al., 2016). Even though this leads to optimism regarding the transformation of the energy industry, the growing concerns over man-made global warming show that the penetration of renewable energy to the power grid has been gradual and insufficient to cover the increasing global energy demand. As a result, fossil fuels still account for more than three-quarters of global energy use, and it is estimated that they will account for 78% by 2035 (EIA, 2011).

The real challenge may be found in the intermittent and variable nature of renewable energy that cause difficulties in accessing energy when it is needed. If tomorrow's electric power grid is expected to contain a considerable amount of renewable energy, the grid's stability, reliability, and security may be at risk due to intermittent and variable renewable energy generation. In avoiding the exposure to such risks, energy storage technology, including battery storage, will play a crucial role in the decades to come. Therefore, it's modeling for long-term economic and policy analysis becomes an integral issue.

In the current paper, I consider an economy with a capacity to store electricity and investigate the implications of this capacity on economic welfare. In particular, I study the electricity generation and storage decisions when the industry demand and supply schedules can take different forms, such as a convex demand and supply schedules, and show how such decisions are affected by the industry and technological characteristics. Focusing on the convexity of the demand and supply schedules allows me to demonstrate the influence of precautionary behavior (e.g., prudence) on economic decisions. In view of the analytical results, my second aim is to fully solve the problem numerically, then calculate the long-run tendencies of the economic variables, such as the steady-state mean values for fossil fuel energy and energy storage. The present literature on the economics of energy storage, which questions the relationship between precautionary behavior and industry cost structures, and alternative sources of energy and energy storage is in its infancy. Accordingly, the current study contributes to the relevant literature by explicitly considering intermittent renewable energy and balancing services coming from energy storage activities.

The remainder of the paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model and evaluates it under different scenarios. Calibration and simulation results are presented in Section 4. Section 5 concludes. The description of the numerical method is presented in the appendix.

2. Literature Review

Crampes and Moreaux (2001) develop an economic model that focuses on storage in the form of reservoirs for hydropower generation, which have a deterministic supply and compete with a thermal producer. The authors address the optimal energy mix and examine its compatibility with market mechanisms when the two producers compete. They show that optimal energy generated from the thermal station is determined by the industry-specific costs and the intertemporal specification of utility.

In a two-period framework, Crampes and Moreaux (2010) consider the optimal use of a pumped storage facility that consists of thermal and hydro energy technologies. In their model, hydro energy is generated from controlled inflows that require energy from the thermal technology. After solving for the optimal allocation, they show that there are social gains from storing water in an off-peak interval (where more energy from the thermal source is generated than consumed), which can then be used in the peak interval (where energy consumption will be more than energy generation).

Considering various cases such as fossil fuel or renewable energy generation with pumped hydroelectric storage[1], Førsund (2012) examines the economic fundamentals of energy storage in a two-period model. Given the growing interest in Norwegian hydroelectric reservoirs on the grounds that they will allow for a higher penetration of renewable energy into the European power grid, the paper also examines the effect of trade in electricity between regions. It finds that unless there are sufficiently large interconnection systems, the price differentials between the regions diminish. As a consequence, this reduces the scope for trade.

When there is a certain number of large conventional plants that have to be online (such as combined cycle gas turbines or the equivalent), intermittent wind energy and a planning horizon of 36 hours (hence one model period constitutes one hour), Tuohy and O'Malley (2011) show that, when modeling energy generation and dispatch of the power system, accounting for the intermittency is important in capturing the benefit of the flexibility offered by pumped storage. Accordingly, intermittent wind makes energy storage more attractive, and its role becomes more significant when wind power is curtailed due to high wind.

The role of hydro storage in enabling a greater penetration of renewable energy into the grid has been investigated in Kanakasabapathy (2013), where the author looks at the impact of pumped storage energy trading on the sum of consumer and producer surplus of the individual market in a static model. The results show that while energy trading by pumped storage plants improve welfare in general, the economic implications for consumers and individual energy generators can be different.

In Korpaas et al. (2003) a method for the scheduling and operation of energy storage for wind power is presented. Solving the optimization problem using dynamic programming, they show that energy storage enables wind power plant owners to take advantage of variations in the spot price.

In a stylized model of energy investment and generation with two sources of energy, Ambec and Crampes (2012) address the optimal energy mix and analyze the optimal capacity investments in the absence of a storage technology. Hence, the focus is on the economics of the interplay between thermal and intermittent renewable energy and their capacities. After characterizing the optimal energy dispatch and capacities, they look at the economic policies that achieve first-best and second-best policies in decentralized markets.

In Van de Ven et al. (2011), the focus is on the decisions to satisfy the demand either directly from the grid or from the energy stored in batteries when the energy demand and prices are variable. Modeling the problem as a Markov decision process, they calculate a threshold to which the battery is charged whenever it is below the threshold, and discharged whenever it is above.

[1] Presently, pumped-storage hydropower (PSH) is considered the most mature method for electricity storage. Of the 140 gigawatts of large-scale energy storage that are currently installed in the electricity grids worldwide, over 99% corresponds to PSH (IEA, 2014). PSH comprises thermal and hydraulic technologies. The system requires two reservoirs with differing elevations. The uphill reservoir (i.e., a mountain lake) is used to generate electricity by allowing the water to fall and turn the blades of turbines. The lower-level reservoir is used to collect water, which is then pumped to the uphill reservoir using thermal systems.

This study, while sharing several characteristics of these papers, will depart from them in a significant way. First of all, in the presence of intermittency and balancing services, I investigate analytically the conditions that will cause welfare improvements when energy is stored, and show how the convexity of the demand and supply schedules can stimulate energy storage decisions. Secondly, I solve numerically for the optimal energy mix and storage decisions, i.e., the optimal decision rule, which I then supplement with Monte Carlo simulations in order to evaluate the long-run tendencies of the economic variables.

3. Model

Consider an infinite horizon economy with a representative consumer. There is a single commodity, i.e., energy, which can be supplied from fossil fuels, renewables, and energy storage systems:

$$Q_t = Q_{dt} + z_t Q_{ct} - R_t,$$

where Q_t is energy consumption, Q_{dt} is fossil fuel energy, Q_{ct} is renewable energy, $z_t \in [0,1]$ is current weather condition (normalized to one) that is known prior to taking economic decisions, and R_t represents the energy storage decision. When R_t is positive, energy is stored to be used in the following periods. When R_t is negative, previously stored energy is used.

The equation of motion for the stored energy is the following:

$$S_{t+1} = \phi S_t + R_t$$

where S_t is the level of stored energy at time. Whenever energy is stored, a certain percentage of it will be lost in time. This is captured by the round-trip efficiency parameter, $\phi \in (0,1)$, which is the ratio of the energy recovered to the initially stored energy.

The timing of the model is depicted in Figure 1. At the beginning of period t, the economy inherits stored energy; S_t. Having observed S_t and the weather conditions z_t, the fossil fuel and renewable energy decisions, Q_{dt} and Q_{ct}, respectively, are made. After taking into account the loss in stored energy, $(1-\phi)S_t$, and Q_{dt} and Q_{ct}, the levels for energy storage, R_t, and therefore, energy consumption, Q_t, are decided. I assume that the power grids are smart (Ambec and Crampes, 2012; Van de Ven et al., 2011). Therefore, production and consumption almost coincide so that no energy is lost in this process. Given the energy storage decision, the level of energy that will be stored and transferred into period *t*+1 is $S_{t+1} = \phi S_t + R_t$.

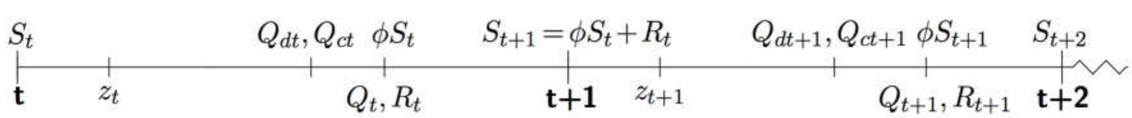

Figure 1: *Timing of the model.*

We assume that energy demand is stationary (Førsund, 2007, Ch. 9). $\mathrm{U}(\mathrm{Q_t})$, is the per period utility function, which is thrice continuously with $\mathrm{U}' > 0,\ \mathrm{U}'' < 0$, and $\mathrm{U}''' \geq 0$. Preferences over energy consumption take the additively separable form given by:

$$\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t U(Q_t)\right] \qquad (1)$$

where $0 < \delta < 1$ is the discount factor and $\mathbb{E}[\cdot]$ denotes the expectation operator.

The cost function of fossil fuel energy generation, $\mathrm{C_d}(\mathrm{Q_d})$, is thrice differentiable with $\mathrm{C}_\mathrm{d}' > 0$, $\mathrm{C}_\mathrm{d}'' \geq 0$, and $\mathrm{C}_\mathrm{d}''' \geq 0$. When the unit cost is constant, one can relate this to a constant-cost industry. On the other hand, when the cost function is convex, this resembles an increasing-cost industry. Moreover, when the third-order derivative of the cost function is strictly positive, that is, the marginal cost is convex, there is an increasingly increasing-cost industry. Given that there is a unique merit order of using individual generators, so that first the power plants with the lower marginal costs of energy generation would be brought online (like a coal-fired power plant), followed by costlier ones (such as a natural gas power plant with carbon capture and storage), a convex marginal cost function is a plausible assumption. This is also confirmed by studies, which recover cost function estimates for electricity generation based on bidding behavior (Bunn et al., 2000; Wolak, 2003). Lastly, $\mathrm{C_c}(\mathrm{Q_c})$ is the cost function for the renewable energy generation. As the cost structure of renewable energy will be discussed later on, I do not make any assumption regarding its functional form at this stage.

When solving the energy generation problem, my aim is to maximize Eq. (1), the intertemporal welfare of the representative agent, through energy generation and storage decisions. For $\mathrm{S_0}$ being the inherited energy, and for $\mathrm{z_0}$ being the initial weather condition, the planner's problem, formulated in the form of a Bellman equation, is the following:

$$\begin{aligned}
V(S_t, z_t) = &\max_{\{Q_t, Q_{dt}, Q_{ct}, R_t, S_{t+1}\}} \{U(Q_t) - C_d(Q_{dt}) - C_c(Q_{ct}) + \delta \mathbb{E}[V(S_{t+1}, z_{t+1})]\} \\
\textit{subject to} \quad & Q_t = Q_{dt} + z_t Q_{ct} - R_t, \\
& S_{t+1} = \phi S_t + R_t, \\
& \overline{Q}_d \geq Q_{dt} \geq 0, \\
& \overline{Q}_c \geq Q_{ct} \geq 0, \\
& \overline{S} \geq S_t \geq 0, \\
& S_0 \geq 0, \text{ and } 1 \geq z_0 \geq 0,
\end{aligned} \qquad (2)$$

where $V(S_t, z_t)$ is the value function, which is the maximum attainable sum of the current and future rewards given the current (inherited) level of stored energy and current weather conditions. Moreover, $\overline{Q}_d$, $\overline{Q}_c$, and $\overline{S}$ are the capacity constraints for fossil fuel energy, renewable energy, and energy storage, respectively. I assume a sufficiently large capacity for fossil fuel energy throughout the analysis such that it never binds. Furthermore, in this section, I only focus on cases in which dispatchable generators always supply the residual load (Joskow, 2011; Tsitsiklis and Xu, 2015). Thus, $Q_{dt} > 0$. I will relax this assumption when I numerically investigate the solution. Future weather conditions, $z' \equiv z_{t+1} \in [0,1]$ are imperfectly known *ex ante,* and the surrounding uncertainty is removed only at the end of the current period; i.e., after Q_{dt}, Q_{ct}, and R_t are determined.

Once the renewable energy system is installed, the unit cost of generating renewable energy becomes so low that it can be considered as zero (Ambec and Crampes, 2012; Evans et al., 2013; Førsund and Hjalmarsson, 2011). Hence, for $C_c'(Q_c) = 0$, and it is optimal to operate the renewable energy at its capacity at all times: $Q_{ct} = z_t\overline{Q}_c$ for $t = 0,1,2,\ldots,\infty$.[2]

Given that $\mathrm{F(z)}$, which is the cumulative distribution function of z over the compact support $[0,1]$, and that the model parameters are time invariant, the problem is stationary. Thus, the problem faced by the planner at every period is identical: $\mathrm{V_t(S,z) = V_{t+y}(S,z)}$ for all $\mathrm{y} > 0$. Unless it causes confusion, I shall drop the time subscripts and use primes to denote next-period values (not to confuse with partial derivatives). Then the dynamic stochastic decision problem has the following structure. At every period, the planner observes the state of the economy, $(\mathrm{S,z})$, i.e. ,how much energy storage has been inherited, and the state of the weather conditions, how strong the wind blows or the sun shines. The benevolent planner then decides on the optimal actions $\mathrm{Q, Q_d, R}$, and S'. Therefore, the planner searches for an optimal decision rule $\{\mathrm{Q^*(S,z), Q_d^*(S,z), R^*(S,z), S'^*(S,z)}\}$ that solves $\mathrm{V(S,z)}$.

The problem is not fully tractable analytically. Therefore, I leave the problem of finding the optimal decision rule to the numerical analysis section. Nevertheless, I can simplify the problem to identify the conditions under which storing energy is optimal and gain intuition that I can benefit from when interpreting the numerical results. To do this, I consider a scenario in which $S = S' = 0$, and ask whether a marginal increase in S', and therefore, R, is welfare improving.

Using a second-order Taylor approximation, and owing to the fact that $Q_d > 0$ and, therefore, $U'(Q) = C_d'(Q_d)$, the welfare effect of increasing energy storage, S', marginally from zero when $S = 0$ is as follows (see Appendix A for the calculations):

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} \cong -U'\left(\mathrm{Q}_d(0,z) + z\overline{\mathrm{Q}}_\mathrm{c}\right) + \delta\phi\left(\mathrm{C}_d'(\mathrm{Q_d}(0,\overline{z})) + \frac{1}{2}\overline{\mathrm{Q}}_c^2\left[\frac{(C_d'')^3}{(C_d''-U'')^3}U''' + \frac{(-U'')^3}{(C_d''-U'')^3}C_d'''\right]\sigma^2\right) \quad (3)$$

where $\overline{\mathrm{z}}$ and σ^2 are the expected value and variance of the random variable z', respectively. From Eq. (3), I can establish the following proposition:

Proposition 1. *If the cost of engaging in energy storage is sufficiently low and the benefit expected from storing energy is sufficiently high, energy storage is welfare improving. Convexity in the marginal utility, and convexity in the marginal cost function of fossil fuel energy generation, and the degree of intermittency are factors that foster energy storage decisions.*

Notice from the expression given by (3) that the value on the RHS diminishes when $U''' = 0$. Therefore, the convexity of marginal utility is a crucial factor that increases the willingness of the economy to engage in energy storage. One other thing that can be noticed from expression (3) is that a convex marginal cost of fossil fuel energy does play a significant role in determining the impact of uncertainty on the optimal energy storage strategy. It can be seen that even when $U''' = 0$, a non-negative C_d''' is necessary for “precautionary " saving of energy. Notice also that a higher

[2] The only cost in generating renewable energy is actually the opportunity cost of not generating more energy than the renewable energy capacity.

volatility in renewable energy increases the expected (social) benefit from energy storage. Lastly, risk aversion and convexity in the cost function play crucial roles for the results. The main reason for this is that uncertainty is welfare deteriorating for the society when the utility function is concave, $U'' < 0$, and the cost function is convex, C_d''>0.

The decision to engage in energy storage given the uncertainty in renewable energy relates to the literature on precautionary saving, where a positive third-order derivative of the utility function governs the precautionary behavior. The analysis regarding the precautionary saving under uncertainty was first introduced by Leland (1968) and Sandmo (1970). A modern treatment of precautionary saving can be found in Kimball (1990), where he coins the term 'prudence' when the marginal utility of consumption is convex and shows that prudence is sufficient for a demand in precautionary savings in standard intertemporal models of consumption.

The convexity of the marginal cost function is an important property of the electricity industry. This property owes to the fact that there is a unique merit order of using individual generators. Accordingly, the power plants with the lower marginal costs of energy generation are the first to be brought online (like a coal-fired power plant). These power plants are followed by others with higher unit cost of producing energy, such as a condensing plant and gas turbine, and a natural gas power plant with carbon capture and storage. As a result, an increasing and a convex curve can successfully characterize the industry supply. Such property can also be identified empirically. For instance, using auction data, Wolak (2003) recovers a convex marginal cost function estimate in the Australian electricity market.

Note that as there are no externalities or other distortions in the model, the competitive equilibrium quantities correspond to the social planner's allocation. Therefore, the results from the analysis of the social planner's problem can be carried to a decentralized equilibrium. Assuming that the consumers have identical preferences, and modeling their behavior by a representative consumer, the marginal utility function can be denoted by $P = U'(Q)$ where $Q \equiv Q(P)$ is the aggregate demand function of electricity given the electricity tariff, P.[3] Additionally, the marginal cost function $C_d'(Q_d)$ is the aggregate supply function of electricity generated by fossil fuels.

To have better a understanding of the results and their implications, let us focus on a special case, and consider that energy is only supplied by fossil fuels; i.e., $\overline{Q}_c = 0$. In this case, when S' is increased marginally from zero, the following welfare effect occurs:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)\big(C_d'(Q_d(0,0))\big) < 0 \tag{4}$$

From (4), I can establish the following corollary:

Corollary 1. *When the economy does not have access to renewable energy, storing energy is welfare deteriorating.*

The intuition is as follows. Without renewable energy, the electricity needed the storage systems will be obtained from the fossil fuel energy industry. Then the unit cost of storing energy becomes $C_d'(Q_d(0,0))$. When energy is stored, its present value adjusted for the discount factor

[3] I consider a quasi-linear utility function over electricity consumption, and a numéraire commodity. Accordingly, $U(Q)$, is the monetary value of utility derived from Q kilowatt-hour of electricity consumption. Such preferences are standard in economic theory when discussing issues related to a single market in a general equilibrium framework.

and the loss in energy becomes $\delta\phi C_d'(Q_d(0,0))$. Comparing the cost of storing energy to its value adjusted for the discount factor and the round-trip efficiency, it is seen that energy storage is suboptimal. As a result, energy consumption, Q, equals fossil fuel energy generation, Q_d, at every period.

Suppose now that there is renewable energy capacity in the economy. As another special case, assume away the intermittency problem. Thus, z' is constant at every period; i.e., $z = z' = \mathbb{E}[z] = \mathbb{E}[z']$. Therefore, $\sigma^2 = 0$. From (3) one arrives at the following outcome:

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)\left(C_d'\big(Q_d(0,z)\big)\right) < 0, \tag{5}$$

which allows us to establish the following corollary:

Corollary 2. *In an economy with fossil fuel and renewable energy, storing energy is welfare deteriorating in the absence of an intermittency problem.*

Accordingly, intermittency in renewable energy, and hence, uncertainty in the levels of energy generated by the renewable energy capacity, is the cause that assigns a positive value to energy storage in this model. Without the uncertain generation of energy from the renewable sources, it will only be welfare deteriorating to engage in energy storage.

Considering again that renewable energy is intermittent and variable, let us assume a constant-cost fossil fuel energy industry. Thus, the cost function is linear, and the fossil fuel energy industry supply curve can be characterized by a horizontal supply curve. In this case, Eq. (3) becomes

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} = -(1-\delta\phi)c_d < 0, \tag{6}$$

where $c_d = C_d'\big(Q_d(0,z)\big) > 0$, which is a constant, is the marginal cost of generating fossil fuel energy when the fossil fuel energy industry supply function is linear. This result leads us to the following corollary:

Corollary3. *When the fossil fuel energy cost function is linear, i.e., there is a constant-cost fossil fuel energy industry, storing energy is welfare deteriorating, and therefore, never optimal. The positive third-order derivative of the utility function loses its impact on storage decisions.*

The intuition is that in an economy in which the fossil fuel energy is the source for energy storage, the present value adjusted for the discount factor and the loss in energy becomes $\delta\phi c_d$, which is smaller than the marginal cost of storing energy, c_d. It can then be seen from (6) that energy storage turns out suboptimal. Hence, although the renewable energy is stochastic, there is indeed no real risk in the economy as long as the dirty carrier has no barriers to produce energy in the following period. Therefore, storage technology will not be employed even if it is perfectly efficient; i.e., even if $\phi = 1$.

Notice, however, that this result is valid only for cases where fossil fuel energy is always generated, i.e., $Q_d > 0$. Consider a case in which renewable energy capacity is sufficiently high such that fossil fuel power plants would be taken offline from time to time. When such a setup is present, the unit cost of storing energy can become sufficiently low when the renewable energy generation is sufficiently high, making energy storage beneficial for the society. We will consider such cases in the next section, where we solve the problem numerically.

Now assume that there is only renewable energy capacity. Thus, $\mathrm{Q_d} = 0$. Here, I assume that $\mathrm{z} \in (0,1]$ (Helm and Meier, 2016). Therefore, no matter how small it is, there is always some renewable energy generation. The welfare effect of a marginal increase in S' yields

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} \cong -U'\left(z\overline{Q}_c\right) + \delta\phi U'\left(\overline{z}\overline{Q}_c\right) + \frac{1}{2}\delta\phi\overline{Q}_c^2 U'''(\overline{z}\overline{Q}_c)\sigma^2 \tag{7}$$

To fix ideas, suppose that the current realization of z coincides with its expected future realization, i.e., $z = \overline{z}$. Then,

$$\left.\frac{\partial\{\cdot\}}{\partial S'}\right|_{S'=0} \cong (1-\delta\phi)U'\left(\overline{z}\overline{Q}_c\right) + \frac{1}{2}\delta\phi\overline{Q}_c^2 U'''(\overline{z}\overline{Q}_c)\sigma^2 \tag{8}$$

One sees that $U''' \geq 0$ is a necessary condition for storage to be optimal in this case.

3.1. Discussion

Regarding the relationship between the volatility of renewable and energy storage, one comes across similar results in the operations research and economics literature. Tuohy and O'Malley (2011) argue that intermittency increases the benefit driven from the flexibility offered by pumped hydroelectric storage and makes energy storage more attractive. The numerical results in Evans et al. (2013) also confirm the positive relationship between variance of renewable energy and storage (stored water, in particular); that is, storage becomes more welfare enhancing with higher uncertainty. The fact that the demand schedule is linear in Evans et al. (2013), that is, $P'' = U'''$, however, suggests that the quantitative results can change once a convex demand schedule is considered. Bobtcheff (2011) numerically demonstrates that a benevolent planner keeps more water in a reservoir when faced with higher uncertainty and explains that this action is due to prudence. Nevertheless, the author does not present a formal analysis. In Bobtcheff (2011), the marginal cost of fossil fuel energy generation is constant, $C_d'(Q_d) = c_d$, and not subject to any capacity constraints. As can be understood from my analysis, it is the convexity in the demand schedule that leads to higher levels of energy storage when the economy is confronted with a higher volatility in renewable energy.

3.1.1. Numerical Analysis

My purpose with the numerical analyses and simulations in this section is not to provide a comprehensive quantitative evaluation. Rather than that, I want to highlight the roles different industry cost and market demand structures, can play in an economy with fossil fuel and renewable energy, and energy storage capacities. In solving the dynamic stochastic decision problem given by (2), I employ dynamic programming based on Bellman's principle of optimality: regardless of the decisions taken to enter a particular state in a particular stage, any optimal policy has the property that the remaining decisions given the stage resulting from the current decision must constitute an optimal policy. Hence, I look for an optimal decision rule $\{Q^*(S,z), Q_d^*(S,z), Q_c^*(S,z), R^*(S,z), S'^*(S,z)\}$, which solves $V(S,z)$. To make sure that the numerical problem has a solution and this solution is unique, I establish the contraction property of the dynamic program in Appendix B.1.

Suppose that there exists an economy in which the level of energy consumption is $\mathrm{Q} = 450\mathrm{MWh}$ (megawatts per hour). To begin with, suppose that the demand is met by a fossil fuel power plant.[4] Thus, the fossil fuel energy generation, $\mathrm{Q_d}$, equals energy consumption, Q: $\mathrm{Q} = \mathrm{Q_d} = 450\mathrm{MWh}$.

In the economy, the energy demand is assumed to be stable. I, therefore, focus on weekly data: $\mathrm{Q} = \mathrm{Q_d} = 450\mathrm{MWh} = 450 \times 24 \times 7\mathrm{MWw} = 75.6\mathrm{GWw}$, where w and GW stand for week and gigawatt, respectively. Note that $1\mathrm{GW} = 1000$ MW. For ease of notation, I drop 'per time period' notation and focus only on the thermal units. I take an annual discount rate of 5%. This corresponds to a weekly discount factor, $\delta = 0.9991$.

For the fossil fuel power plant, I assume that the ramp-up level equals $\underline{\mathrm{Q}}_\mathrm{d} = 8.4\mathrm{GW}$, which corresponds to 50MW per hour. The capacity constraint for fossil fuel power generation is given by $\overline{Q}_d = 100.8\mathrm{G}W$,corresponding to 600MW, which, in the simulations, will not be binding as $\mathrm{Q} = 75.6\mathrm{GW}$.

In the simulations, I will make use of a constant relative risk aversion (CRRA) utility function, $\mathrm{Q}^{1-\gamma}/(1-\gamma)$, where γ and $\gamma + 1$ are the coefficients of relative risk aversion and relative prudence, respectively. I take $\gamma = 2$.[5] From the necessary first-order condition with respect to Q_d, given by (13a) in the appendix, I get $\mathrm{Q}^{-\gamma} = C'_d(Q_d)$. Assuming a linear cost function for fossil fuel energy, $C'_d(Q_d) = c_l Q_d$, where c_l is a constant, one gets, $c_l = \mathrm{Q}^{-\gamma}$. For $\mathrm{Q} = 75.6\mathrm{GW}$, $c_l = 0.000175\mathrm{UoN}$ (units of the numéraire). If, however, the cost function is quadratic, I have $C'_d(Q_d) = c_q Q_d^2$, where c_q is another constant. Finally, for a cubic cost function $C'_d(Q_d) = c_c Q_d^3$, where c_c is also a constant.

In order to be consistent in the analysis, I assume that when the fossil fuel energy generation is at the ramp-up level, $\mathrm{Q_d} = \underline{\mathrm{Q}}_\mathrm{d}$, the marginal costs are equal among the different cost functions. This gives

$$c_l = 2c_q \underline{\mathrm{Q}}_\mathrm{d} = 3c_c \underline{\mathrm{Q}}_d^2.$$

Accordingly, $c_l = 0.000175\mathrm{UoN}$, $c_q = 1.0417 \times 10^{-5}\mathrm{UoN}$ and $c_c = 8.2672 \times 10^{-7}$. For $Q_d > \underline{\mathrm{Q}}_\mathrm{d}$, it can be seen that $c_l < 2c_q Q_d < 3c_c Q_d^2$.

Suppose that a wind farm with a maximum capacity of $\overline{Q}_c = 100.8\mathrm{G}W$, which corresponds to 600MW per hour is then introduced to the economy.[6] Moreover, the economy has access to an energy storage capacity of 100MW, corresponding to $\overline{S} = 16.8GW$ per week.[7] I first assume that 1% of stored energy would be lost every week, hence $\phi = 0.99$. I address the effects of different round-trip efficiency parameters by making a sensitivity analysis in Appendix D.

As is discussed in the Appendix for method description (Appendix C.1), I approximate

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[4] Although I do not aim for a comprehensive quantitative evaluation, it is possible to find a range of examples to associate with 450MWh of energy consumption. As an example, see Kaldellis et al. (2012).

[5] Heal (2009) argues that [2,6] is a reasonable range for the relative risk aversion parameter.

[6] The Fantanele-Cogealac Wind Farm, which opened in 2012 in Romania, and the Whitelee Wind Farm, which opened in 2012 in the United Kingdom, have capacities of 108GW and 90.5GW, respectively.

[7] Considering battery storage, even though such a capacity is not present as of today, it is achievable given the current battery technology. The biggest battery storage capacity exists in west Texas located at 153MW Notrees wind farm where 36MW battery storage system became operational in December 2012. The 36MW battery storage is a scalable assembly of thousands of 12 volt, 1 kWh, dry cell batteries based on a proprietary formula of alloys including copper, lead and tellurium.

the expected value for the intermittent renewable energy production using Gaussian quadrature nodes and weights. In determining the weights and nodes (normalized wind speed), I make use of a beta distribution defined on the interval [0, 1] and parametrized by two positive shape parameters, a and b. As an example, for $a = 2$ and $b = 2$, the probability density function, $f(z)$, for the beta distribution looks like the one in Figure 2.

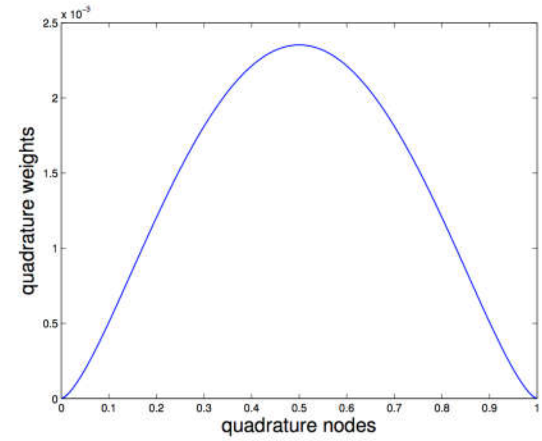

Figure 2: *Beta probability density function for the (normalized) wind speed (a=2, b=2).*

Finally, in evaluating the long-run steady-state behavior of the controlled economic process, I make use of Monte Carlo Simulations (see Appendix C.1).

4.2. Discussion

Figure (3) presents the optimal decision rules for three different (linear, quadratic and cubic) cost functions. To be consistent with my earlier analysis, I present only the decision rules regarding the fossil fuel energy generation, $Q_d^*(S, z)$, and energy storage that will be transferred to the next period, $S'^*(S, z)$.

Considering the case with the linear cost function in generating the fossil fuel energy one can see that when the wind strength is highest, i.e., $z = 1$, and $z\overline{Q}_c = 100.8GW$, then it is optimal to generate the fossil fuel energy at its ramp-up level (see Figure (3a)-i). It is also optimal to store energy up to its capacity, $16.8GW$, which is an outcome independent of the level of stored energy in this case (see Figure (3a)-ii). Furthermore, when the wind strength is less than 0.5, all stored energy will then be consumed, which is a result independent of how much energy was transferred into the current period.

The optimal decision rules for the two remaining cases are quite distinct. In line with Proposition 1, one can see that the costlier it gets to generate the fossil fuel energy, the lower the corresponding generation levels and the higher the level of energy transferred into the next period.[8] For example, if $z = 0.5$ and there is no stored energy, $S' = 0, 5.2, 6.9$GW for a constant – , increasing – and increasingly increasing – cost fossil fuel energy industry,

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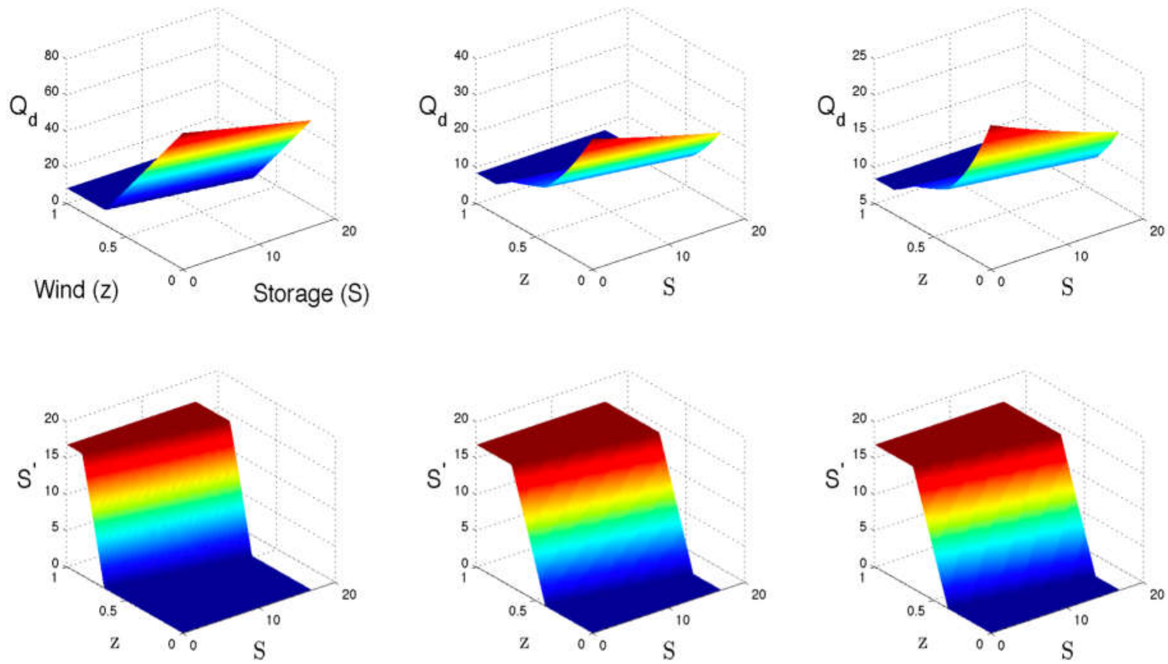
[8] For all variations of z and S, while the fossil fuel energy generation takes its lowest values, the energy levels transferred to the next period are the highest for a cubic cost function, i.e., $C_d''' > 0$.

respectively.



(a) *Linear cost function* **(b)** *Quadratic cost function* **(c)** *Cubic cost function*

Figure 3: *Optimal decision rules for fossil fuel energy generation, Q_d, and energy storage, S', for different cost functions.*

A lower level of stored energy for each pair of z and S when the cost function is linear can be attributed to the lower opportunity cost of not storing energy in the current period: if the wind power is low, and energy is not stored, then, in case it is required, the cost of generating the required energy from fossil fuels will not be too costly. However, this is not necessarily the case when the cost function is nonlinear: if there is no stored energy and suddenly the wind ceases to blow, then the economy would have to incur greater costs to get the desired level of energy from fossil fuels.

Having solved for the optimal decision rules, I can examine the long-run tendencies of the model variables. Here, I aim at computing the steady-state mean values for the model variables and analyze how they respond to different specifications of the cost function and model parameters.

In doing this, I simulate the representative paths for the model variables using Monte Carlo simulations. Given that I work with a stationary distribution, i.e., that the transition probabilities are time-invariant, I can argue that the problem possesses a steady-state distribution so that I can calculate the steady-state mean values for the economic variables.

Assuming three different cost functions in generating fossil fuel energy, the results of the simulations are summarized by Figure (4a). As expected from the previous discussion regarding the optimal decision rule, the fossil fuel steady-state (SS) mean levels are the smallest, approximately 10GW, for the case with the cubic-cost function. On the contrary, the SS mean value for the stored energy is the highest, 10.2GW for the same case. Moreover, when one considers the long-run tendencies given that the cost structure of the fossil fuel energy industry is constant, i.e., a linear cost function, I see that the fossil fuel energy SS mean takes its highest value, $27GW$, while the stored energy gets much lower, approximately $2GW$. In line with Proposition 1, the simulation results show the impact a positive third-order derivative of the cost function can have on energy storage decisions.

Another fundamental result I got previously was the effect of prudence on precautionary energy storage decisions. In looking at the effect of a more prudent economy, I take $\gamma = 3$. The results indicate that a higher level of prudence can alter the results significantly. Compared to the previous cases with different cost structures, a higher level of prudence can indeed result in a much higher level of SS energy savings, even if the cost function is linear (see Figure 4b).

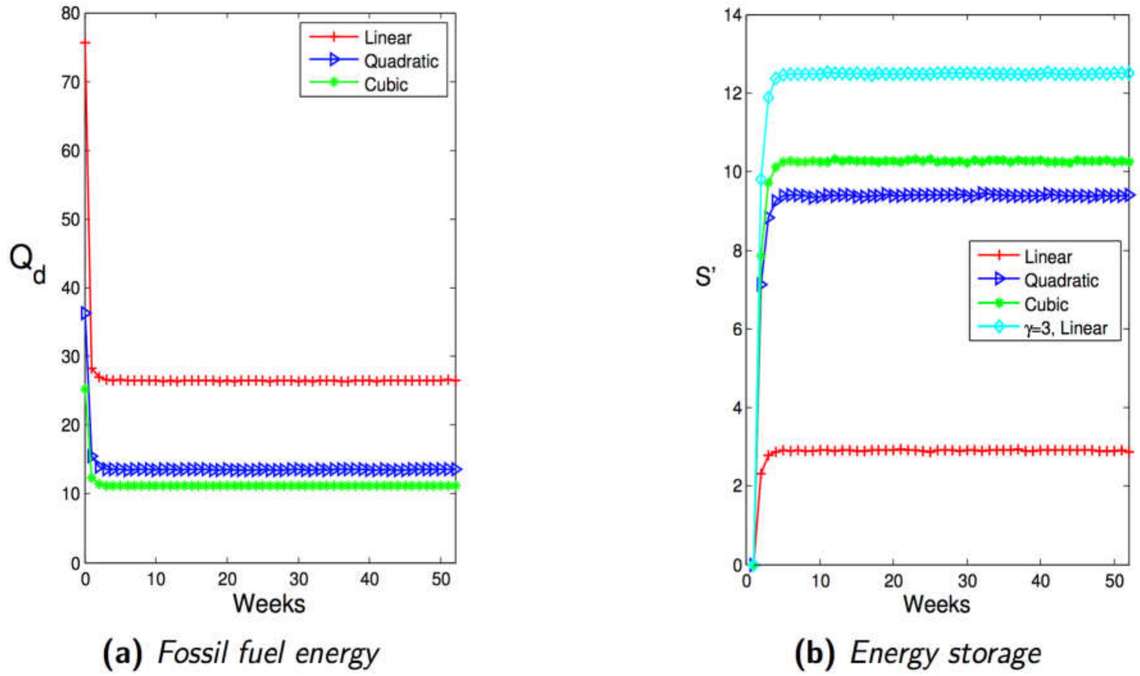


Figure 4: *Steady-state analysis - mean values*

5. Conclusion

In line with the global efforts to reduce CO2 emissions, renewables have an extensive potential to substitute for the fossil fuels. However, they also have their shortcomings. One of them, maybe the most crucial one, is the intermittency problem that can jeopardize immediate access to energy. One technology considered to alleviate, or even cause the intermittency problem to be negligible, is energy storage. Yet the economics of energy generation lacks the treatment of intertemporal welfare decisions in the presence of intermittent renewable energy and energy storage technology. This may become a serious drawback, as without taking this into account, long-term analysis and the policy decisions in this respect can be biased and even misleading.[9]

By approaching the problem both analytically and numerically, I attempted to fill this gap. The

[9] It is also important to note that the long-term policy suggestions of assessment models need be taken with a grain of salt not only because they are big abstractions of complex dynamics, but also the intermittency problem (thus, shorter time periods) and, with it, the energy storage decisions are excluded. This can have cogent influence on the ongoing research in assessment modeling and climate change, as their calculations and conclusions extend to the near and distant futures.

analytical results show the conditions where a convex demand schedule can have considerable effect on industry-wide energy storage. I also show how the cost structures, including the third-order derivative of the cost function in generating the fossil fuel energy, can influence energy storage decisions. Utilizing numerical simulations, I then calculated the optimal decision rules, i.e., optimal policy functions, which are vital in navigating decisions regarding how much energy to generate from fossil fuels and how much to use from stored energy (or how much to store). I benefited from these policy functions when analyzing long-term tendencies, or alternatively, the steady-state mean levels, of the economic variables.

The results not only reveal that prudence and a third-order derivative of the cost function are important for energy storage decisions, but also show that a prior knowledge of the prudence level and the cost-structure of the fossil fuel industry can be quite fundamental in the optimal management of energy sources. Such knowledge will also be crucial when evaluating energy investment decisions.

The present study can be extended in several directions. First, one can extend the current model by taking into account investment decisions in capacities. It is also interesting to incorporate a climate module and investigate the effects of climate change, and hence, the climate policies on the use of fossil fuels, intermittent renewable energy, and energy storage. One can also consider R&D investments and technological change, and analyze how the use of different energy sources and their technologies evolve over time depending on both climate and R&D policies. Last but not least, a further investigation of the effects of prudence and the cost structures on the economic decisions can be quite important not only in the literature in energy economics but also in the literature on prudence in general. The decentralization of the optimal allocation decisions by market mechanisms and the investigation of how allocations are modified when risk attitudes and time preferences change is another interesting avenue.

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Appendices

A. Derivation of Eq. (3)

The Bellman equation is the following:

$$V(S,z) = \max_{Q_{dt}} \left\{ U\left(Q_d + z\overline{Q}_c - S' + \phi S\right) - C_d(Q_d) + \delta \mathbb{E}[V(S',z')] \right\}\Big|_{S'=0} \quad (12)$$

for which the first order condition (FOC) with respect to (w.r.t) $\mathrm{Q_d}$ yields

$$U'(Q) - C_d(Q_d) \leq 0, \quad (13)$$

with equality whenever $\mathrm{Q_d}>0$. Let the optimal decision (the optimal response function) be $\mathrm{Q_d}(\mathrm{S}, \mathrm{z})$. The welfare effect of S' when it is increased marginally from zero can be shown as

$$\frac{\partial V(S,z)}{\partial S\prime}\Big|_{S'=0} = -U'\left(Q_d(S,z) + z\overline{Q}_c + \phi S\right) + \delta\ \mathbb{E}\left[\frac{\partial V(S',z')}{\partial S\prime}\right]\Big|_{S'=0}. \quad (14)$$

From the Envelope Theorem, only the direct effect of a marginal change in the state variable matters on the value function. Given that I evaluate the problem when $S' = 0$, the derivative of the associated value function w.r.t S shows:

$$V_1(S,z) = \phi U\left(Q_d + z\overline{Q}_c - S' + \phi S\right),$$

where $\mathrm{V_1}(S,z)$ is the derivative of the value function w.r.t its first argument S. This is the Benveniste-Scheinkman (Envelope Theorem) condition. Iterating this one period forward, and plugging the result in (14) yields:

$$\frac{\partial V(S,z)}{\partial S\prime}\Big|_{S'=0} = -U'\left(Q_d(0,z) + z\overline{Q}_c + \phi S\right) + \delta\phi\mathbb{E}\left[U'(Q_d(0,z') + z'\overline{Q}_c)\right]. \quad (15)$$

As I restrict the analysis to $S = 0$, and hence, assume no inherited energy, then from (15) I arrive at the following expression:

$$\frac{\partial V(S,z)}{\partial S\prime}\Big|_{S'=0} = -U'\left(Q_d(0,z) + z\overline{Q}_c\right) + \delta\phi\mathbb{E}\left[U'(Q_d(0,z') + z'\overline{Q}_c)\right]. \quad (16)$$

Let $g(z') \stackrel{\text{def}}{=} C_d'\left(Q_d(0,z')\right)$. Taking the expectation of a second-order Taylor approximation around $\overline{z}$ gives

$$\mathbb{E}[g(z')] \simeq g(\overline{\mathrm{z}}) + \frac{1}{2} g''(\overline{\mathrm{z}})\sigma^2\,. \quad (17)$$

In the following, I will calculate $g''(\overline{\mathrm{z}})$. First, $g'(z') = C_d''\left(Q_d(0,z')\right)\frac{\partial Q_d(0,z')}{\partial z\prime}$, where

$$\frac{\partial Q_d(0,z')}{\partial z\prime} = \frac{U''(Q(0,z\prime))\overline{Q}_c}{C_d''(Q_d(0,z')) - U''(Q(0,z\prime))} < 0. \quad (18)$$

Following (18) one gets

$$g''(\mathrm{z}') = C_d'''\left(\frac{\partial Q_d(0,z')}{\partial z'}\right)^2 + C_d''\frac{\partial^2 Q_d(0,z')}{\partial z'^2},$$

and

$$\frac{\partial^2 Q_d(0,z')}{\partial z'^2} = \frac{\overline{Q}_c^{\,2}}{(C_d''-U'')^3}\left(C_d''^2 U''' - U''^2 C_d'''\right),$$

where U''' is the third-order derivative of the utility function.

These outcomes allow me to write

$$g''(\mathrm{z}') = \overline{Q}_c^{\,2}\left[\frac{(C_d'')^3}{(C_d''-U'')^3}U''' + \frac{(-U'')^3}{(C_d''-U'')^3}C_d'''\right].$$

Using the results from the second-order Taylor approximation, and owing to the fact that $Q_d > 0$, and therefore, $U'(Q) = C_d'(Q_d)$, the welfare effect of increasing S' marginally from zero when $S = 0$ will yield the desired expression given by Eq. (3).

B. Applying Blackwell's sufficient conditions for a contraction to the model

Blackwell's sufficient conditions for a contraction

The right-hand side of a Bellman equation is a mapping of the value function $V(\cdot)$ and $V = TV$ is a fixed point of the mapping, where T is a function mapping V into itself. For there to be a unique solution to the dynamic programming problem, one needs show that the mapping for the Bellman equation is a contraction mapping. In showing this, one makes use of Blackwell's sufficient conditions for a contraction.

Theorem *(Blackwell's sufficient conditions for a contraction) Let* $X \subseteq R^l$, *and let* $B(X)$ *be a space of bounded functions* $f: X \to R$, *with supremum norm* $\|\cdot\|_\infty$. *Let* $T: B(X) \to B(X)$ *be an operator satisfying*

1. *(Monotonicity) for* $f, g \in B(X)$ *and* $f\ (x) \le g(x), \forall x \in X$, *implies* $(Tf\ )(x) \le (Tg)(x)$, $\forall\ x \in X$;
2. *(Discounting) there exists some* $\delta \in (0,1)$ *such that*

$$[T(f + a)](x) \le (Tf)(x) + \delta a,\ all\ f \in B(X), a \ge 0,\ x \in X.$$

Then T is a contraction with modulus δ.[10]

In the following, I prove that the energy generation and storage model I work with satisfies Blackwell's sufficient conditions for a contraction.

Proposition *The energy generation and storage model, satisfies Blackwell's sufficient conditions for a contraction.*

Proof. Looking at the equation of motion for stored energy, S, one can see that it takes its maximum value when energy consumption is null and $\mathrm{z} = 1$; i.e., $S^{max} = (\mathrm{Q_d} + \mathrm{Q_c})/(1 - \phi)$. This defines the state space $X \subseteq [0, \overline{Q}_d + \overline{Q}_c] \subseteq R$ and $B(X)$ the function space of the bounded functions $f : X \to R$ with supremum norm.

In the energy storage problem, I defined an operator T by:

[10] $(f + a)(x)$ is the function defined by $(f + a)(x) = f\ (x) + a$. For the proof I refer the reader to Stokey (1989).

$$(TV)(S,z) = \max_{\{Q,Q_d,Q_c,R,S\prime\}} \{U(Q) - C_d(Q_d) - C_c(Q_c) + \delta\mathbb{E}[V(S',z')]\}$$

If $\mathrm{V}(\mathrm{S}',\mathrm{z}') \leq \widehat{\mathrm{V}}(\mathrm{S}',\mathrm{z}')$ for all values of S', then the objective function for which $\mathrm{T}\widehat{\mathrm{V}}$ is the maximized value is uniformly higher than the function for which TV is the maximized value, which makes the monotonicity requirement obvious.

The discounting requirement is satisfied from the following:

$$\begin{aligned}\big(T(V+a)\big)(S,z) &= \max_{\{Q,Q_d,Q_c,R,S'\}} \{U(Q) - C_d(Q_d) - C_c(Q_c) + \delta\mathbb{E}[V(S',z') + a]\} \\ &= \max_{\{Q,Q_d,Q_c,R,S'\}} \{U(Q) - C_d(Q_d) - C_c(Q_c) + \delta\mathbb{E}[V(S',z')]\} + \delta a \\ &= (TV)(S,z) + \delta a.\end{aligned}$$

□

Accordingly, there exists a unique fixed point for the mapping of the value function, i.e., a unique solution to the dynamic programming problem.

C. Numerical implementation of the model

Method description

I solve the dynamic stochastic decision problem by collocation method. In doing this, I approximate the value function by an approximant $\widetilde{\mathrm{V}}(\mathrm{S})$ that is parameterized by and solved for a vector of parameters, $\boldsymbol{\beta}$.

Abstracting from intermittency, a function can be approximated by a combination of n linearly independent basis functions, $\{\psi_i\}_{i=0}^{n}$, and basis coefficients, $\{\beta\}_{i=0}^{n}$, where n represents the number of collocation points:

$$\mathrm{F(x)} \approx \tilde{\mathrm{F}}\mathrm{(x)} = \sum_{i=1}^{n} \beta_i \psi_i(x).$$

The interpolation problem in one dimension is then to find $\{\beta\}_{i=0}^{n}$, satisfying F at n interpolation points. In vector notation this can be written as the following:

$$F(\boldsymbol{x}) = \Psi(\boldsymbol{x})\boldsymbol{\beta}$$

where $\boldsymbol{\Psi}(\boldsymbol{x}) = [\psi_1(x)\ \psi_2(x)\ \psi_3(x) \ldots \psi_{n+1}(x)]$ is the Chebyshev Vandermonde matrix, $\boldsymbol{\beta} = [\beta_1\ \beta_2\ \beta_3 \ldots \beta_{n+1}]'$ and $\boldsymbol{x} = [x_1\ x_2\ x_3 \ldots x_{n+1}]'$,

$$\boldsymbol{\Psi}(\boldsymbol{x}) = \begin{bmatrix} \psi_1(x_1) & \cdots & \psi_{n+1}(x_1) \\ \vdots & \ddots & \vdots \\ \psi_1(x_{n+1}) & \cdots & \psi_{n+1}(x_{n+1}) \end{bmatrix}$$

Similarly, in approximating a value function, I search for a coefficient vector, β, that ensures that the approximant satisfies the Bellman equation and the equilibrium conditions at the n collocation nodes (one can think of collocation nodes as discrete states of the economy).
In the current energy consumption and storage problem, I approximate a bivariate function, $\mathrm{V}(\mathrm{S},\mathrm{z})$, as the planner considers the amount of stored energy and weather conditions before taking decisions. Therefore, I apply the collocation method solution strategy in a multidimensional setting (i.e.,

multidimensional interpolation).

I numerically solve Eq. (2) which is simplified to give:

$$V(S,z) = \max_{\{Q_d,S'\}} \left\{ \frac{\left(Q_d + z\overline{Q}_c - S' + \phi S\right)^{1-\gamma}}{1-\gamma} - C_d(Q_d) + \delta \mathbb{E}[V(S',z')] \right\}$$
$$subject\ to\quad \overline{Q}_d \geq Q_d \geq 0,$$
$$\overline{S} \geq S \geq 0.$$

I approximate the value function using Chebyshev polynomials. Chebychev basis polynomials in combination with Chebychev interpolation nodes can yield extremely well- conditioned interpolation collocation equations that one can accurately and efficiently solve. For a discussion regarding Chebychev basis functions and nodes, I refer the reader to Judd (1992), Judd (1998) and Miranda and Fackler (2002). When approximating the value function using the Chebyshev polynomials, I discretize z and S into K ($\mathrm{z_k}$ for $\mathrm{k} = (1,2,\ldots \mathrm{K})$) into n collocation nodes. I determine the basis function coefficients for each z and S. For n basis functions, there are going to be n basis coefficients, and given K different weather states, the computational problem is to solve for $\mathrm{K} \times \mathrm{n}$ coefficients. Let us denote these coefficients by $\beta = [\beta_1\ \beta_2\ \beta_3\ \ldots\ \beta_K]'$,where, for example, $\beta_2 = [\beta_{1,2}\ \beta_{2,2}\ \ldots\ \beta_{n,2}]'$.

For each state of the weather, z_k, and for each level of stored energy, S_i, the approximant is formed as follows:

$$V(S_i,z) \approx \tilde{V}(S_i,z) = \sum_{j=1}^{n} \beta_{j,z} \Psi_j(S_i).$$

Given $V(S_i,z)$, I form the approximant to $V(S_i',z_k')$ as well. In doing this for S_i and z_k, I need to compute the level for the stored energy in the period ahead, S', and energy generation today Q_d given the intervals $\overline{S} \geq S \geq 0$ and $\overline{Q}_d \geq Q_d \geq 0$. Using these boundaries, I construct a grid for fossil fuel energy, $\{Q_{di,z_k,l}\}_{l=1}^{n}$, and energy storage, $\{S'_{i,z_k,l}\}_{l=1}^{n}$.

Given the approximants of the value function, I have $(K\ \times\ n)$ equations in $(K\ \times\ n)$ unknowns:

$$\sum_{j=1}^{n} \beta_{j,z} \Psi_j(S_i) = \max_{\{Q_d,S'\}} \left\{ \frac{\left(Q_{di,z_k,l} + z\overline{Q}_c - S' + \phi S\right)^{1-\gamma}}{1-\gamma} - C_d(Q_d) + \delta \sum_{k=1}^{K} \sum_{l=1}^{n} \omega_k\, \beta_{j,z} \Psi_j(S'_{i,z_k,l}) \right\}_{l=1}^{n}$$

where, in approximating the integral operation, I replaced the continuous random variable $\mathrm{z_k}$ with its discrete counterpart ω_k, the weight functions, using Gaussian quadrature scheme. The weight functions are defined over the interval K. (For a weight function defined on an interval K, $\int_\mathrm{K} \omega(\mathrm{z})\mathrm{zdz} \approx \sum_{\mathrm{k}=1}^{\mathrm{K}} \omega_\mathrm{k}\, \mathrm{z_k}$.) The expected value of the renewable energy generation can be numerically computed as follows:

$$\mathbb{E}\left[z\overline{Q}_c\right] = \int_\mathrm{K} z\overline{Q}_c \omega(z)\mathrm{dz} \approx \sum_{\mathrm{k}=1}^{\mathrm{K}} \omega_\mathrm{k}\, \mathrm{z_k} \overline{Q}_c$$

For $k\ =\ 1,2,\ldots K$, quadrature nodes z_k and the corresponding weights ω_k are selected such that 2K moments are satisfied.

Above, I showed the approximant for $V(S_i', z_k')$ in its explicit form:

$$V(S_i', z_k') \approx \tilde{V}(S_i', z_k') = \sum_{j=1}^{n} \beta_{j,z_k} \psi_j(S'_{i,z_k,l})$$
$$= \sum_{j=1}^{n} \beta_{j,z_k} \mathrm{T}_{j-1}\left[2\left(\frac{S'_{i,z_k,l} - \underline{S}}{\overline{S} - \underline{S}}\right) - 1\right]$$

for $l = \{1,2,\dots,n\}$, where $\psi_j(S'_{i,z_k,l}) = \mathrm{T}_{j-1}\left[2\left(\frac{S'_{i,z_k,l} - \underline{S}}{\overline{S} - \underline{S}}\right) - 1\right]$ are the Chebyshev polynomial basis functions. $\overline{S}$ and $\underline{S}(= 0)$ denote the upper and lower bounds for energy storage, respectively.

Having explained how the polynomial interpolation can work, I now explain the procedure of how to calculate the basis function coefficients, $\beta = [\beta_1\ \beta_2 \dots\ \beta_K]$. First, I need to make a guess for the initial values of the basis functions' coefficients: $\beta^0 = [\beta_1^0\ \beta_2^0 \dots\ \beta_K^0]$. We then need to construct a grid of Chebyshev nodes, $u_{n\times1}$, and convert them into grid of stored energy, S. The mapping looks like the following:

$$\boldsymbol{u} \to \boldsymbol{S} \in [\overline{S}, \underline{S}], \boldsymbol{S} = \frac{\overline{S} + \underline{S}}{2}\boldsymbol{I} + \frac{\overline{S} + \underline{S}}{2}\boldsymbol{u}$$

where $\boldsymbol{I}$ is a vector of ones: $\boldsymbol{I}_{n\times1}$.

For $k = \{1,2,\dots,K\}$ and $i = \{1,2,\dots,n\}$, I construct a feasible grid of energy generation Q_d and S_i' using Chebyshev nodes:

$$\boldsymbol{u} \to \boldsymbol{Q}_d \in \left[\underline{Q}_d, \overline{Q}_d\right], \boldsymbol{Q}_{\boldsymbol{d}i,z_k} = \frac{\overline{Q}_d - \underline{Q}_d}{2}\boldsymbol{I} + \frac{\overline{Q}_d - \underline{Q}_d}{2}\boldsymbol{u}$$
$$\boldsymbol{u} \to \boldsymbol{S}' \in [\overline{S}, \underline{S}], \boldsymbol{S}'_{i,z_k} = \frac{\overline{S}}{2}\boldsymbol{I} + \frac{\overline{S}}{2}\boldsymbol{u}$$

where $\underline{S} = 0$.

For $\boldsymbol{S}'$, I have the Chebyshev Vandermonde matrix: $\boldsymbol{\Psi}(\boldsymbol{S})$. Then

$$\tilde{V}(\boldsymbol{S}, \boldsymbol{z}) = \frac{\left(\boldsymbol{Q}_{d,z} + \boldsymbol{z}\overline{Q}_c - \boldsymbol{S}' + \phi\boldsymbol{S}\right)^{1-\gamma}}{1-\gamma} - C_d(\boldsymbol{Q}_{d,z}) + \delta \sum_{k=1}^{K} \omega_k \boldsymbol{\psi}(\boldsymbol{S}')\boldsymbol{\beta}_k^0.$$

Taking the maximal entries in $\tilde{V}(S, z)$, I can construct $\tilde{V}(\beta^0)$, and update the coefficients according to Newton-Raphson method (see Judd,1998):

$$\beta' = \boldsymbol{\beta} - [\boldsymbol{\psi} - \tilde{V}^j(\boldsymbol{\beta})]^{-1}[\boldsymbol{\psi}\boldsymbol{\beta} - \tilde{V}(\boldsymbol{\beta})]$$

where $\tilde{V}^j(\beta)$ is the Jacobian of the approximant. One can then use the iterative update rule until the following difference gets smaller than a predetermined tolerance level, ε:

$$\boldsymbol{\beta}' - \left(\boldsymbol{\beta} - \left[\boldsymbol{\psi} - \tilde{V}^j(\boldsymbol{\beta})\right]^{-1}[\boldsymbol{\psi}\boldsymbol{\beta} - \tilde{V}(\boldsymbol{\beta})]\right) < \varepsilon$$

Long-run analysis

After solving for the collocation coefficients, β, one can estimate the evolution of the variables in the model. Using the grid, I constructed for the stored energy S, the solution to the model gives us an implicit policy rule: $S' = g(S, z)$. (Given S_i and z_k, I know what $S'_{i,k}$ is.)

By satisfying the convergence criteria, I also solve for S'. I can use these values to estimate the policy (transition) rule, hence solve for the Chebyshev function coefficients, φ:

$$S' = \psi\varphi \rightarrow \varphi = (\psi'\psi)^{-1}\psi' S'$$

Using these coefficients, one can pick a random sequence for weather conditions z_t for $t = 1, 2, \ldots, T$. One can then generate another sequence for S_{t+1}:

$$S_{t+1} = \psi(S_t)\varphi$$

Suppose that I do this N times (for N large) by generating N pseudorandom sequences for z. (Pseudorandom sequences are sequences that display some properties satisfied by random variables, such as zero serial correlation and correct frequency of runs, although none satisfy all properties of an i.i.d random sequence (Judd, 1992).) Given the policy functions I calculated, $S'(S, z)$ and $Q_d(S, z)$, and the initial states S_0 and z_0, I can then generate a representative path from the N paths. Calculating the average value from the various pseudorandom sequences, one would get representative paths for the model variables in the long run. This procedure is called a Monte Carlo Simulation.

Numerical implementation

I solve dynamic programming equation (2) by using collocation method and update the collocation coefficients according to the Newton's method (see the subsection entitle Method description). The predetermined tolerance level for the convergence criterion is 1×10^{-7}. I construct a 40 Chebychev polynomial basis functions by forming 40 collocation nodes (4 nodes along *S* and 10 nodes along *S* dimension) and 40 basis function coefficients. The Beta distribution for the intermittent wind is approximated by Gauss-Legendre quadrature with 20 nodes.

The code is written in Matlab. In generating and evaluating the Chebychev polynomials, and doing the Monte Carlo simulations, I use CompEcon toolbox described in Miranda and Fackler (2002).

D. Sensitivity analysis: Round-trip efficiency parameter

The analysis in this section are based on an economy, in which $\overline{Q}_d = 100.8$GW, $\overline{Q}_c = 100.8$GW, $\underline{Q}_d = 8.4$GW, $\overline{S} = 16.8$GW, $\underline{S} = 0, \gamma = 2, \rho = 0.05, \phi = 0.99$, quadratic cost function, $C_d(Q_d) = c_q Q_d^2$, where $c_q = 1.0417 \text{ x } 10^{-5}$ UoN (see p.11).

From Figure 5 one can see that all scenarios discern the same pattern and display similar qualitative features; i.e., during the first few periods energy is accumulated and stays roughly on its long-run expected level. However, the lower the round-trip efficiency parameter is, the smaller is energy storage, i.e., lower levels of ϕ imply less enthusiastic storage policies. Accordingly, when $\phi \leq 0.4$ energy storage becomes suboptimal.

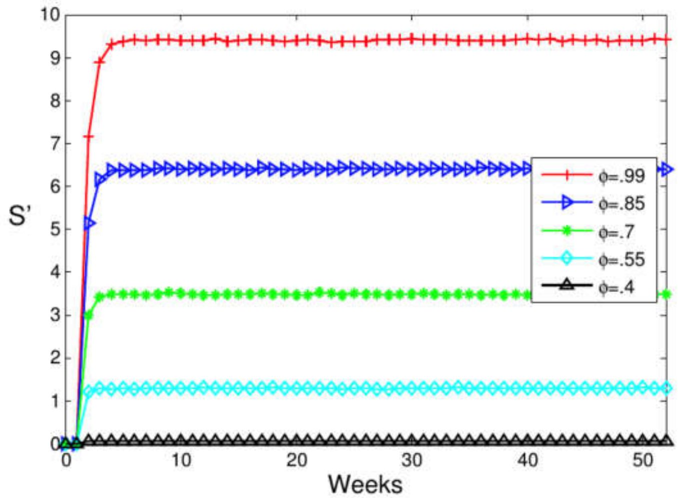

Figure 2: *Sensitivity analysis for the round-trip efficiency parameter,* ϕ