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An Application of the Modified Expansion Method to Nonlinear Partial Differential Equation

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Abstract. In this article, the travelling wave solutions of the medium equal width equation are obtained using the modified expansion method. The solution functions were obtained by selecting the appropriate parameters. It has been checked that these functions provide the MEW equation. Density, two and three dimensional graphics of the obtained solutions and other mathematical operations were found with the Mathematica software program. When the resulting solution functions are examined, it is determined that they include trigonometric, topological and singular soliton properties.

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1. Introduction

The solutions of nonlinear partial differential equations have an important place in engineering, physics and real life. Therefore, there are various methods in the literature for obtaining solutions of these equations. Some of these, the (G'/G) expansion method [1,2], the generalized Kudryashov method [3], the Fourier spectral method [4], the sine-Gordon expansion method [5–7] and so on. In this study, we will investigate travelling wave solutions of medium equal width equation by the modified expansion function method (MEFM) [8].

The medium equal width equation can be defined as follow [9],

$$u_t + 3u^2 u_x - d u_{xxt} = 0.$$

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2. Modified Expansion Function Method

We consider the following nonlinear partial differential equation;

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{tx}, ...) = 0. \tag{2.1}$$

where $u(x, t)$ is unknown function.
The following travelling wave transformation:

$$u(x, t) = U(\xi), \xi = v(x - ct). \tag{2.2}$$

where v,c are constants and can be determined later.
Substituting Eq. (2.2) into Eq. (2.1), it gives the following nonlinear ordinary differential equation;

$$N(U, U', U'', U''', ...) = 0. \tag{2.3}$$

According to the MEFM, the requested solution is as follows:

$$U(\xi) = \frac{\sum_{i=0}^{m} A_i [e^{-\vartheta(\xi)}]^i}{\sum_{j=0}^{n} B_j [e^{-\vartheta(\xi)}]^j} = \frac{A_0 + A_1 e^{-\vartheta} + ... + A_m e^{-m\vartheta}}{B_0 + B_1 e^{-\vartheta} + ... + B_n e^{-n\vartheta}}. \tag{2.4}$$

where A_i,B_j, $(0 \le i \le m, 0 \le j \le n)$.
Using the homogeneous balance principle, the m and n positive integer values are obtained.

$$\upsilon'(\xi) = e^{-\vartheta(\xi)} + \mu e^{\vartheta(\xi)} + \lambda. \tag{2.5}$$

Eq. (2.5) has the following families of solutions [10].
Family 1: When $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right).$$

Family 2: When, $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$\vartheta(\xi) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right).$$

Family 3: When, $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu > 0$,

$$\vartheta(\xi) = -\ln\left(\frac{\lambda}{e^{\lambda(\xi+E)} - 1}\right).$$

Family 4: When, $\mu \neq 0, \lambda \neq 0, \lambda^2 - 4\mu = 0$,

$$\vartheta(\xi) = \ln\left(-\frac{2\lambda(\xi + E) + 4}{\lambda^2(\xi + E)}\right).$$

Family 5: When, $\mu = 0, \lambda = 0, \lambda^2 - 4\mu = 0$,

$$\ln(\xi + E).$$

Substituting Eq. (2.4) and its derivatives into Eq. (2.3), we get an algebraic equation system. This system has been solved by the computational programme and then the solutions of the medium equal width equation have been obtained.

3. Application

Consider the following travelling wave transformation:

$$u(x,t) = U(\xi), \xi = v(x - ct).$$

The following nonlinear ordinary differential equation is obtained,

$$cdv^2 U'' + U^3 - cU = 0. \tag{3.1}$$

If the balancing procedure is applied to Eq. (3.1),we have the following relationship

$$m = n + 1.$$

Choosing $n = 1$ we get $m = 2$,the Eq. (2.4) is obtained for m and n values as follows;

$$U(\xi) = \frac{A_0 + A_1 e^{-\vartheta} + A_2 e^{-2\vartheta}}{B_0 + B_1 e^{-\vartheta}} \tag{3.2}$$

After all derivative terms from Eq. (3.2) have been found and required arragements have been done, we have found an algebraic equation system. With the aid of computation programme, we have solved the system to obtain the following coefficients.

$$A_0 = \frac{\sqrt{c}\lambda B_0}{\sqrt{\lambda^2 - 4\mu}}, A_1 = \frac{\sqrt{c}\,(2B_0 + \lambda B_1)}{\sqrt{\lambda^2 - 4\mu}}, A_2 = \frac{2\sqrt{c}B_1}{\sqrt{\lambda^2 - 4\mu}}, k = \sqrt{\frac{2}{-d\,(\lambda^2 - 4\mu)}}$$

Substituting these coefficients into Eq. (3.2), we have the following solutions:

Family 1: When, $\mu \neq 0, \lambda^2 - 4\mu > 0$,

$$u_1(x,t) = \frac{\sqrt{c}(\lambda^2 - 4\mu + \lambda\sqrt{\lambda^2 - 4\mu}Tanh[\psi])}{\sqrt{\lambda^2 - 4\mu}(\lambda + \sqrt{\lambda^2 - 4\mu}Tanh[\psi]} \tag{3.3}$$

where $\psi = \frac{1}{2}\sqrt{\lambda^2 - 4\mu}(\xi + E)$.

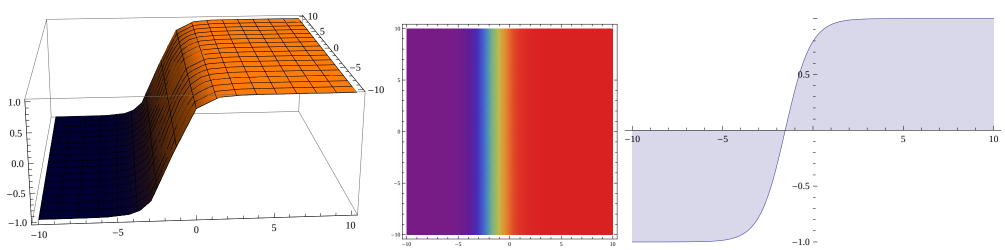

Figure 1. The 3D, density graphic and the 2D surface of Eq. (3.3) in $\lambda = 2, \mu = 0.25, c = 1, d = -1, v = \sqrt{\frac{2}{3}}, \beta = 0.6, B_0 = 0.35, E = 0.55$ and $t = 1$

Family 2: When, $\mu \neq 0, \lambda^2 - 4\mu < 0$,

$$u_2(x,t) = (\frac{\sqrt{c}(\lambda^2 - 4\mu - \lambda\sqrt{-\lambda^2 + 4\mu}Tanh[\upsilon])}{\sqrt{\lambda^2 - 4\mu}(\lambda - \sqrt{-\lambda^2 + 4\mu}Tanh[\upsilon])} \tag{3.4}$$

where $\upsilon = \frac{1}{2}\sqrt{-\lambda^2 + 4\mu}(\xi + E)$.

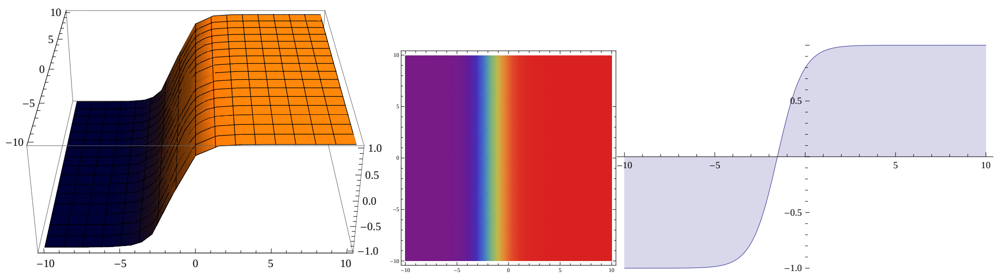

Figure 2. The 3D, density graphic and the 2D surface of Eq. (3.4) in $\lambda = 2, \mu = 0.25, c = 1, d = -1, v = \sqrt{\frac{2}{3}}, \beta = 0.6, B_0 = 0.35, E = 0.55$ and $t = 1$

Family 3: When, $\mu = 0, \lambda \neq 0, \lambda^2 - 4\mu > 0$

$$u_3(x,t) = \frac{\sqrt{c}\lambda Coth(\frac{\lambda}{2}(\xi + E))}{\sqrt{\lambda^2}} \tag{3.5}$$

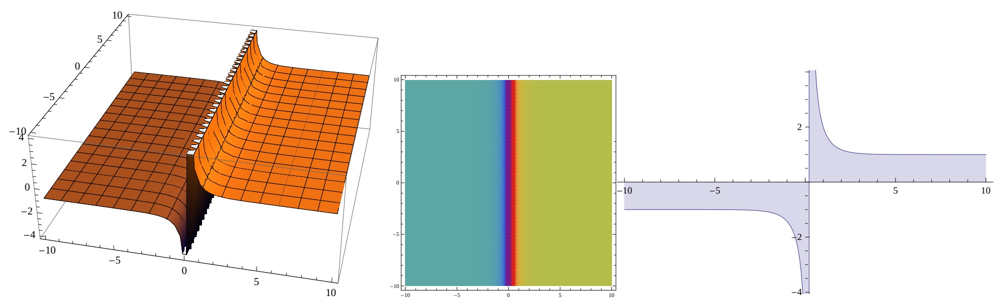

Figure 3. The 3D, density graphic and the 2D surface of Eq. (3.5) in $\lambda = 2, \mu = 0.25, c = 1, d = -1, v = \frac{\sqrt{2}}{3}, \beta = 0.6, B_0 = 0.35, E = 0.55$ and $t = 1$

Remark 3.1. The obtained coefficients have not given solutions for Family-4 and Family-5.

4. Conclusion

In this study, we have obtained travelling wave solutions of medium equal width equation by using modified expansion function method. The results show that the modified expansion function method is a very effective mathematical method for solving nonlinear partial differential equations. The obtained solutions have been checked by a software programme.

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