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Original Article

On Micro Topological Spaces

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Abstract — Every year different type of topological spaces are introduced by many topologist. Now a days available topologies are supra topology, ideal topology, bitopology, fuzzy topology, Fine topology, nano topology and so on. Nano topology introduced by Thivagar, using this nano topology we introduced micro topology and also study the concepts of micro-pre open sets and micro-semi open sets and some of their properties are investigated.

Keywords — *Micro Topology, Micro-pre open sets, Micro-semi open sets, Micro continuous, Micro pre continuous, Micro semi continuous.*

1 Introduction

In 1963 Kelly [4] introduced Bitopological spaces, In 1983, Mashhour [6] et al. introduced the supra topological spaces. In 1965, Zadeh [9] introduced the concept of fuzzy sets, the study of fuzzy topological spaces which had been introduced by Chang [2] in 1968. The concept of ideal in topological space was first introduced by Kuratowski. They also have defined local function in ideal topological space. Further in 1990 Hamlett and Jankovic [3] investigated further properties of topological space.

Powar and Rajak [7] introduced fine topological spaces in the year 2012. Nano topology introduced by Thivagar [5] in the year 2013. Nano topology based on the concept of lower approximation, upper approximation and boundary region. Nano topology have maximum five nano open sets and minimum three nano open sets including U , ϕ suppose we want add some more open sets, for that time we can use Levine's simple extension concept in nano topology we can extend some more open sets that topology is called micro topology. Every nano topology is micro topology. In this paper, introduce micro topology, micro pre open sets, micro semi open sets and some of their properties are investigated.

2 Preliminary

Let us recall the following definition, which are useful in the sequel.

Definition 2.1. Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2. Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$ satisfies the following axioms

1. $U, \phi \in \tau_R(X)$
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
3. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is called the nano topology on U with respect to X . The space $(U, \tau_R(X))$ is the nano topological space. The elements of are called nano open sets.

3 Micro Topological Spaces

In this section, I introduce and study the properties of Micro topological spaces.

Definition 3.1. $(U, \tau_R(X))$ is a nano topological space here $\mu_R(X) = \{N \cup (N' \cap \mu)\}$: $N, N' \in \tau_R(X)$ and called it Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 3.2. The Micro topology $\mu_R(X)$ satisfies the following axioms

1. $U, \phi \in \mu_R(X)$
2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$

3. The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X . The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological spaces and The elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Example 3.3. $U = \{1, 2, 3, 4\}$, with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\} \subseteq U$, $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$ Then $\mu = \{3\}$. Micro-O= $\mu_R(X) = \{U, \phi, \{1\}, \{3\}, \{1, 3\}, \{2, 4\}, \{2, 3, 4\}, \{1, 2, 4\}\}$

Example 3.4. $U = \{a, b, c, d\}$, with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{b, d\} \subseteq U$, $\tau_R(X) = \{U, \phi, \{b, d\}\}$ and then $\mu = \{b\}$. Micro-O= $\mu_R(X) = \{U, \phi, \{b\}, \{b, d\}\}$

Example 3.5. Let $U = \{p, q, r, s, t\}$, $U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$. Let $X = \{p, q\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$. Then $\mu = \{t\}$. Then Micro-O= $\mu_R(X) = \{U, \phi, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}, \{q, r, s, t\}\}$

Definition 3.6. The Micro closure of a set A is denoted by $\text{Micro-cl}(A)$ and is defined as $\text{Mic-cl}(A) = \cap \{B : B \text{ is Micro closed and } A \subseteq B\}$. The Micro interior of a set A is denoted by $\text{Micro-int}(A)$ and is defined as $\text{Mic-int}(A) = \cup \{B : B \text{ is Micro open and } A \supseteq B\}$.

Definition 3.7. For any two Micro sets A and B in a Micro topological space $(U, \tau_R(X), \mu_R(X))$,

1. A is a Micro closed set if and only if $\text{Mic-cl}(A) = A$
2. A is a Micro open set if and only if $\text{Mic-int}(A) = A$
3. $A \subseteq B$ implies $\text{Mic-int}(A) \subseteq \text{Mic-int}(B)$ and $\text{Mic-cl}(A) \subseteq \text{Mic-cl}(B)$
4. $\text{Mic-cl}(\text{Mic-cl}(A)) = \text{Mic-cl}(A)$ and $\text{Mic-int}(\text{Mic-int}(A)) = \text{Mic-int}(A)$
5. $\text{Mic-cl}(A \cup B) \supseteq \text{Mic-cl}(A) \cup \text{Mic-cl}(B)$
6. $\text{Mic-cl}(A \cap B) \subseteq \text{Mic-cl}(A) \cap \text{Mic-cl}(B)$
7. $\text{Mic-int}(A \cup B) \supseteq \text{Mic-int}(A) \cup \text{Mic-int}(B)$
8. $\text{Mic-int}(A \cap B) \subseteq \text{Mic-int}(A) \cap \text{Mic-int}(B)$
9. $\text{Mic-cl}(A^C) = [\text{Mic-int}(A)]^C$
10. $\text{Mic-int}(A^C) = [\text{Mic-cl}(A)]^C$

4 Micro-Pre-Open Sets

In this section, I define and study about micro-pre-open sets some of their properties are analogous to those for open sets.

Definition 4.1. Let $(U, \tau_R(X), \mu_R(X))$ be a micro topological space and $A \subset U$. Then A is said to be micro-pre-open if $A \subseteq \text{Mic-int}(\text{Mic-cl}(A))$ and micro-pre-closed set if $\text{Mic-cl}(\text{Mic-int}(A)) \subseteq A$.

Example 4.2. Let $U = \{p, q, r, s, t\}$, $U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$. Let $X = \{p, q\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$. Then $\mu = \{t\}$. $\text{Micro-O} = \mu_R(X) = \{U, \phi, \{p\}, \{t\}, \{p, t\}, \{p, q, r, s\}, \{q, r, s\}, \{q, r, s, t\}\}$. Clearly $A = \{q, r, s\}$ is Micro-pre open.

Theorem 4.3. Every Micro-open set is Micro-pre open.

Proof. Let A be Micro-open. Then $A \subseteq \text{Mic-int}(\text{Mic-int}A)$. Since $\text{Mic-int}(\text{Mic-int}A) \subseteq \text{Mic-int}(\text{Mic-cl}A)$, it follows that $A \subseteq \text{Mic-int}(\text{Mic-cl}A)$. Hence A is Micro-pre open. Converse of the above Theorem need not be true. \square

Example 4.4. Let $U = \{i, j, k, l, m\}$ $U/R = \{\{i\}, \{j, k, l\}, \{m\}\}$. Let $X = \{j, k\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{j, k, l\}\}$. Then $\mu = \{i\}$. Then $\text{Micro-O} = \{U, \phi, \{i\}, \{i, j, k, l\}, \{j, k, l\}\}$. Clearly $A = \{i, j, k, m\}$ is Micro-pre open but not Micro-open.

Theorem 4.5. 1. Arbitrary union of Micro-pre open sets is Micro-pre open.
2. Arbitrary intersection of Micro-pre closed sets is Micro-pre closed.

Proof. 1. Let $\{A_\alpha | \alpha \in I\}$ be the family of Micro-pre open sets in X . By Definition 3.6, for each α , $A_\alpha \subseteq \text{Mic-int}(\text{Mic-cl}(A_\alpha))$, this implies that $\cup A_\alpha \subseteq \cup(\text{Mic-int}(\text{Mic-cl}(A_\alpha)))$. Since $\cup(\text{Mic-int}(\text{Mic-cl}(A_\alpha))) \subseteq \text{Mic-int}(\cup \text{Mic-cl}(A_\alpha))$ and $\text{Mic-int}(\cup \text{Mic-cl}(A_\alpha)) = \text{Mic-int}(\text{Mic-cl}(\cup A_\alpha))$, this implies that $\cup A_\alpha \subseteq \text{Mic-int}(\text{Mic-cl}(\cup A_\alpha))$. Hence $\cup A_\alpha$ is Micro-pre open.

2. Let $\{B_\alpha | \alpha \in I\}$ be a family of Micro-pre closed sets in X . Let $A_\alpha = B_\alpha^C$, then $\{A_\alpha | \alpha \in I\}$ is a family of Micro-pre open sets. By (i), $\cup A_\alpha = \cup B_\alpha^C$ is Micro-pre open. Consequently $(\cap B_\alpha)^C$ is Micro-pre open. Hence $(\cap B_\alpha)$ is Micro-pre closed. \square

Remark 4.6. Finite intersection of Micro-pre open sets need not be Micro-pre open.

Example 4.7. In Example 4.4 $\{i, l\}$ and $\{j, l\}$ are Micro-pre open sets, but $\{i, l\} \cap \{j, l\} = \{l\}$ is not Micro-pre open.

Theorem 4.8. In a Micro topological space $(U, \tau_R(X), \mu_R(X))$ the set of all Micro-pre open sets form a generalized topology.

Proof. proof follows from Remark 4.6, Theorem 4.3 and Theorem 4.5. \square

Definition 4.9. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro-topological space. An element $x \in A$ is called Micro-pre interior point of A , if there exist a Micro-pre open set H such that $x \in H \subset A$.

Definition 4.10. The set of all Micro-pre interior points of A is called the Micro-pre interior of A , and is denoted by $\text{Micro-pre-int}(A)$.

Theorem 4.11. 1. Let $A \subset (U, \tau_R(X), \mu_R(X))$ Then $\text{Micro-pre int } A$ is equal to the union of all Micro-pre open set contained in A .

2. If A is a Micro-pre open set then $A = \text{Micro-pre int } A$.

Proof. 1. We need to prove that, $\text{Micro-pre int } A = \cup\{B | B \subset A, B \text{ is Micro-pre open set}\}$. Let $x \in \text{Micro-pre int } A$. Then there exist a Micro-pre open set B such that $x \in B \subset A$. Hence $x \in \cup\{B | B \subset A, B \text{ is Micro-pre open set}\}$. Conversely, suppose $x \in \cup\{B | B \subset A, B \text{ is Micro-pre open set}\}$, then there exist a set $B_o \subset A$ such that $x \in B_o$, where B_o is Micro-pre open set. i.e., $x \in \text{Micro-pre int } A$. Hence $\cup\{B | B \subset A, B \text{ is Micro-pre open set}\} \subset \text{Micro-pre int } A$. So $\text{Micro-pre int } A = \cup\{B | B \subset A, B \text{ is Micro-pre open set}\}$.

2. Assume A is a Micro-pre open set then $A \in \{B | B \subset A, \text{ Micro-pre open set}\}$, and every other element in this collection is subset of A . Hence by part (1) $\text{Micro-pre int } A = A$. □

Theorem 4.12. 1. $\text{Micro-pre int } (A \cup B) \supset \text{Micro-pre int } A \cup \text{Micro-pre int } B$.

2. $\text{Micro-pre int}(A \cap B) = \text{Micro-pre int } A \cap \text{Micro-pre int } B$.

Proof. 1. The fact that $\text{Micro-pre int } A \subset A$ and $\text{Micro-pre int } B \subset B$ implies $\text{Micro-pre int } A \cup \text{Micro-pre int } B \subset A \cup B$. Since Micro-Pre interior of a set is Micro-Pre open, $\text{Micro-pre int } A$ and $\text{Micro-pre int } B$ are pre open. Hence by Theorem 4.5(1), $\text{Micro-pre int } A \cup \text{Micro-pre int } B$ is Micro-Pre open and contained in $A \cup B$. Since $\text{Micro-pre int}(A \cup B)$ is the largest Micro-pre open set contained in $A \cup B$, it follows that $\text{Micro-pre int } A \cup \text{Micro-pre int } B \subset \text{Micro-pre int}(A \cup B)$.

2. Let $x \in \text{Micro-pre int}(A \cap B)$. Then there exist a Micro-pre open set H , such that $x \in H \subset (A \cap B)$. That is there exist a Micro-pre open set, such that $x \in H \subset A$ and $x \in H \subset B$. Hence $x \in \text{Micro-pre int } A$ and $x \in \text{Micro-pre int } B$. That is $x \in \text{Micro-pre int } A \cap \text{Micro-pre int } B$. Thus $\text{Micro-pre int}(A \cap B) \subset \text{Micro-pre int } A \cap \text{Micro-pre int } B$. Retracing the above steps, we get the converse. □

Definition 4.13. $(U, \tau_R(X), \mu_R(X))$ be a Micro-topological space. Let $A \subset X$. The intersection of all Micro-pre closed sets containing A is called Micro-pre closure of A and it is denoted by $\text{Micro-pre cl}(A)$. $\text{Micro-pre cl}(A) = \cap\{B | B \supset A, B \text{ is Micro-pre closed set}\}$.

Remark 4.14. 1. $\text{Micro-pre cl}(A)$ is also a Micro-pre closed set.

2. $\text{Micro-precl}(A)$ is smallest Micro-pre closed set containing A .

Theorem 4.15. Every Mic-closed set is Micro-pre closed.

Proof. Let A be Mic-closed, then by Theorem 4.5, we have $\text{Mic-cl}(\text{Mic-cl } A) \subseteq A$. Since $\text{Mic-cl}(\text{Mic-int}A) \subseteq \text{Mic-cl}(\text{Mic-cl}A) \subseteq A$, A is Micro-pre closed. Converse of the above Theorem need not be true. \square

Example 4.16. $U = \{1, 2, 3, 4\}$, with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\} \subseteq U$, $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$. Then $\mu = \{3\}$. $\text{Micro-O} = \{U, \phi, \{1\}, \{3\}, \{1, 3\}, \{2, 4\}, \{2, 3, 4\}, \{1, 2, 4\}\}$. Then $A = \{1, 2, 3\}$ is Micro-pre closed but not Micro-closed.

Theorem 4.17. A is Micro-pre closed if and only if $A = \text{Micro-pre cl}(A)$.

Proof. $\text{Micro-pre cl}(A) = \cap \{B/B \supseteq A, B \text{ is Micro-pre closed set}\}$. If A is a Micro-pre closed set then A is a member of the above collection and each member contains A . Hence their intersection is A and $\text{Micro-precl}(A) = A$. Conversely, if $A = \text{Micro-precl}(A)$, then A is Micro-pre closed by Remark 4.14. \square

5 Micro-Semi Open Sets

Definition 5.1. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro-topological space and $A \subset U$. Then A is said to be Micro-semi open if $A \subseteq \text{Mic-cl}(\text{Mic-int}A)$ Micro-semi closed. If $\text{Mic-int}(\text{Mic-cl } A) \subseteq A$.

Example 5.2. $U = \{a, b, c, d\}$, with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ $X = \{b, d\} \subseteq U$, $\tau_R(X) = \{U, \phi, \{b, d\}\}$ and then $\mu = \{a\}$. Then $\text{Micro-O} = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$ Clearly $A = \{b, d\}$ is Micro-semi open.

Theorem 5.3. 1. Every Micro-open set is Micro-semi open.

2. Every Mic-closed set is Micro-semi closed.

Proof. 1. If A is Micro-open set then by then, $A \subseteq \text{Mic-int}(\text{Mic-int}A)$. Since $\text{Mic-int}(\text{Mic-int}A) \subseteq \text{Mic-cl}(\text{Mic-int}A)$, $A \subseteq \text{Mic-cl}(\text{Mic-int}A)$. Hence A is Micro-semi open.

2. If A is Mic-closed set then by Theorem 4.5, we have $\text{Mic-cl}(\text{Mic-cl}A) \subseteq A$. Since $\text{Mic-int}(\text{Mic-cl } A) \subseteq \text{Mic-cl}(\text{Mic-cl}A)$, $\text{Mic-int}(\text{Mic-cl } A) \subseteq A$. Hence A is Micro-semi closed. \square

Remark 5.4. Converse of the above Theorem need not be true.

Example 5.5. In Example 5.2 clearly $A = \{b, d\}$ is Micro-semi open Clearly $A = \{b, d\}$ is Micro-semi closed, but not Mic-closed.

6 Continuous Functions in Micro-Top. Spaces

Definition 6.1. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be two Micro-topological spaces. A function $f : U \rightarrow V$ is called Micro-continuous function if $f^{-1}(H)$ is Micro-open in U for every Micro-open set H in V.

Example 6.2. Let $U = \{p, q, r, s, t\}$, $U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$. Let $X = \{p, q\} \subseteq U$. The $\tau_R(X) = \{U, \phi, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$. Then $\mu = \{q\}$. Then Micro-O = $\mu_R(X) = \{U, \phi, \{p\}, \{q\}, \{p, q\}, \{p, q, r, s\}, \{q, r, s\}\}$. Let $V = \{1, 2, 3, 4, 5\}$, $V/R = \{\{1, 2, 3\}, \{4\}, \{5\}\}$. Let $X = \{1, 2\} \subseteq U$. Then $\tau'_R(X) = \{V, \phi, \{1, 2, 3\}\}$. Then $\mu = \{4\}$. Then Micro-O = $\mu'_R(X) = \{V, \phi, \{4\}, \{1, 2, 3, 4\}, \{1, 2, 3\}\}$. $f : U \rightarrow V$ be a function defined as $f(p)=4, f(q)=2, f(r)=3, f(s)=1$. Micro-open sets in U are $\{p\}, \{q\}, \{p, q\}, \{p, q, r, s\}, \{q, r, s\}$ and Micro-open sets in V are $\{4\}, \{1, 2, 3, 4\}, \{1, 2, 3\}$. Therefore for every Micro-open set H in V, $f^{-1}(H)$ is Micro-open set in U. Then f is Micro-continuous function.

Definition 6.3. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be the two Micro-topological space. A function $f : U \rightarrow V$ is called Micro-continuous at a point $a \in U$ if for every Micro-open set H containing $f(a)$ in V, there exist a Micro-open set G containing a in U, such that $f(G) \subset H$.

Theorem 6.4. $f: U \rightarrow V$ is Micro-continuous if and only if f is Micro-continuous at each point of U.

Proof. Let $f: U \rightarrow V$ be Micro-continuous. Let $a \in U$, and H be a Micro-open set in V containing $f(a)$. Since f is Micro-continuous, $f^{-1}(H)$ is Micro-open in U containing a. Let $G = f^{-1}(H)$, then $f(G) \subset H$, and $f(a) \in G$. Hence f is continuous at a. Conversely, suppose f is Micro-continuous at each point of U. Let H be Micro-open set in V. If $f^{-1}(H) = \phi$ then it is Micro-open. So let $f^{-1}(H) \neq \phi$. Take any $a \in f^{-1}(H)$, then $f(a) \in H$. Since f is Micro-continuous at each point there exist a Micro-open set G_a containing a such that $f(G_a) \subset H$. Let $G = \bigcup \{G_a | a \in f^{-1}(H)\}$. Claim: $G = f^{-1}(H)$. If $x \in f^{-1}(H)$ then $x \in G_x \subset G$. Hence $f^{-1}(H) \subset G$. On the other hand, suppose $y \in G$ then $y \in G_x$ for some x and $y \in f^{-1}(H)$. Hence $G \subset f^{-1}(H)$. Since G_x is Micro-open, by definition 6.3 G is Micro-open and hence $G = f^{-1}(H)$ is Micro-open for every Micro-open set H in V. Hence f is Micro-continuous. \square

Theorem 6.5. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be two Micro-topological spaces. Then $f: U \rightarrow V$ is Micro-continuous function if and only if $f^{-1}(H)$ is Micro-closed in U, whenever H is Micro-closed in V.

Proof. Let $f: U \rightarrow V$ is Micro-continuous function and H be Micro-closed in V. Then H^C is Micro-open in V. By hypothesis $f^{-1}(H^C)$ is Micro-open in U, i.e., $[f^{-1}(H)]^C$ is Micro-open in U. Hence $f^{-1}(H)$ is Micro-closed in U whenever H is Micro-closed in V. Conversely, suppose $f^{-1}(H)$ is Micro-closed in U whenever H is Micro-closed in V. Let U is Micro-open in V then G^C is Micro-closed in V. By assumption $f^{-1}(G^C)$ is Micro-closed in U. i.e., $[f^{-1}(G)]^C$ is Micro-closed in X. Then $f^{-1}(G)$ is Micro-open in U. Hence f is Micro-continuous. \square

Theorem 6.6. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be two Micro-topological space. Then $f: U \rightarrow V$ is Micro-continuous function if and only if $f(\text{Micro-cl}A) \subset \text{Micro-cl}[f(A)]$.

Proof. Suppose $f: U \rightarrow V$ is Micro-continuous and $\text{Micro-cl}[f(A)]$ is Micro-closed in V . Then by $f^{-1}(\text{Micro-cl}[f(A)])$ is Micro-closed in U . Consequently, $\text{Micro-cl}[f^{-1}(\text{Micro-cl}[f(A)])] = f^{-1}(\text{Micro-cl}[f(A)])$. Since $f(A) \subset \text{Micro-cl}[f(A)]$, $A \subset f^{-1}(\text{Micro-cl}[f(A)])$ and $\text{Micro-cl}(A) \subset \text{Micro-cl}(f^{-1} \text{Micro-cl}[f(A)]) = f^{-1}(\text{Micro-cl}[f(A)])$. Hence $f(\text{Micro-cl}(A)) \subset \text{Micro-cl}[f(A)]$. Conversely, if $f(\text{Micro-cl}(A)) \subset \text{Micro-cl}[f(A)]$ for all $A \subset U$. Let F be Micro-closed set in V , so that $\text{Micro-cl}(F) = F$... (1) By hypothesis, $f(\text{Micro-cl}(f^{-1}(F))) \subset \text{Micro-cl}[f(f^{-1}(F))] \subset \text{Micro-cl}(F)$, then by (1), $\text{Micro-cl}(f^{-1}(F)) \subset F$. It follows that $\text{Micro-cl}(f^{-1}(F)) \subset f^{-1}(F)$. But always $f^{-1}(F) \subset \text{Micro-cl}(f^{-1}(F))$, so that $\text{Micro-cl}(f^{-1}(F)) = f^{-1}(F)$. Hence $f^{-1}(F)$ is Micro-closed in U and f is continuous by Theorem 6.4. \square

Theorem 6.7. Let $((U, \tau_R(X), \mu_R(X)))$, $((V, \tau'_R(X), \mu'_R(X)))$ and $((W, \tau''_R(X), \mu''_R(X)))$ be three Micro-topological spaces. If $f: U \rightarrow V$ and $g: V \rightarrow W$ are Micro-continuous mappings then $g \circ f: U \rightarrow W$ is also Micro-continuous.

Proof. Let G be a Micro-open set in W . Since by g is Micro-continuous, $g^{-1}(G)$ is Micro-open set in V . Now, $(g \circ f)^{-1}G = (f^{-1} \circ g^{-1})G = f^{-1}(g^{-1}(G))$. Take $g^{-1}(G) = H$ which is Micro-open in V , then $f^{-1}(H)$ is Micro-open in U , since by f is Micro-continuous. Hence $g \circ f: U \rightarrow W$ is Micro-continuous function. \square

7 Micro-Pre Continuous and Micro-Semi Continuous Functions

Definition 7.1. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be two Micro-topological spaces, then $f: U \rightarrow V$ is Micro-pre continuous if $f^{-1}(V)$ is Micro-pre closed in U whenever V is Micro-closed.

Theorem 7.2. Every Micro-continuous function is Micro-pre continuous

Proof. Let $f: U \rightarrow V$ be Micro-continuous. i.e., $f^{-1}(H)$ is Micro-closed in U , whenever H is Micro-closed in V . By Theorem 4.11, every Micro-closed set is Micro-pre closed, and hence $f^{-1}(V)$ is Micro-pre closed in U whenever H is Micro-closed in V . Hence $f: U \rightarrow V$ be Micro-pre continuous \square

Definition 7.3. Let $((U, \tau_R(X), \mu_R(X)))$ and $((V, \tau'_R(X), \mu'_R(X)))$ be two Micro-topological space, then $f: U \rightarrow V$ is Micro-semi continuous if $f^{-1}(H)$ is Micro-semi closed in U whenever H is Micro-closed in V .

Theorem 7.4. Every Micro-continuous function is Micro-semi continuous.

Proof. Let $f: U \rightarrow V$ be Micro-continuous. i.e., $f^{-1}(H)$ is Micro-closed in U , whenever H is Micro-closed in V . By Theorem 5.3 (2), every Micro-closed set is Micro-semi closed. This implies that $f^{-1}(H)$ is Micro-semi closed in U whenever H is closed in V . Hence $f: U \rightarrow V$ be Micro-semi continuous. \square

8 Conclusion

Every year many topologist introduced different type of topological spaces. In this paper i introduced Micro topological spaces and discussed properies and applications of Micro pre open sets, Micro semi open sets. This shall be extended in the future Research with some applications

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