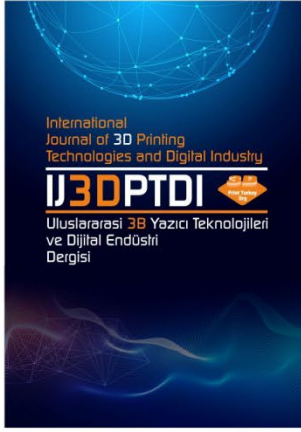




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ADAPTIVE BERNSTEIN POLYNOMIAL BASED TRACKING CONTROL FOR CARTESIAN POSITIONING MECHANISMS IN 3D PRINTING MOTION STAGES

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ABSTRACT

Accurate motion tracking is a key requirement in three-dimensional printing systems, where deviations of the planar positioning stage directly degrade the accuracy of the deposited path. This study proposes an adaptive Bernstein polynomial based nonlinear tracking controller for a class of two axis Cartesian positioning mechanisms whose axis dynamics are uncertain and contain unmodeled effects. The design exploits the finite duration of a printing motion by representing the lumped uncertainty that appears in the error dynamics as a normalized time signal along the trajectory, rather than requiring its explicit model. The Bernstein coefficients are updated online from the tracking error, yielding a compact approximation structure tailored to finite horizon printing trajectories. The controller is supported by a Lyapunov based stability analysis, which proves uniform ultimate boundedness of the closed loop error signals and relates the resulting bound to the feedback gain, the approximation residual, and the leakage term. Simulations on a CoreXY printing mechanism illustrate the tracking performance and the online uncertainty compensation of the proposed scheme under a demanding disturbance comprising ripple and external effects. A comparison with a nonadaptive baseline further demonstrates that the proposed adaptive structure reduces the cumulative tracking error while maintaining the same cumulative control effort. The proposed scheme uses only axis level position and velocity measurements together with a fixed number of adaptive coefficients, and it requires no model identification stage, so it is suitable for implementation in an embedded motion controller.

Keywords: Cartesian Positioning Mechanism, Bernstein Polynomials, Nonlinear Adaptive Control, Trajectory Tracking, Lyapunov Methods, 3D Printing.

1. INTRODUCTION

Three-dimensional (3D) printing builds a part by depositing material along prescribed planar and vertical motions, and the quality of the printed part is strongly affected by the accuracy with which these motions are executed [1-2]. In fused deposition modeling systems, the nozzle or the build platform is commonly driven by a planar Cartesian positioning stage. Deviations of this stage from the commanded trajectory may appear as dimensional and path following errors. Accurate trajectory tracking of the planar positioning stage is therefore a central requirement for improving print quality [3-6]. Recent studies further show that the dimensional precision and the mechanical performance of printed parts remain sensitive to part geometry and process settings, which keeps

accurate motion generation an active requirement in current printing practice [7-8].

The planar motion stage of a 3D printer is usually realized as a two axis Cartesian mechanism, including belt driven Cartesian and CoreXY configurations, whose dynamic behavior has been investigated in several studies [9-11]. Although each axis can often be approximated by a linear inertia damping model, practical mechanisms are subject to unmodeled dynamics that are difficult to model accurately and may vary with the operating conditions [12]. As a result, a fully model based controller may not provide the desired tracking performance unless these unmodeled dynamics are compensated during operation [13]. Closed loop strategies that monitor the deposition

process and correct faults during printing have also been reported, which indicates a broader trend toward feedback based accuracy improvement in 3D printing systems [14].

Several motion stage architectures used in 3D printers and planar positioning systems can be considered within the same class of two axis Cartesian positioning mechanisms. In serial Cartesian arrangements, planar motion is generated through orthogonal translational axes, which provides a direct kinematic relationship between the commanded Cartesian coordinates and the corresponding axis motions and facilitates the construction of an axis level nominal model [9]. However, depending on the placement of the actuators and transmission elements, this arrangement may increase the moving mass and thereby limit the achievable dynamic response. Parallel belt driven architectures, such as H-frame or H-Bot mechanisms, address this issue by locating the actuators on the fixed frame and transmitting motion to the end effector through coupled belt paths. These mechanisms have therefore been investigated as high speed XY positioning systems [15-16]. In 3D printing applications, the same architectural features have also motivated software compensation strategies for reducing layout induced positioning errors [12]. CoreXY, T-Bot, and related planar mechanisms can be viewed as alternative transmission arrangements that pursue the same objective of generating two axis Cartesian motion through compact actuation and transmission structures. Although these mechanisms differ in their kinematic mappings, actuator placement, and transmission topology, they share the need for accurate trajectory tracking under modeling uncertainty. This motivates a control formulation based on the lumped error dynamics, which avoids dependence on a detailed architecture specific model while retaining applicability to a broader class of Cartesian positioning mechanisms.

Adaptive and approximation based control methods provide a systematic way to compensate for uncertain dynamics without requiring their exact mathematical form [17-19]. Neural network, fuzzy, and learning based approximation schemes have been widely used for uncertainty compensation in adaptive control, including applications to positioning systems [20-21]. These approaches provide

flexible approximation capabilities, but their complexity depends on the selected approximation architecture. Bernstein polynomials, by contrast, provide a finite dimensional basis directly defined on a compact interval, making them suitable for representing uncertainty signals over the bounded duration of a printing motion. Bernstein polynomials are well suited to this purpose because they form a nonnegative basis on a compact interval and satisfy the partition of unity property [22]. These features provide a compact approximation structure for signals defined over a normalized finite interval. For a single printing motion, the lumped uncertainty appearing along the closed loop trajectory can thus be treated as a time signal over the normalized printing interval and approximated by a Bernstein expansion with online adapted coefficients [23-25]. Recent applications of Bernstein type operators in robust adaptive schemes, including observer based impedance control of cooperative electrically driven manipulators, confirm the continuing interest in this approximation family [26].

Motivated by these observations, this study proposes an adaptive Bernstein polynomial based nonlinear tracking controller for a class of two axis Cartesian positioning mechanisms used in 3D printing motion stages. The central idea is to exploit the finite duration of a printing motion and represent the lumped uncertainty in the error dynamics as a normalized time signal rather than requiring its explicit model. The Bernstein coefficients are adjusted online through an adaptation law that is derived together with the control input from a Lyapunov based stability analysis. The design guarantees uniform ultimate boundedness of the closed loop error signals, with a practical bound characterized by the feedback gain, the approximation residual, and the leakage term. The controller is evaluated in simulation on a CoreXY positioning mechanism following a representative printing contour under a demanding disturbance comprising ripple and external effects.

The scientific contribution and the novelty of this study can be stated as follows. In contrast to previous adaptive Bernstein polynomial based control studies addressing robot manipulators and broader classes of uncertain nonlinear systems, this study formulates the

Bernstein approximation structure for the trajectory tracking dynamics of a Cartesian positioning mechanism representative of a 3D printing motion stage. The accompanying Lyapunov analysis establishes uniform ultimate boundedness, with an ultimate bound that reflects the feedback gain, the approximation residual and the leakage term. In addition, the finite duration of a printing motion is exploited so that the lumped uncertainty is approximated as a function of normalized time rather than of the full state, which fixes the approximation domain to a known compact interval and keeps the number of adaptive coefficients constant and moderate. The practical contribution is that the controller uses only axis level position and velocity measurements and a constant kinematic map, involves no model identification stage, and consists of algebraic operations that are light enough for an embedded motion controller, so the scheme is applicable to belt driven stages of the CoreXY class. The fair comparison methodology employed in Section 5 further provides a reproducible procedure for assessing adaptive controllers on printing stages under realistic measurement noise. The Lyapunov argument would apply to any uniformly bounded basis, and the specific advantage of the Bernstein family is that its coefficients are the values of the approximated signal at the nodes of the normalized interval. The controller structure itself is generic to planar positioning mechanisms, and the printing context enters through the finite and known duration of a commanded motion and through the repetition of the same contour.

Notation. Throughout this paper, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the n -dimensional real vector space, and $\mathbb{R}^{m \times n}$ denotes the set of real matrices of dimension $m \times n$. For a vector x , $\|x\|$ denotes the Euclidean norm. For a matrix A , $\|A\|_F$ denotes the Frobenius norm, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of a symmetric positive definite matrix A , $\text{tr}\{\cdot\}$ denotes the trace operator, I_n denotes the $n \times n$ identity matrix, and $\text{diag}(\cdot)$ denotes the operator that forms a diagonal matrix from its arguments.

2. SYSTEM MODEL

Consider a planar two axis Cartesian positioning mechanism that moves in the

horizontal plane, and let $q(t) \in \mathbb{R}^2$ denote the vector of the two axis positions. The joint space dynamics are written as

$$M\ddot{q} + B\dot{q} + \Delta = \tau \quad (1)$$

where $M \in \mathbb{R}^{2 \times 2}$ and $B \in \mathbb{R}^{2 \times 2}$ are constant, symmetric, and positive definite inertia and damping matrices, respectively, $\tau(t) \in \mathbb{R}^2$ is the control input vector, and $\Delta(q, \dot{q}, t) \in \mathbb{R}^2$ denotes the unmodeled dynamics and the external disturbances acting on the mechanism. The term $\Delta(q, \dot{q}, t)$ is assumed to be continuous and bounded on the operating region, that is, there exists an unknown positive constant $\bar{\Delta}$ such that $\|\Delta(q, \dot{q}, t)\| \leq \bar{\Delta}$, which is a regional and not a global bound. In the absence of this term the model is linear, so this term is what makes the tracking problem nontrivial. The individual sources of this term are not modeled separately, and the design is based on its aggregate effect at the axis level.

3. ERROR SYSTEM DEVELOPMENT AND CONTROL DESIGN

The control objective is to design the input $\tau(t)$ so that the axis position vector $q(t)$ tracks a sufficiently smooth desired trajectory $q_d(t) \in \mathbb{R}^2$, without requiring exact knowledge of the inertia and damping matrices or an explicit model of the unmodeled term $\Delta(q, \dot{q}, t)$. The desired trajectory is assumed to have bounded first and second time derivatives, and the design is developed under full state feedback. The position tracking error $e(t) \in \mathbb{R}^2$, and the filtered tracking error $r(t) \in \mathbb{R}^2$ are defined as

$$e \triangleq q_d - q \quad (2)$$

$$r \triangleq \dot{e} + \alpha e \quad (3)$$

where $\alpha \in \mathbb{R}^{2 \times 2}$ is a positive definite gain matrix. Differentiating Equation (3), premultiplying by M , and substituting the dynamic model in (1) yields

$$M\dot{r} = M(\ddot{q}_d + \alpha\dot{e}) + B\dot{q} + \Delta - \tau \quad (4)$$

Collecting the terms that depend on the desired trajectory and on the unknown dynamics into a single vector $F(t) \in \mathbb{R}^2$,

$$F \triangleq M(\ddot{q}_d + \alpha\dot{e}) + B\dot{q} + \Delta \quad (5)$$

the error dynamics reduce to

$$M\dot{r} = F - \tau \quad (6)$$

The signal $F(t)$ is unknown, since it contains $\Delta(t)$ and the uncertain model terms. Along the closed loop trajectory of a single finite duration printing motion, $t \in [0, T_f]$, this lumped uncertainty is a continuous time signal, and rather than approximating it as a general function of the state, it is represented as a time signal on a normalized interval by a Bernstein polynomial expansion. For a sequence of motions the normalized argument is restarted at the beginning of each motion, so that the basis is always evaluated on the unit interval. Introducing the normalized time

$$\vartheta \triangleq \frac{t}{T_f} \in [0,1] \quad (7)$$

the unknown vector is approximated on $[0,1]$ by a Bernstein polynomial expansion. The Bernstein basis polynomials of degree N are

$$b_{k,N}(\vartheta) \triangleq \binom{N}{k} \vartheta^k (1-\vartheta)^{N-k}, \quad k = 0, 1, \dots, N \quad (8)$$

and the basis vector $\psi(\vartheta) \in \mathbb{R}^{N+1}$ is

$$\psi(\vartheta) \triangleq [b_{0,N}(\vartheta) \quad \dots \quad b_{N,N}(\vartheta)]^T \quad (9)$$

The basis functions are nonnegative and sum to unity, so $\|\psi(\vartheta)\| \leq 1$ for all $\vartheta \in [0,1]$. The continuous signal $F(t)$ is then represented over the normalized interval by the approximation

$$F = \Lambda^T \psi(\vartheta) + \epsilon \quad (10)$$

where $\Lambda \in \mathbb{R}^{(N+1) \times 2}$ is the constant coefficient matrix generated by the Bernstein operator, whose rows are the values of the lumped uncertainty at the nodes of the normalized interval, and $\epsilon(t) \in \mathbb{R}^2$ is the bounded approximation residual satisfying $\|\epsilon\| \leq \bar{\epsilon}$ for an unknown positive constant $\bar{\epsilon} \in \mathbb{R}$. Bounds on the coefficient matrix and on the residual are thus determined by the operating region and by the degree, and they do not depend on the online estimate. The Bernstein estimate $\hat{F}(t) \in \mathbb{R}^2$ of $F(t)$ is

$$\hat{F} \triangleq \hat{\Lambda}^T \psi(\vartheta) \quad (11)$$

with $\hat{\Lambda}(t) \in \mathbb{R}^{(N+1) \times 2}$ the estimated coefficient matrix. The control input is designed as

$$\tau = \hat{\Lambda}^T \psi(\vartheta) + K_r r \quad (12)$$

where $K_r \in \mathbb{R}^{2 \times 2}$ is a symmetric positive definite control gain matrix, and the coefficient matrix is updated according to the update law

$$\dot{\hat{\Lambda}} = \Gamma_{\hat{\Lambda}} (\psi(\vartheta) r^T - \sigma \hat{\Lambda}) \quad (13)$$

where $\Gamma_{\hat{\Lambda}} \in \mathbb{R}^{(N+1) \times (N+1)}$ is a symmetric positive definite adaptation gain and $\sigma \in \mathbb{R}$ is a small positive constant that prevents the estimates from drifting. Defining the coefficient estimation error $\tilde{\Lambda}(t) \in \mathbb{R}^{(N+1) \times 2}$

$$\tilde{\Lambda} \triangleq \Lambda - \hat{\Lambda} \quad (14)$$

and substituting the control law in (12) into Equation (6) while using Equations (10) and (14) yields the closed loop error dynamics

$$M\dot{r} = \tilde{\Lambda}^T \psi(\vartheta) + \epsilon - K_r r. \quad (15)$$

The error dynamics in (15) are driven by the coefficient estimation error and by the approximation residual, and they are used in the stability analysis presented next.

4. STABILITY ANALYSIS

Theorem 1. Consider the mechanism in (1) with the control law designed in (12) and the adaptation law designed in (13) over the printing interval, with symmetric positive definite gain matrices K_r and $\Gamma_{\hat{\Lambda}}$ and a small positive leakage constant $\sigma \in \mathbb{R}$. If the feedback gain satisfies

$$\lambda_{\min}(K_r) > \delta \quad (16)$$

for a positive analysis parameter $\delta \in \mathbb{R}$, then the tracking error $e(t)$, the filtered error $r(t)$, and the coefficient estimation error $\tilde{\Lambda}(t)$ are uniformly ultimately bounded.

Proof. Consider the nonnegative-valued function

$$V \triangleq \frac{1}{2} r^T M r + \frac{1}{2} \text{tr}\{\tilde{\Lambda}^T \Gamma_{\hat{\Lambda}}^{-1} \tilde{\Lambda}\}. \quad (17)$$

Differentiating (17) and substituting the closed loop dynamics in (15) and the adaptation law designed in (13) yields

$$\dot{V} = r^T \tilde{A}^T \psi(\vartheta) - \text{tr}\{\tilde{A}^T \psi(\vartheta) r^T\} + r^T \epsilon - r^T K_r r + \sigma \text{tr}\{\tilde{A}^T \hat{A}\} \quad (18)$$

Since $r^T \tilde{A}^T \psi(\vartheta) = \text{tr}\{\tilde{A}^T \psi(\vartheta) r^T\}$, canceling the common terms yields

$$\dot{V} = -r^T K_r r + r^T \epsilon + \sigma \text{tr}\{\tilde{A}^T \hat{A}\} \quad (19)$$

Utilizing Young's inequality, the last two sign indefinite terms on the right hand side of (19) are bounded as

$$r^T \epsilon \leq \delta \|r\|^2 + \frac{\bar{\epsilon}^2}{4\delta} \quad (20)$$

and, writing $\hat{A} = A - \tilde{A}$,

$$\sigma \text{tr}\{\tilde{A}^T \hat{A}\} \leq -\frac{\sigma}{2} \|\tilde{A}\|_F^2 + \frac{\sigma}{2} \|A\|_F^2 \quad (21)$$

Substituting these bounds into the time derivative of $V(t)$ yields

$$\dot{V} \leq -\gamma \|r\|^2 - \frac{\sigma}{2} \|\tilde{A}\|_F^2 + \nu \quad (22)$$

where $\gamma = \lambda_{\min}(K_r) - \delta \in \mathbb{R}$ and $\nu = \frac{\sigma}{2} \|A\|_F^2 + \frac{\bar{\epsilon}^2}{4\delta} \in \mathbb{R}$ are positive constants, γ being positive by the gain condition (16). In view of (17), inequality (22) can be rewritten as

$$\dot{V} \leq -\eta V + \nu \quad (23)$$

with $\eta = \min\{2\gamma/\lambda_{\max}(M), \sigma\lambda_{\min}(\Gamma_A)\}$. Solving the differential inequality in (23) yields

$$V(t) \leq V(0)\exp(-\eta t) + \frac{\nu}{\eta}(1 - \exp(-\eta t)) \quad (24)$$

so that $V(t)$ is bounded, and the boundedness of the remaining closed loop signals follows from standard signal chasing arguments [27]. The solution in (24) is valid on the printing interval and provides the explicit bound, which limits the filtered error and the coefficient estimation error through (17). The ultimate bound can be reduced by increasing the feedback gain and by decreasing the approximation residual through an appropriate Bernstein degree, subject to the usual trade-offs involving control effort, conditioning, and leakage. Hence $r(t)$ and $\tilde{A}(t)$

are uniformly ultimately bounded. Finally, the filtered error definition (3) is a stable linear relation driven by $r(t)$ with α positive definite, so the uniform ultimate boundedness of $r(t)$ carries over to the tracking error $e(t)$. This concludes the proof of Theorem 1.

5. SIMULATION RESULTS

The controller is assessed in simulation on a planar two axis belt driven positioning stage with CoreXY kinematics. The parameters given below describe a lumped two axis stage at the motor side and are not taken from a specific machine.

The simulated plant includes nonlinear, coupled, and time varying uncertainty components in order to assess the tracking and compensation capability of the adaptive Bernstein structure. These uncertainty components are used only to generate the plant response and are not supplied to the controller. The motor coordinates $q(t)$ and the Cartesian coordinates $p(t) \triangleq [x, y]^T$ are related by the constant kinematic map

$$p = f q, \quad f \triangleq \frac{r_p}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (25)$$

with pulley radius $r_p = 0.02$ m. The desired contour is specified in the Cartesian coordinates and then mapped to the motor coordinate reference $q_d(t)$ through the inverse of this constant transformation. The controller is implemented in the motor coordinate dynamics, while the Cartesian space trajectory is used only to visualize the resulting printing path. The reference trajectory is selected as an ellipse in the Cartesian space as

$$p_d(t) \triangleq \begin{bmatrix} R_x \cos(2\pi\vartheta) \\ R_y \sin(2\pi\vartheta) \end{bmatrix} \quad (26)$$

with semi-axes $R_x = 30$ mm and $R_y = 20$ mm, repeated over two periods of $T_f = 5$ s. The inertia and damping matrices are $M = \begin{bmatrix} 1.2 & 0.15 \\ 0.15 & 0.9 \end{bmatrix}$ and $B = \text{diag}(0.8, 0.7)$. The normalized time is restarted at the beginning of each of the two motions, while the adapted coefficients are retained.

A challenging composite disturbance is injected in the simulation. It combines several nonlinear effects and is given by

$$\Delta \triangleq \Delta_f + \Delta_r + \Delta_c + d \quad (27)$$

where $\Delta_f = \sigma_v \dot{q} + [f_c + (f_s - f_c) \exp(-(\dot{q}/v_s)^2)] \tanh(\dot{q}/\phi)$ is a smooth Stribeck inspired friction component used to represent velocity dependent friction effects in the simulated plant, $\Delta_r = a \sin(k_r q + \phi_r)$ is a position dependent ripple, $\Delta_c = \kappa \dot{q}_1 \dot{q}_2 [1, -1]^T$ is a cross axis term, and $d(t)$ is an exogenous component containing two sinusoids per axis together with a smooth step load applied at $t = T_f$, that is, at the start of the second printing period. It should be emphasized that the disturbance model (27) is introduced only for simulation purposes. The controller uses neither the structure nor the parameters of this disturbance and relies only on the measured position and velocity signals. The numerical values used are $f_c = [1.5, 1.2]^T$, $f_s = [2.0, 1.6]^T$, $v_s = 0.6$, $\sigma_v = [1.5, 1.2]^T$, $\phi = 2.0$, $a = [0.8, 0.6]^T$, $k_r = [1.5, 1.5]^T$, $\phi_r = [0, 1.0]^T$, $\kappa = 0.3$, the two exogenous components $0.6 \sin(2\pi t/T_f) + 0.4 \sin(4\pi t/T_f)$ and $0.5 \sin(2\pi t/T_f + 1) + 0.3 \sin(4\pi t/T_f)$, and a load step of magnitude $[2.0, 1.5]^T$ smoothed by $(1 + \tanh((t - T_f)/0.05))/2$.

The control gains are selected as $\alpha = 8I_2$ and $K_r = 10I_2$, the adaptation gain as $\Gamma_\lambda = 300I_{12}$, and the leakage constant as $\sigma = 8 \times 10^{-4}$. The Bernstein degree is selected to provide sufficient approximation flexibility while keeping the number of adaptive coefficients moderate. Increasing the degree may reduce the approximation residual but also increases the number of estimated parameters and may affect the numerical conditioning of the basis, and the value $N = 11$ is used here. The closed loop is integrated at a fixed rate of 1 kHz, and the position and velocity feedback are corrupted by zero mean Gaussian measurement noise with standard deviations $\sigma_q = 2 \times 10^{-4}$ rad and $\sigma_{\dot{q}} = 0.15$ rad/s, representative of the encoder

resolution and of finite difference velocity estimation at this sampling rate. The mechanism starts from rest at an interior point of the contour, with initial joint configuration $q(0) = [1.25, 1.45]^T$ rad and $\dot{q}(0) = [0, 0]^T$ rad/s.

To quantify the contribution of the online adaptation, the proposed controller is compared with a nonadaptive PD type baseline. This baseline is obtained by removing the Bernstein approximation term from the control law and retaining only the fixed gain filtered error feedback term, which acts as a proportional derivative action through the definition of $r(t)$. Both controllers share the same feedback structure and are driven by the identical disturbance and the same measurement noise realization. A fixed gain controller can reject disturbances only by raising its feedback gain, which under measurement noise also inflates the control effort, so the two controllers are compared at equal actuation, the baseline gain being tuned until its integrated control effort equals that of the proposed controller. Three integral measures are evaluated over the simulation horizon $T = 2T_f$, namely the integrated tracking error J_e , the integrated control effort J_τ , and the integrated Cartesian error J_c , whose definitions and values are reported in Table 1. With the same control effort, the proposed controller reduces the two integral error measures by about 58%. Improvements in tracking accuracy without hardware modification have likewise been reported for adaptive gain control on an H frame stage [16] and for learning based feedforward compensation in 3D printing applications [12, 13]. The contour normal error is a component of the Cartesian error vector and is therefore bounded by its norm, while its explicit decomposition into contour normal and contour tangential parts is outside the scope of this study.

Table 1. Performance metrics over the simulation horizon for the baseline and the proposed controller at equal control effort.

Performance metric	Baseline	Proposed
$J_e \triangleq \int_0^T \ e\ dt$ [rad·s]	0.455	0.190
$J_\tau \triangleq \int_0^T \ \tau\ dt$ [Nm·s]	71.5	71.5
$J_c \triangleq \int_0^T \ p_d - p\ dt$ [mm·s]	6.44	2.69

The Cartesian space tracking over one printing period is shown in Figure 1, where both controllers follow the desired contour, and the joint coordinates are compared in Figure 2. Both the proposed and the baseline controllers keep the motion close to the desired contour despite the nonlinear and time varying disturbances, with the proposed controller achieving closer tracking.

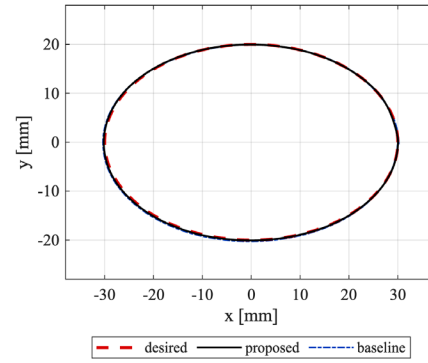


Figure 1. Cartesian space tracking over one printing period for the proposed and baseline controllers.

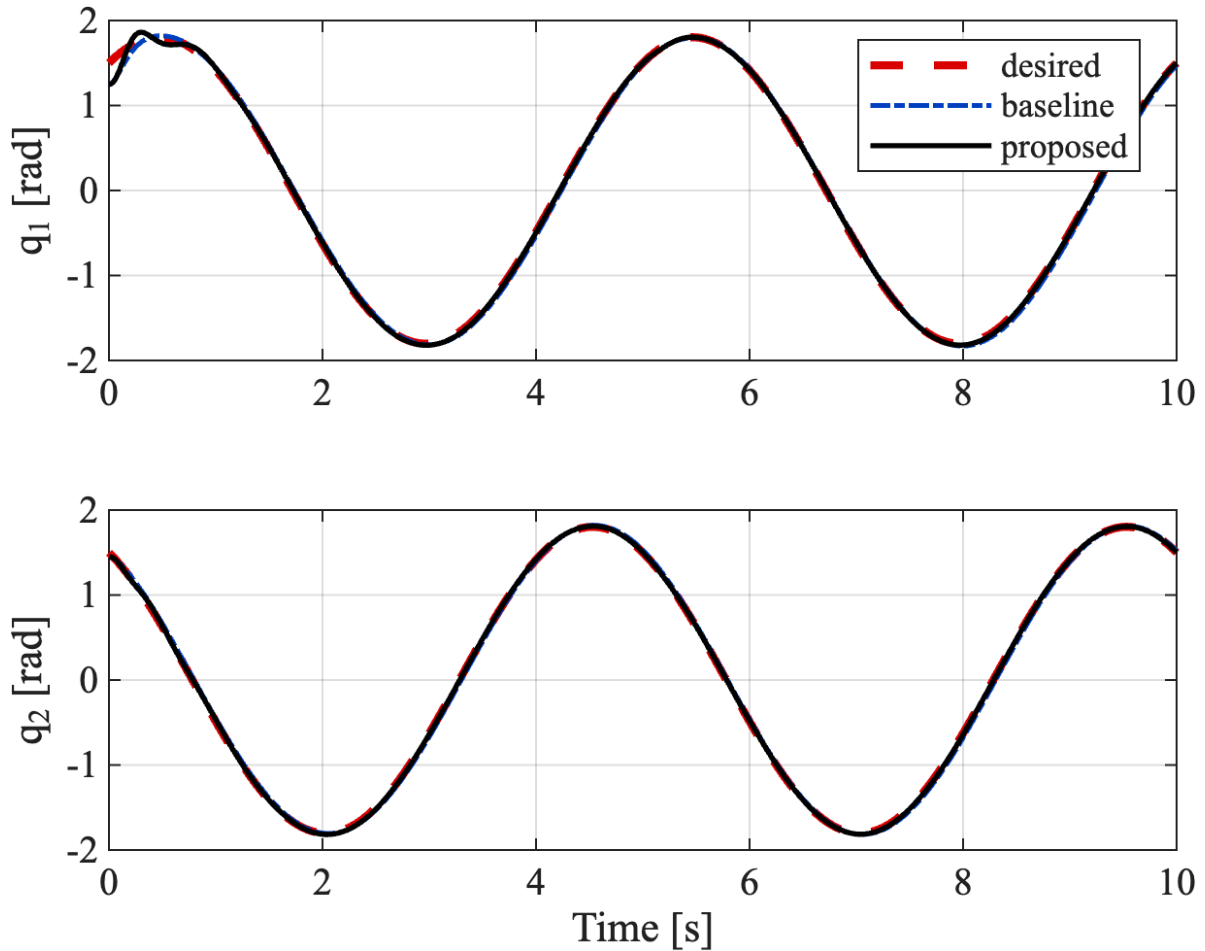


Figure 2. Joint positions for the proposed and baseline controllers and desired trajectories.

The joint tracking errors are given in Figure 3 and the control inputs in Figure 4. At equal control effort, the proposed controller produces markedly smaller tracking errors than the baseline, whose higher feedback gain cannot reject the time varying disturbance as effectively as the adaptive feedforward. The

proposed control inputs remain bounded and smoother than those of the baseline, which exhibits stronger noise induced fluctuations at its higher gain. The largest errors occur during the initial transient and around the smooth load step, after which the proposed controller settles to a small bounded error.

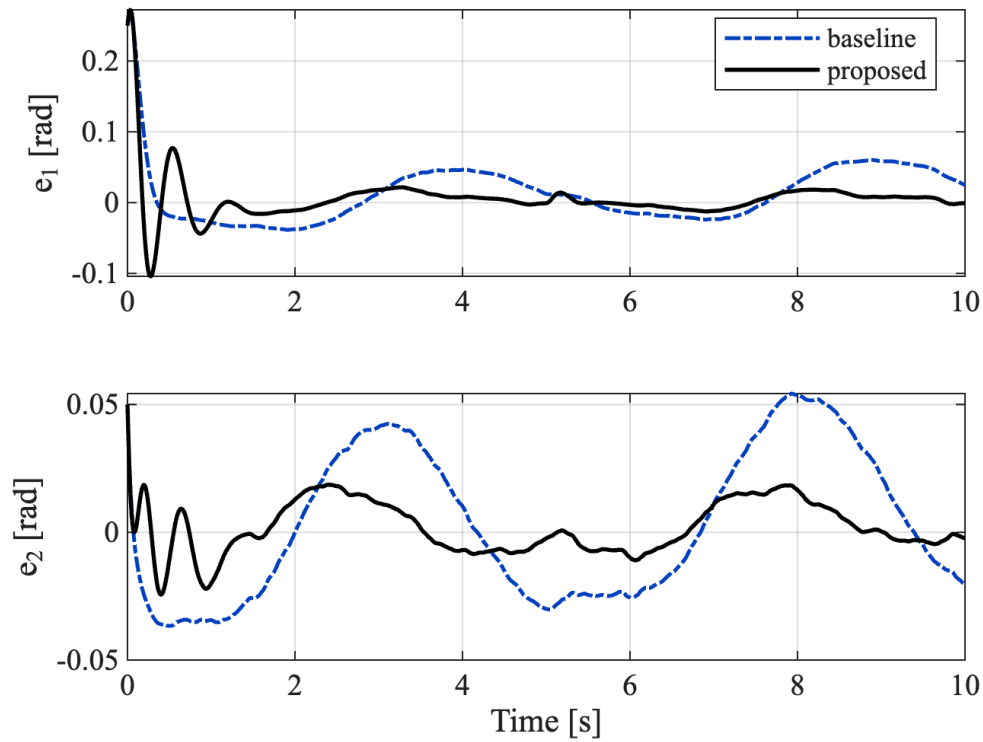


Figure 3. Joint tracking errors for the proposed and baseline controllers.

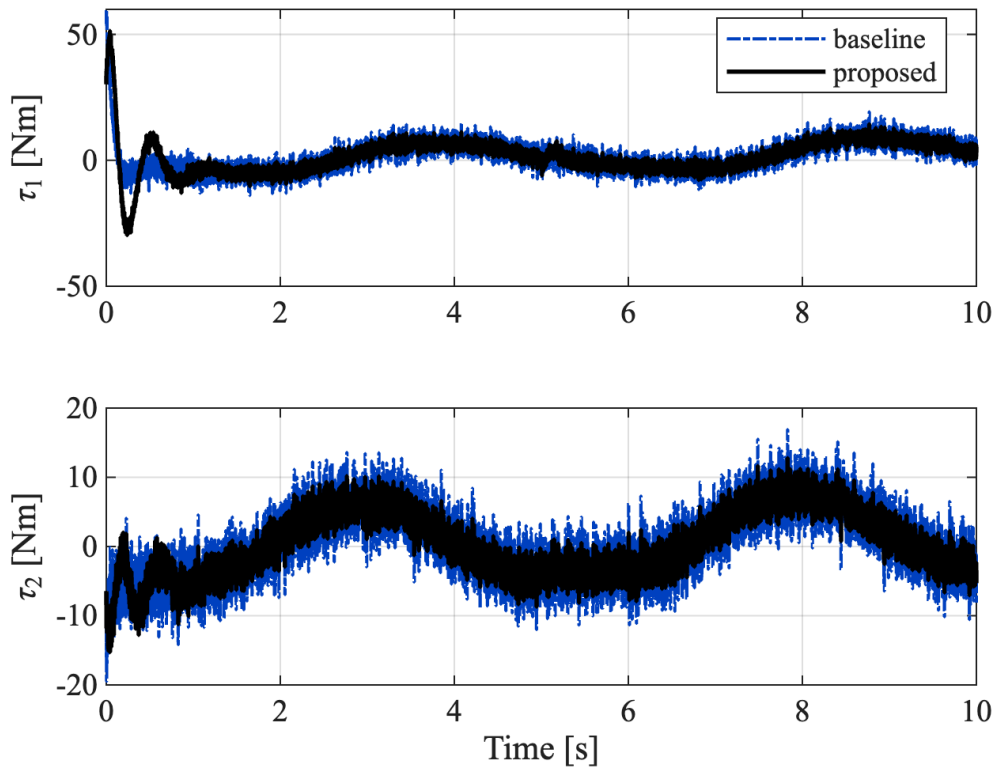


Figure 4. Control inputs for the proposed and baseline controllers.

Figure 5 compares the lumped uncertain term with its Bernstein estimate for the proposed controller, and the estimated coefficients are shown in Figure 6. The Bernstein estimate captures the dominant behavior of the lumped uncertainty and follows its main variations, including the response after the smooth load

step. This indicates that the online coefficient adaptation follows the lumped uncertainty over the finite printing motion. The Bernstein estimate follows the repeated uncertainty pattern, and the estimated coefficients remain bounded throughout the motion.

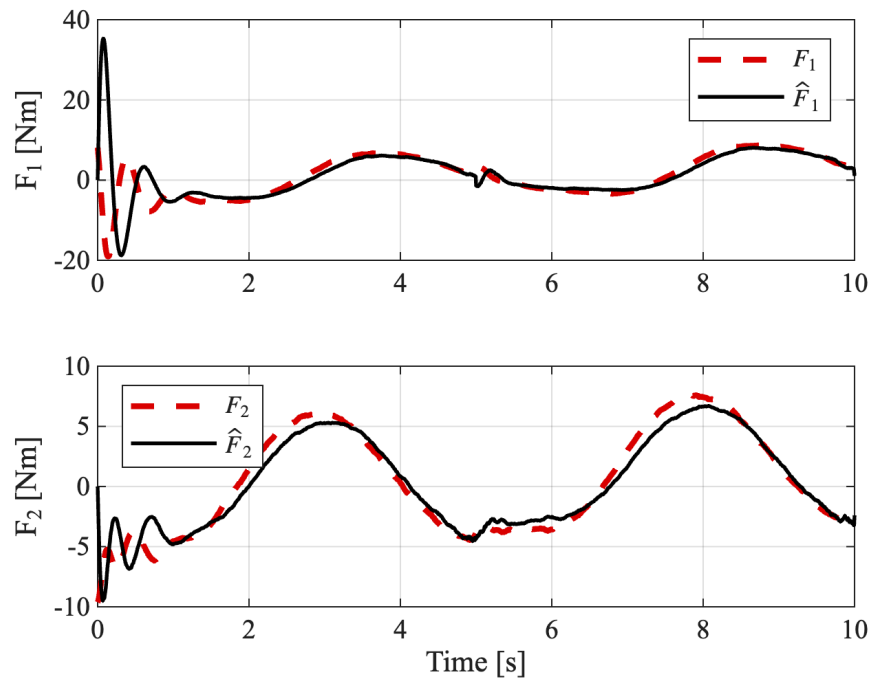


Figure 5. Uncertain term and its Bernstein polynomial based estimate for the proposed controller.

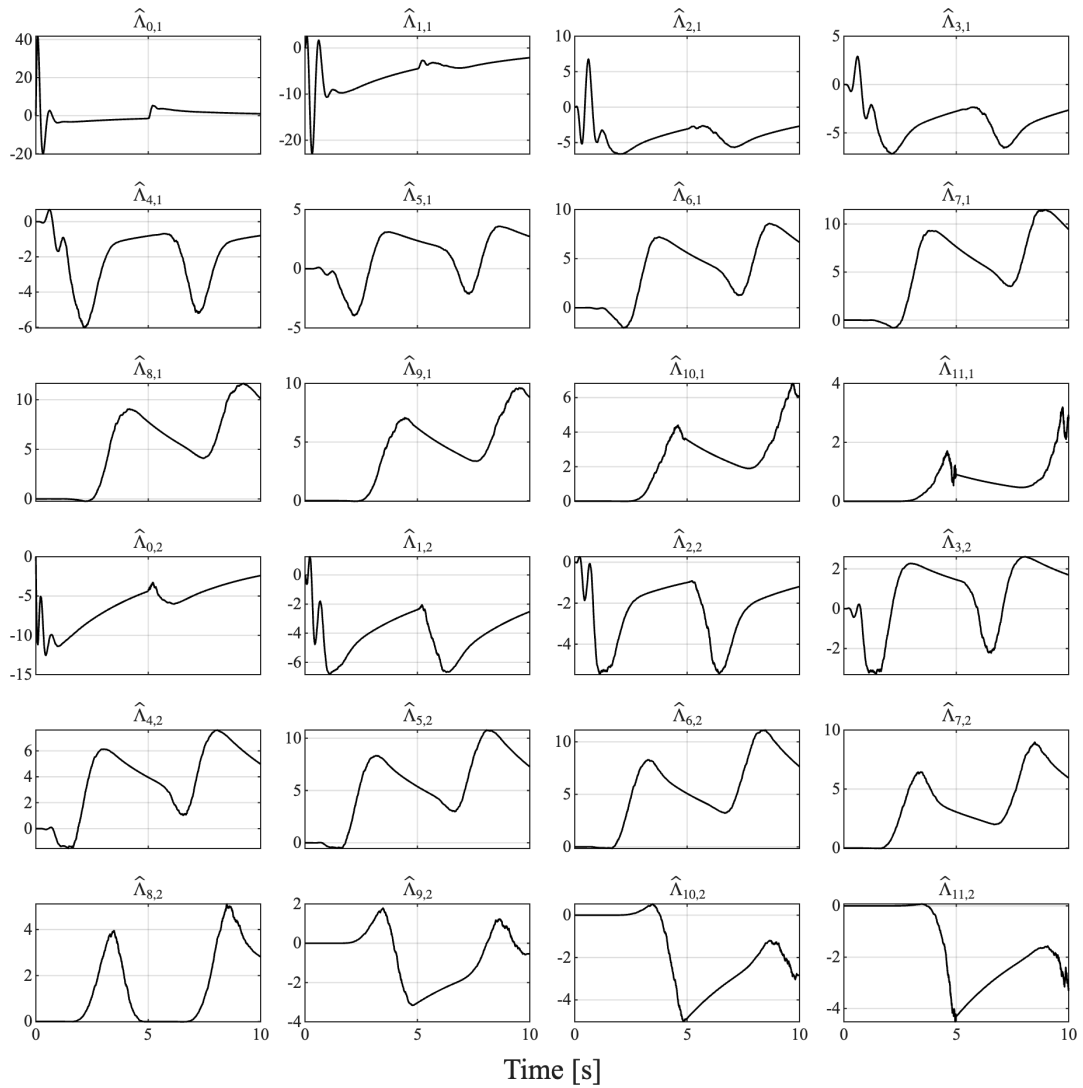


Figure 6. Estimated Bernstein coefficients for the proposed controller.

6. DISCUSSION

The findings of Section 5 can be positioned within the existing literature on planar positioning stages. Model based compensation methods for belt driven stages, such as filtered B spline feedforward for racking motion [12] and physics guided data driven feedforward [13], achieve accurate tracking by identifying or learning a model of the stage before operation. The proposed scheme differs in that the compensation signal is constructed online during the motion from the tracking error alone, so no training data and no identification experiment are required. Compared with the adaptive gain scheme reported for an H frame stage [16], which adjusts the feedback gains, the Bernstein structure adds an explicit feedforward estimate of the lumped uncertainty, and this estimate is the only structural difference between the two controllers compared in Section 5. This behavior agrees with the general observation in approximation based adaptive control that a suitable feedforward term relieves the feedback gain from the burden of disturbance rejection [17-19].

The fair comparison is carried out under measurement noise, which reflects practical printing stages where encoder quantization and finite difference velocity estimation are unavoidable. Since a high feedback gain amplifies the noise content of the control signal and inflates the control effort without improving the accuracy, the equal effort condition limits how far the baseline gain can be raised, while the adaptive Bernstein feedforward remains smooth and improves the tracking at the same actuation level.

A further observation concerns the selection of the Bernstein degree. Increasing the degree reduces the attainable approximation residual on the normalized interval, yet it also increases the number of adapted coefficients and degrades the numerical conditioning of the basis. This trade off is consistent with the properties of the Bernstein basis documented in the literature [22], and it suggests that the degree should be treated as a tuning parameter tied to the duration and complexity of the printing motion rather than as a quantity to be maximized.

A printing process consists of many toolpath segments of different durations, and the formulation applies to one of them at a time,

with the horizon taken from the duration of the commanded motion and the normalized argument restarted at each segment boundary. The adapted coefficients may be retained across segments or reset at each of them, which is a choice left to the implementation.

From an application perspective, the proposed controller addresses a need that is visible across the recent 3D printing literature, where the dimensional precision and the quality of printed parts are shown to depend strongly on the fidelity of the executed motion [3, 7-8]. Since the scheme relies only on axis level measurements, a constant kinematic map and a fixed number of adapted coefficients, it is suitable for integration into printer based mechatronic systems with embedded control [28]. The simulation based methodology adopted in this study is also in line with the broader trend of assessing control strategies on a virtual model of the plant before deployment, as exemplified by digital twin oriented studies [29], and the disturbance model of Section 5 was designed to make this assessment demanding rather than favorable.

7. CONCLUSION

This study proposed an adaptive Bernstein polynomial based nonlinear tracking controller for a class of two axis Cartesian positioning mechanisms used in 3D printing motion stages. The main idea was to exploit the finite duration of a printing motion and approximate the lumped uncertainty in the error dynamics as a normalized time signal using Bernstein polynomials with online adapted coefficients. The controller and adaptation law were derived through a Lyapunov based stability analysis, which established uniform ultimate boundedness of the closed loop error signals.

Simulation results on a CoreXY positioning mechanism illustrated the tracking performance of the proposed scheme under demanding, time varying uncertainty. In a comparison carried out under measurement noise and with equal control effort, the adaptive scheme reduced the cumulative tracking errors by about 58% relative to a nonadaptive baseline. The Bernstein estimate captured the dominant behavior of the lumped uncertainty during the finite printing motion. Future work will focus on experimental validation on a fused deposition modeling platform, extensions to

constrained Cartesian space tracking, and combining robust integral of the sign of the error feedback with Bernstein polynomial adaptation to pursue stronger asymptotic tracking guarantees.

The main contributions of this study are the finite horizon Bernstein compensation structure for Cartesian printing stages, the Lyapunov based ultimate boundedness proof with an explicit gain condition, and the noise aware equal effort evaluation methodology. The study also has limitations that delimit the scope of the conclusions. The results are obtained in simulation on a planar two axis model with constant inertia and damping matrices, the model parameters are representative rather than identified from a specific machine, and effects that add dynamic states or violate the actuation limits are not modeled. The controller is moreover formulated for a single finite duration motion, so repeated segments are handled by restarting the normalized time argument. Future studies will address these limitations by incorporating flexible transmission dynamics, by extending the formulation to consecutive printing segments, and by validating the controller experimentally on a CoreXY stage, in addition to the directions stated above.

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