Communication in Mathematical Modeling and Applications
http://ntmsci.com/cmma

Boundary layer flow of a nanofluid in an inclined wavy wall with convective boundary condition

A. Yusuf[1], *G. Bolarin*[1], *M. Jiya*[1], *Y. M. Aiyesimi*[1] *and G. T. Okedayo*[2]

[1] Mathematics Department, Federal University of Technology, PMB 65, Minna, Nigeria, Niger State, Nigeria
[2] Mathematics Department, Ondo State University of Science and Technology, Okiti-Pupa, Ondo State, Nigeria

Received: 28 June 2018, Accepted: 25 October 2018
Published online: 13 December 2018.

Abstract: This problem focuses on the laminar flow of a nanofluid in an inclined permeable parallel walls. We assume that the lower wall is wavy while the upper wall is flat with Dufour effects, Soret effects, and a magnetic field effect with boundary conditions been convective. The rectangular coordinate system has been used to present the model for this problem. It also incorporates the effect of thermophoresis parameter and Brownian motion. The obtained similarity solution is dependent on the thermophoresis number (N_t), Darcy number (Da), Magnetic parameter (M), Dufour (DU) number, Soret (Sr) number, Brownian motion (N_b), Lewis number (Le), Prandtl number (P_r). It is found that at the wavy wall, the fluid flow back.

Keywords: Thermophoresis, nanofluid, nano particles, boundary layer, thermo-difussion, difussion thermo, channel flow, wavy wall and Adomian decomposition method.

1 Introduction

The study of energy or heat transfer from irregular surfaces, in particular, wavy configurations has become extensive, for their widespread applications during past years. Preventing thermal boundary layers formation and promoting capability of the fluid motion near the surface for corrugated and roughened geometries increase the heat transfer rates, thereby resulting in enhancement of heat transfer performance. Among the various applications of such heat transfer surfaces are wall undulation, plate heat exchangers, micro-electronic devices, design of solar collectors, film vaporization in combustion chambers and cross-hatching on ablative surfaces, transpiration cooling of return automobiles and rocket boosters.

In recent times, the joint free and force convection considering both mass and heat transfer within a perpendicular corrugated porous duct with moving heat waveswas analyzed and investigated by Muthuraj and Srinivas (2010). Using the perturbation method, the influence of the various pertinent parameters, namely, the Schmidt number, Hartmann number, and the porosity constraint, over the flow fields with mass and heat transfer characteristics were explained. Umavathi and Shekar (2011) investigated the combined convection and heat transfer fluid flow via a long perpendicular corrugated channel filled with permeable material, employing linearization method. They assumed long wave approximation for perturbation solution. Gireesha and Mahanthesh (2013) conducted an analytical approach for combined mass and heat transfer of an unsteady magnetohydrodynamics visco-elastic flow via an irregular vertical passage with coupled boundary condition using perturbation technique. They analyzed the influence of different salient constraint such as Sherwood and Biot number on velocity and temperature fields. Kumar and Umavathi (2013) conducted the perturbation technique to a problem of time independent two-dimensional convective (natural) flow in a permeable medium amid a long perpendicular undulated wall and parallel flat wall with heat source utilizing a Walters

© 2018 BISKA Bilisim Technology

* Corresponding author e-mail: yusuf.abdulhakeem@futminna.edu.ng

fluid (model B/). They discussed the significance of the flow and energy transmission features, namely, the rate of heat transfer at both walls and the skin friction in detail. Umavathi and Shekar (2014) conducted the same technique on convection (mixed) of the laminar flows in a wavy-perpendicular passage filled with two immix able viscous fluids. They found that the viscosity parameter, grashof number, conductivity ratio and geometry ratio increase the momentum component parallel to the flow direction.

Influence of magnetic field on copper oxide nanofluid with heat transmission in a closed system was considered by Shiekholeslami et al. (2014). It was observed the influence of energy source length and Hartmann number is quite evident with increase in Rayleigh number.

Magnetohydrodynamic boundary layer flow of a Nanofluid over an extending sheet rooted in a darcian Permeable media with radiation was studied by Aiyesimi et al. (2015a). They observed that when prandtl number reduces, the heat diffuses faster from the system.

Lately Aiyesimi et al. (2015b) considered the convective boundary-layer flow of a nanofluid over an extending sheet with radiation by extending the model of Khan and Pop (2010). They observed that the nano-fraction buoyancy and thermal buoyancy increases the fluids momentum, energy, and nano-fraction. It is proper to extend the work of Aiyesimi et al (2015a) over an inclined permeable wavy channel with magnetic field, Duffour and Soret properties and use the Adomian Decompositon Method (stocktickerADM) to obtain the analytical solution of the model.

This study is a new advancement in the literature in which an analytical solution of a nanofluid in an inclined permeable wavy channel with Soret and Duffour effects is proposed using the Adomian Decomposition Method.

2 Problem formulation

Consider a time independent, 2-D boundary-layer flow of a nanofluid in a channel inclined at angle Θ. located at $y = a\cos(Lx)$ is the wavy wall while at$y = h$ the other flat wall. Where a is the amplitude of the wavy wall and Lx is a location on the wavy wall. The temperature Thas no constant value at the wavy wall while nanoparticle fraction C have constants value C_0 at $y = aCos(Lx)$ and T_h and C_h at $y = h$ respectively. For the present, we will implement the formulation of Aiyesimi (2015a) in a permeable wavy wall with permeability, magnetic field, Soret, Dufourt, Heat generation effects with convective boundary conditions. The governing equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation

$$\begin{aligned} u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} = &-\frac{1}{\rho_f}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\upsilon\varphi}{k}u-c\varphi u^2-\frac{\sigma B_0^2}{\rho}u \\ &+g\beta(T-T_h)\cos\Theta+g\beta(C-C_h)\cos\Theta. \end{aligned} \tag{2}$$

Energy equation

$$\begin{aligned} u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} =&\alpha(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2})+\frac{Q}{\rho c_p}(T-T_h) \\ &+\tau\left(D_B(\frac{\partial C}{\partial x}\frac{\partial T}{\partial x}+\frac{\partial C}{\partial y}\frac{\partial T}{\partial y})+\frac{D_T}{T_h}\left(\left(\frac{\partial T}{\partial x}\right)^2+\left(\frac{\partial T}{\partial y}\right)^2\right)\right)+\frac{D_M K_T}{C_S C_p}\frac{\partial^2 C}{\partial y^2} \end{aligned} \tag{3}$$

© 2018 BISKA Bilisim Technology

Nanofraction equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_M K_T}{T_M}\right)\left(\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (4)$$

Subject to the boundary conditions:

$$y=aCos(Lx): u=0,\ \ v=0,\ \ -k^*\frac{\partial T}{\partial y}=h^*(T_f-T_h),\ \ C=C_0,\ \ y=h: u\to 0,\ \ T\to T_h,\ \ C\to C_0. \quad (5)$$

The velocity along the x and y axes are respectively u and v, ρ_f is the density of the base fluid, $p$is the fluid pressure, υ is the kinematic viscousity, k^* is the heat conductivity, k is the permeability, σ is the electrical conductivity, $\alpha$is the heat diffusivity, K_T is the heat-distribution ratio, T_f is the convective fluid temperature, h^* is the convective heat transfer coefficient, φ is the porousity, B_0external magnetic field, $Q$is the heat generation, C_p is the specific heat capacity at constant pressure, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient and $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the fluid with ρbeing the density, c is Forchheimer's inertia coefficient and ρ_P is the density of the particles, g is the acceleration due to gravity, $T_M$is the mean fluid temperature, C_S is concentration susceptibility.

The dimensional stream function is define as $(\psi(x,y))$ such that $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$. From Sheikholeslami et al., (2014).

$$\eta=\frac{y}{h},\ \ \psi=axf(\eta),\ \ \theta(\eta)=\frac{T-T_h}{T_0-T_h},\ \text{and}\ \ \chi(\eta)=\frac{C-C_h}{C_0-C_h}(\textbf{??}) \quad (6)$$

where η, $f(\eta)$, $\theta(\eta)$, $\chi(\eta)$ are the non-dimensional distance, velocity, temperature, and nanoparticle concentration.

Neglecting the pressure gradient equations (1) to (5) reduces to the following local similarity solution

$$f'''+Re\left(ff''-(1+\phi)f^2-\left(Da^{-1}+M\right)f'\right)+Gr_{Tx}\theta\cos\Theta+Gr_{Cx}\chi\cos\Theta=0 \quad (7)$$

$$\theta''+Re\Pr f\theta'+\Pr\phi_0\theta+\Pr N_b\chi'\theta'+\Pr N_t\theta^2+DU\Pr\chi''=0 \quad (8)$$

$$\chi''+ReLef\chi'+LeS_r\theta''=0 \quad (9)$$

with corresponding boundary conditions

$$f(0)=0,\ \ f'(0)=0,\ \ \theta'(0)=-B_i(1-\theta(0)),\ \ \chi(0)=1,\ \ f'(1)=0,\ \ \theta(1)=0, \chi(1)=0. \quad (10)$$

in which

$$Gr_{Tx}=\frac{h^3 g\beta(x)(T_0-T_\upsilon)}{a\upsilon},\ \ Gr_{Cx}=\frac{h^3 g\beta(x)(C_0-C_h)}{a^2\upsilon},\ \ \phi=cx\varphi,\ \ Da^{-1}=\frac{h\upsilon\varphi}{ak},\ \ Re=\frac{ha}{\upsilon},\ \ M=\frac{\sigma B_0^2}{a\rho},\ \ P_r=\frac{\upsilon}{\alpha},$$

$$Le=\frac{\upsilon}{D_B},\ \ \phi_0=\frac{h^2 Q}{\alpha\rho C_p},\ \ B_i=\frac{h^*}{k^*}\left(\frac{\upsilon}{a}\right)^{1/2},\ \ N_b=\frac{(\rho c)_p D_B(C_0-C_h)}{(\rho c)_f\upsilon},\ \ N_t=\frac{(\rho c)_p D_T(T_0-T_h)}{(\rho c)_f T_h\upsilon},$$

$$DU=\frac{D_M K_T(C_0-C_h)}{C_S C_p(T_0-T_h)},\ \ S_r=\frac{D_M K_T(T_0-T_h)}{T_M\upsilon(C_0-C_h)}$$

are the Grashof number, Inverse Darcy number, modified Grashof number, inertia coefficient, Gravitational parameter, Renold number, Magnetic Parameter, Lewis number, Prandtl number, heat generation or absorption parameter, Biot number, Brownian motion parameter, thermophoresis parameter, Dufour number, and Soret number respectively.

© 2018 BISKA Bilisim Technology

Table 1: Comparison of Result for $\theta(\eta)$ with the present work for Re=1 and$\text{Pr} = 0.1$.

η	RK-Method	Present Work
0.0	0.9999	1.0000
0.1	0.8988	0.8976
0.2	0.7977	0.7953
0.3	0.6967	0.6931
0.4	0.5960	0.5913
0.5	0.4956	0.4898
0.6	0.3956	0.3889
0.7	0.2960	0.2891
0.8	0.1969	0.1907
0.9	0.0982	0.0941
1	0.0000	0.0000

3 Results and discussion

In order to establish the uniqueness of the present technique, the nonlinear coupled ordinary differential equations (7) to (9) alongside the boundary conditions (10) has been solved by the proposed Modified Adomian Decomposition Methods Ebaid and Al-armani (2013) and results have been compared for different values of η with the Numerical method as shown in Table 1. All other parameters are assumed to be zero except for Re=1 and$\text{Pr} = 0.1$and temperature is assumed to be constant on the lower wall. From the graphs it isobserved that the present method and the Numerical method are in an excellent agreement.

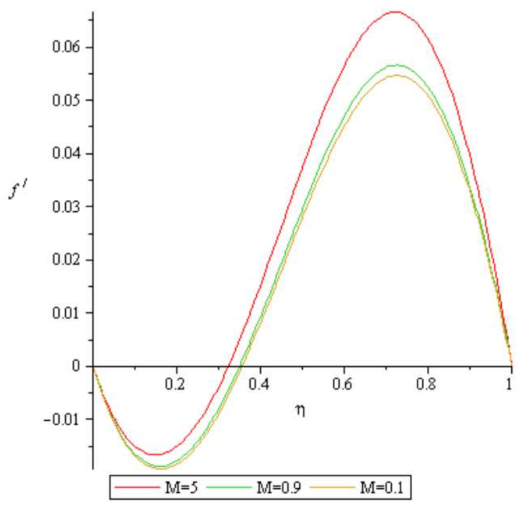

Fig. 1: Effect of magnetic parameter on velocity profile.

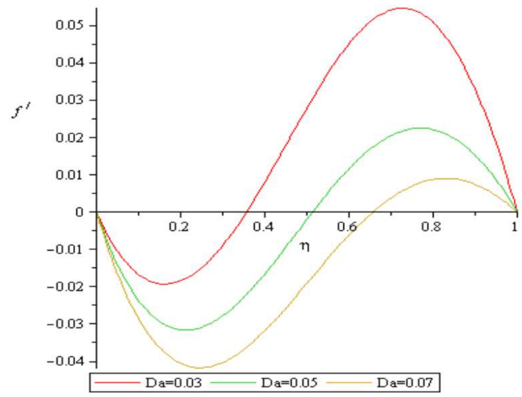

Fig. 2: Effect of Dacian number on velocity profile.

Figure 1 show the effect of magnetic field parameter (M) on the velocity of the fluid. We observed that as M reduces, the velocity profile increases along the wavy wall but rises at the flat wall.

Figures 2 to 3 present the effect of Darcy number(Da) on the velocity profile and temperature profile. We noticed that as the Darcy number increases, the velocity profile boundary thickness reduces, and thermal boundary thickness increases across the boundary layer.

© 2018 BISKA Bilisim Technology

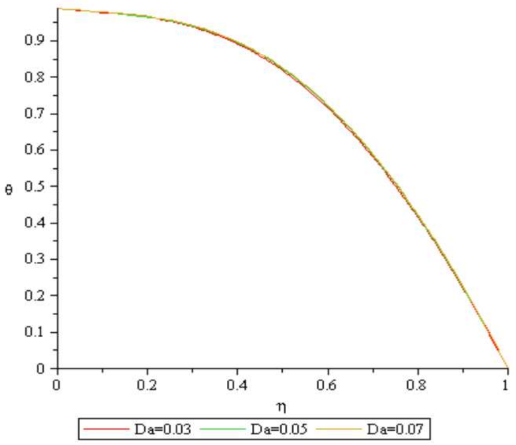

Fig. 3: Effect of Dacian number on temperature profile.

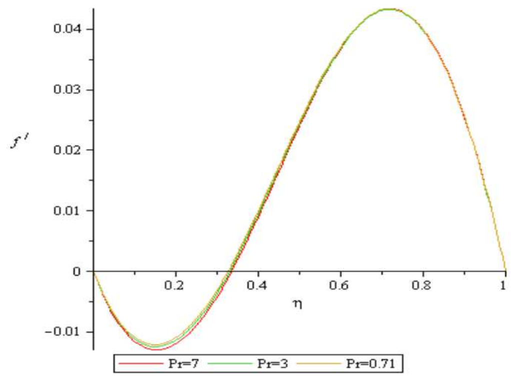

Fig. 4: Effect of Prandtl number on velocity profile.

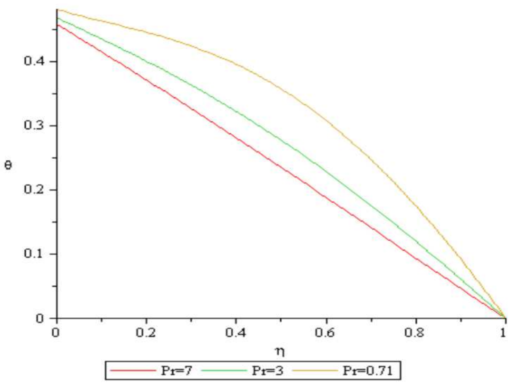

Fig. 5: Effect of Prandtl number on temperature profile.

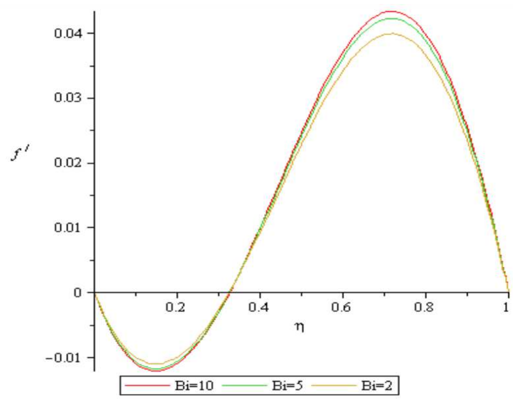

Fig. 6: Effect of Biot number on velocity profile.

Figures 4 to 5 display the influence of Prandtl number (Pr) on the velocity and energy distribution. As the Prandtl reduces the velocity profile also reduces at the region closer to the wavy wall but has no effect at the flat wall region. The temperature distribution increases with reduction in prandtl number.

Figures 6 to 7 displays the effect of Biot number (Bi) on velocity and temperature profiles. We noticed that the fluid velocity at both walls rises with rise in the Biot number while the thermal boundary thickness reduces. It is worthy of note that at $\eta = 0.22$the velocity profile is the same for all values of Biot number.

Figures 8 to 9 depicts the influence of Dufour number (DU) on both the velocity and temperature profiles respectively. Rise in Dufour number raises the velocity profile while the temperature drops as it increases.

Figures 10 to 11 display the effect of soret number (Sr) on velocity and nanofraction profile. As the Soret number increases, the velocity profile increases while the nanofraction profile reduces.

Figures 12 to 14 display the influence of Lewis number (Le) on velocity, temperature and the concentration profiles

© 2018 BISKA Bilisim Technology

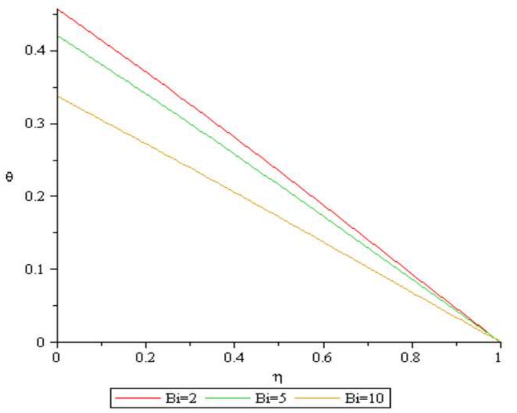

Fig. 7: Effect of Biot number on temperature profile.

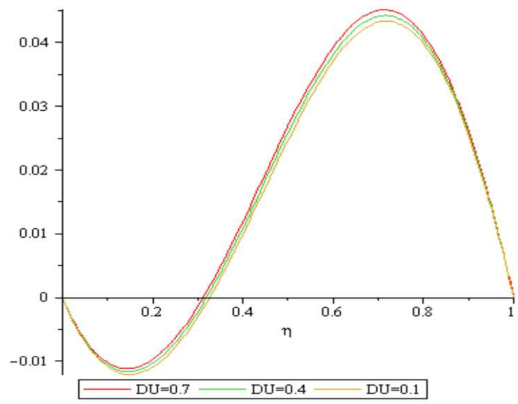

Fig. 8: Effect of Dofour number on velocity profile.

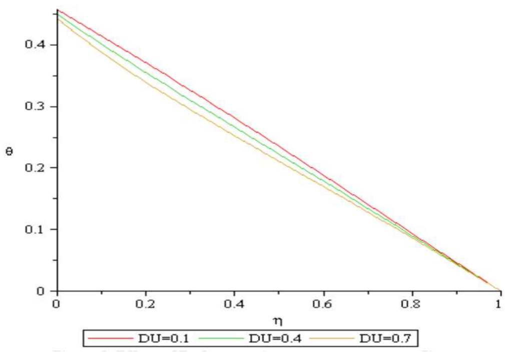

Fig. 9: Effect of Dufour number on temperature profile.

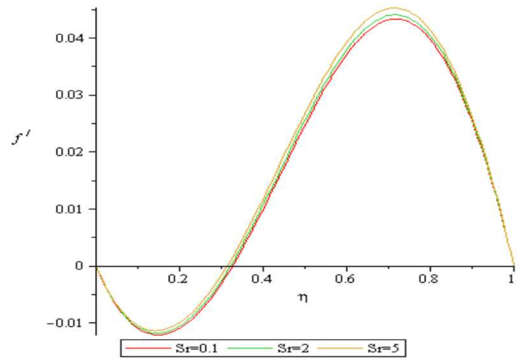

Fig. 10: Effect of Soret number on velocity profile.

respectively. It is observe that reduction in Lewis number causes the velocity profile to reduce and temperature profile increases while the nanofraction profiles is reduced.

Figure 15 shows that as heat generation (ϕ_0) increases from negative to positive so the temperature profile also increases.

4 Conclusion

This problem focuses on the laminar flow of a nanofluid in an inclined permeable parallel walls. We assume that the lower wall is wavy while the upper wall is flat with Dufour effects, Soret effects, and a magnetic field effect with boundary conditions been convective. The rectangular coordinate system has been used to present the model for this problem. It also incorporates the effect of thermophoresis parameter and Brownian motion. The obtained similarity solution is dependent on the thermophoresis number (N_t), Darcy number (Da), Magnetic parameter (M), Dufour (DU) number, Soret (Sr) number, Brownian motion (N_b), Lewis number (Le), Prandtl number (P_r). It is found that at the wavy wall, the fluid flow back. From the results displayed, the following observations were made

(1) The graphs displayed in this work satisfy the boundary conditions.
(2) Results obtained are in good agreement with the Numerical Technique used previously as shown in Table 1. This guaranteed the uniqueness of the method.

© 2018 BISKA Bilisim Technology

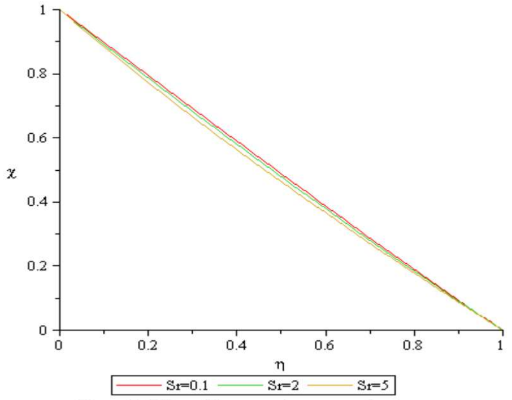

Fig. 11: Effect of Soret number on nanofraction.

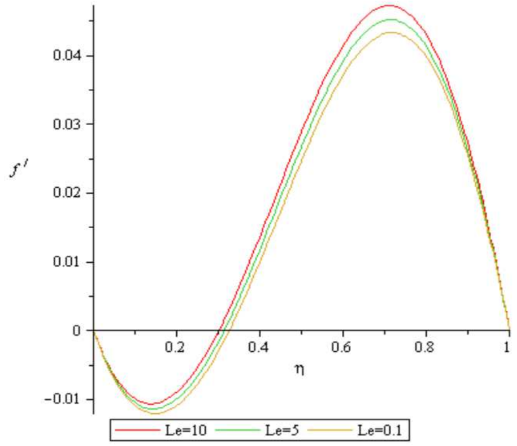

Fig. 12: Effect of Lewis number on velocity profile.

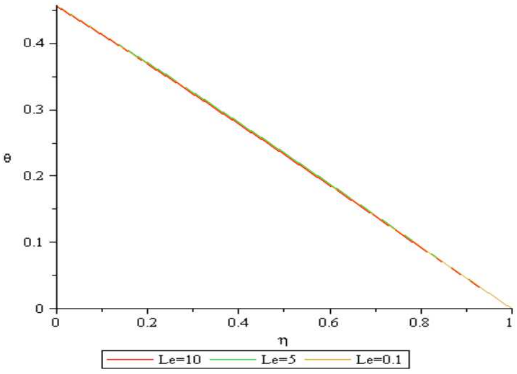

Fig. 13: Effect of Lewis number on temperature profile.

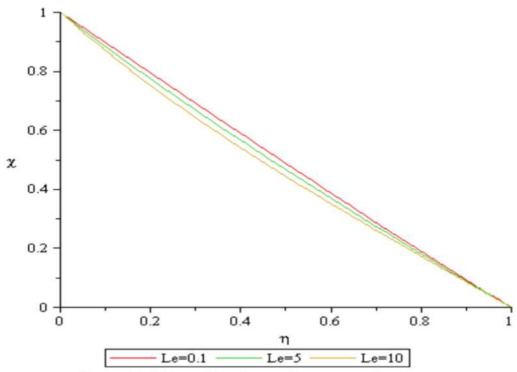

Fig. 14: Effect of Lewis number on nanofracrion.

(3) The problem is solved by taking$Lx = \frac{\pi}{2}$.
(4) Generally on the wavy wall, the fluid velocity is zero, the temperature varies as the parameter as a result of convective heating, and the fluid nanofraction is at maximum while on the flat wall, the velocity, temperature and nanofraction are zeros. Also, at the region close to the wavy wall, there exist a flow back which is the reason the graphs of the velocity profile crossed the negative axis.
(5) It is worthy of note that while some quantities are varied, others were kept constant through-out the work.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

© 2018 BISKA Bilisim Technology

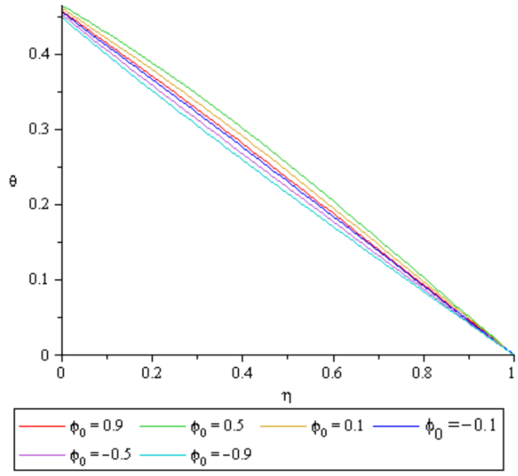

Fig. 15: Effect of heat generation on temperature profile.

References

[1] Y. M. Aiyesimi, A. Yusuf, M. Jiya, "Hydromagnetic Boudary-Layer Flow of a Nanofluid Past a Stretching Sheet Embedded in a Darcian Porous Medium with Radiation". *Nigerian Journal of Mathematics and Applications*, 24, (2015a). 13-29.

[2] Y. M. Aiyesimi, A. Yusuf, & M. Jiya, "An Analytic Investigation of Convective Boundary-Layer Flow Of A Nanofluid Past A Stretching Sheet With Radiation". *Journal of Nigerian Association of Mathematical Physics. 29* 1, (2015b). 477-490.

[3] A. Ebaid, & N. Al-Armani, "A new Approach for a Class of the Blasius Problem via a Transformation and Adomian's Method ". Abstract and Applied Analysis, the Scientific World Journal, 2013

[4] B. J. Gireesha , & B. Mahanthesh, "Perturbation solution for radiating viscoelastic fluid flow and heat transfer with convective boundary condition in nonuniform channel with hall current and chemical reaction". *ISRN Thermodynamics*, (2013). Article ID 935481, 14. http://dx.doi.org/10.1155/2013/935481.

[5] W. A. Khan, & I. Pop, "Boundary-layer flow of a nanofluid past a stretching sheet", Int. J. Heat Mass Transf. 53, 2477-2483 (2010).

[6] A. S. Kherbeet, H. A. Mohammed, & B. H. Salman, "The effect of nanofluids flow on mixed convection heat transfer over microscale backward-facing step", Int. J. Heat Mass Transfer 55, (2012). 5870–5881.

[7] J. P. Kumar, & J. C. Umavathi, "Free convective flow in an open-ended vertical porous wavy channel with a perfectly conductive thin baffle". *Heat Transfer—Asian Research*, (2013). DOI: 10.1002/htj.21118. http://dx.doi.org/10.1002/htj.21118.

[8] S. G. Lekoudis, A. H. Nayfeh, , & W. S. Saric, "Compressible boundary layers over wavy walls". *Physics of Fluids*, 19, (1976). 514–519. http://dx.doi.org/10.1063/1.861507.

[9] M. Lessen, M & S. T. Gangwani, "Effects of small amplitude wall waviness upon the stability of the laminar boundary layer". *Physics of Fluids, 19*, (1976).510–513. http://dx.doi.org/10.1063/1.861515.

[10] R. Muthuraj, & S. Srinivas, "Mixed convective heat and mass transfer in a vertical wavy channel with traveling thermal waves and porous medium". *Computers & Mathematics with Applications, 59*, (2010).3516–3528. http://dx.doi.org/10.1016/j.camwa.2010.03.045.

[11] D. A. S. Rees, & I. Pop, "Free convection induced by a horizontal wavy surface in a porous medium". *Fluid Dynamics Research, 14*, (1994). 151–66. http://dx.doi.org/10.1016/0169-5983 94, 90026-4.

[12] P. N. Shankar, &U. N. Sinha, "The Rayleigh problem for a wavy wall". *Journal of Fluid Mechanics, 77,* (1976). 243–256. http://dx.doi.org/10.1017/S0022112076002097.

[13] M. Sheikholeslami, M. Gorji-Bandpy, R. Ellahi, & A. Zeeshan, " Simulation of MHD CuO–water nanofluid flow and convective heat transfer considering Lorentz forces", J. Mag. Magn. Mater. 369,(2014) 69–80.

[14] J. C. Umavathi, & M. Shekar, "Mixed convection flow and heat transfer in a vertical wavy channel containing porous and fluid layer with traveling thermal waves". *International Journal of Engineering, Science and Technology, 197 3* , (2011). 196-219.

© 2018 BISKA Bilisim Technology

[15] J. C. Umavathi, & M. Shekar, “!Mixed convective flow of immiscible fluids in a vertical corrugated channel with traveling thermal waves”. *Journal of King Saud University – Engineering Sciences, 26*, (2014). 49–68. http://dx.doi.org/10.1016/j.jksues.2012.11.002.

© 2018 BISKA Bilisim Technology