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# NATURAL SEISMIC PROTECTION PROPERTIES OF HISTORICAL "WALLED OBELISK" STRUCTURE IN ISTANBUL: MODELING-NUMERICAL APPROACH

**Sertaç TUHTA** *

Faculty of Engineering, Department of Civil Engineering, Ondokuz Mayıs University, Samsun, Turkey

## ABSTRACT

The "Walled Obelisk" is a 32.77-meter high, 2x2 meter mean square, rough-cut limestone monument that is becoming thinner to the four-sided hill. This monument is in a very sensitive seismic region of the World-Istanbul and remains intact from the time of its construction (approximately tenth century) to the present despite all seismic activity during that time. The current study extracts and investigates a mathematical model of the monument, which reveals that support of the structure incorporates a "natural seismic isolator" that demonstrates behavior similar to current used lead-core rubber-bearings isolator (LRB). The total height of the "natural seismic isolator" is 3.44 m. and consists of three–steps of rough-cut limestone and a massive marble stone that forms four sliding surfaces bonded with Horasan mortar. The components of the "natural seismic isolator" form four sliding surfaces and appear to function similar to LRB's combined layers of steel plates and hard rubber. The massive marble stone appears to function like the lead cylinder core of the LRB. The "natural seismic isolator" demonstrates fully plastic behavior and differs from the LRB by maintaining the monument's upper part in equilibrium during earthquake excitation, which also varies near zero equilibrium. During this study's comparison of the isolator's use of the expression, "similar" is frequent used in this meaning, and the comparison uses a fixed base model of the monument that cannot reciprocate during recovery from earthquakes over the 1000-year history. The proposed model of "natural seismic isolation" from historical experience (for this monument, 1000 years), suggests that the easily constructed natural isolator requires no special maintenance, and is suitable for modern structures in seismic regions.

**Keywords:** Walled Obelisk, Dynamic Analysis, Seismic Protection, Historical Structures

## 1. INTRODUCTION

Örme Dikilitaş is a four-sided monument that narrows from an 8-meter circumference at the base as it rises to its 32.77 height (Figure 1). This structure remains despite exposure to very frequent and strong earthquakes that have destroyed or caused devastating fires in İstanbul. The monument, located next to the Yılanlı Column on the south side of Sultan Ahmet Square consists of rough-cut limestone (Örme Dikilitaş, 2005). The current study creates a structural mathematical model of the monument considers: a) A structural component, acts as a "natural seismic isolator" and consists of three steps of rough-cut limestone and a massive marble stone (enablement) to form four sliding surfaces bonded with Horasan mortar. The height of the steps are approximately 0.41, 0.40, 0.43 meters and in accordance with the plan covers areas of 4.49x4.47 m, 4.047x3.994 m, 3.450x3.295 m, respectively On the three steps, covering an area of 3.235x3.204 m and at a height of 2.2 m is massive marble stone (enablement). The mathematical model of the base structure which acts as a "natural seismic isolator" has a height of 3.44 m; modeling considers this feature. b) The "upper structure" rises to a height of 29.33 m and consists of three parts. The first part's height is 27.735m with a base area of 2.63x2.55 m and an upper surface area of 1.795x1.745 m; second part's a height 1.425 m with a base area of 1.795x1.745 m and an upper surface area of 0.344x0.306 m. Third's part's height is 0.17 m with a base area of 0.344x0.306 m and an upper surface area of 0.146x0.065 m. The "upper structure" is rough- cut limestone (bonded with Horasan mortar), and the modal analysis considers these features.

*Corresponding Author: stuhta@omu.edu.tr
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This study compares the “natural seismic isolator” with a contemporary lead core rubber bearings isolator (LRB), shown in Figure 2. Three storey steps and massive marble stone that form four sliding surfaces using Horasan mortar appears to functions “similar” to the LRB’s combined layers of steel plates and hard rubber. The massive marble stone appears to function in a similar way as the LRB’s lead cylinder core. The “natural seismic isolator” which exhibits fully plastic behavior differs from the LRB by maintaining the monument’s upper part in equilibrium during earthquake excitation which also varieties around zero- equilibrium. Figure 3 shows the monument’s “natural seismic isolator” which has functioned during an approximate 1000 years of earthquakes. As seen in this figure, the 1.24 m height above the base surface the slippage of massive marble stone did not exceed its original position. This suggests the function of the looseness of the special plaster’s thickness and viscosity in between the step and the massive marble stone. Materials used in the monument are marble, limestone and Horasan mortar (Örme Dikilitaş, 2005). Horasan mortar and plaster, prepared by using broken bricks and lime, were the most important binding materials in the history of ancient structures. Horasan (Hydraulic) mortar shows a compressive strength equal to 5.40 MPa and a modulus of elasticity of 3890 MPa (Maria Rosa Valluzzi, Luigia Binda, Claudio Modena, 2005).This study models the Örme Dikilitaş monument’s response to intensive earthquakes as a coupling of base’s “natural seismic isolator” and tower’s “upper structure,” as shown in Figure 2. For analysis, a fixed base model of the monument cannot replicate the recovery from earthquake actions over the monument’s lifetime (1000 years). The fixed-base and isolated structure’s mass, rigidity, damping distributions, and schematic analysis models appear in Figure 4. The analysis places the monument in the first seismic zone, $A_0 = 0.4$, local site class $ZC$ (Turkish Earthquake Code, 2018). The fixed-base model’s natural vibration period, $T_1$, in the first iteration is an assumed to 0.5 sec, with a damping ratio, $\xi_1$ 4%. For the same iteration the assumption for the natural vibration period, $T_I$, of the structure with a base isolator is 2 sec, with a damping ratio $\xi_1$ 10%.

The fixed-base structure mass, damping, and rigidity matrices are:

$$[m_b] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}; \qquad [k_b] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}; \tag{1a, b}$$

$$[c_b] = [m_b]\left(\sum_{i=1,N} \frac{2\xi_i \omega_{0i}}{M_i}\{\Phi\}_i \{\Phi\}_i^T\right)[m_b];$$
(1c)

$$\{\Phi\}_i = \frac{a_{ij}}{\sqrt{\{a\}_j^T [m_b]\{a\}_j}}; \tag{1d}$$

$$M_i = \{\Phi\}_i^T [m_b]\{\Phi\}_i . \tag{1e}$$

Two methods based on various references (Kasimzade A.A., 2006, Kasimzade A.A and Tuhta S., 2004, Naeim F., Kelly J.M., 1999, Skinner R.I., Robinson W.H., McVerry G.H., 1993) provide the method for adopting the mass, damping, rigidity matrices of the structure with the isolator. Building of the $[m],[c],[k]$ matrices with the first method:

$$[m]=\begin{bmatrix} m_I+\sum\limits_{i=1,3} m_i & m_1 & m_2 & m_3 \\ m_1 & m_1 & 0 & 0 \\ m_2 & 0 & m_2 & 0 \\ m_3 & 0 & 0 & m_3 \end{bmatrix};[c]=\begin{bmatrix} [c_I] & 0 \\ 0 & [c_b] \end{bmatrix};\ c_I=2\xi_I\left(\sum_{i=1,N} m_i+m_I\right); \tag{2a, b, c}$$

$$[k]=\begin{bmatrix} k_I & 0 & 0 & 0 \\ 0 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix}. \tag{2d}$$

Building of the $[m],[c],[k]$ matrices with the second method follows:

$$[m]=\begin{bmatrix} m_I & m_1 & m_2 & m_3 \\ m_1 & m_1 & 0 & 0 \\ m_2 & 0 & m_2 & 0 \\ m_3 & 0 & 0 & m_3 \end{bmatrix};[c_b]=\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}; \tag{3a, b}$$

$$[c]=\begin{bmatrix} c_I+c_1 & -c_1 & 0 & 0 \\ -c_1 & c_1+c_2 & -c_2 & 0 \\ 0 & -c_2 & c_2+c_3 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix},\ [k]=\begin{bmatrix} k_I+k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2+k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix}. \tag{3c, d}$$

The total mass of the monument is 506.747 kNsec$^2$/m, and the structural components which constitute the "natural seismic isolator" have a mass of 201.076 kNsec$^2$/m and a height from the ground of 3.44 m. The tower section is the "Upper Structure," which is 29.33 m high from "natural seismic isolator" and has a mass 305.671 kNsec$^2$/m. Examination of structural models (matrices $[m],[c],[k]$ ) uses modal superposition and time history methods, respectively.

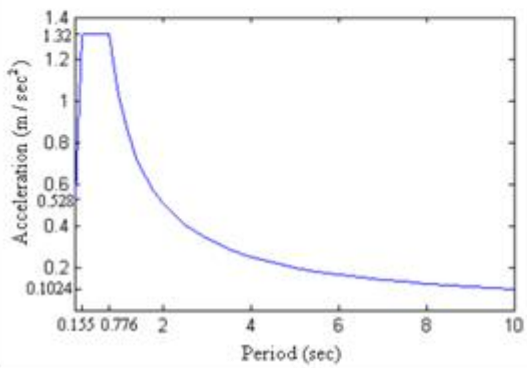



**Figure 1.** Image of "Örme Dikilitaş" monument (Turkey, Istanbul, Sultan Ahmet Square)

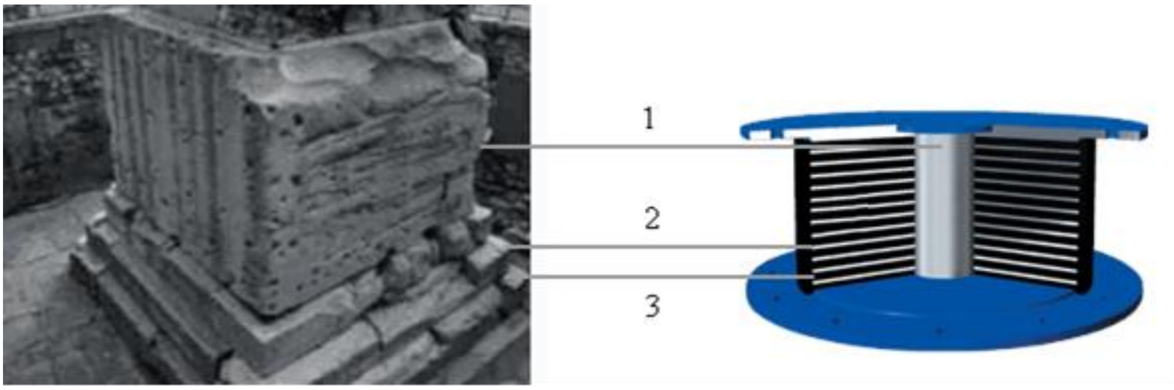



**Figure 2.** Comparison of the related peculiarities of the "Örme Dikilitaş" Structural Foundation's "natural seismic isolator" and the elastomeric seismic isolation system with Lead Core (LRB)

1-Energy Distributor Mass (massive marble stone with dimension 3.235x3.204x2.2 m)
a) Distributes earthquake energy by diminishing its action on the structure
b) Provides wave resistance
2- Rigid layers (consisting of three step rough-cut limestone) providing vertical load capacity and four sliding surfaces
3- Inner layer (1.5 cm weak and soft Horasan mortar materials) provides horizontal flexibility

**Figure 3.** The "Örme Dikilitaş" base component undertook the role of "natural seismic isolator" to dissipate the actions of earthquakes during 1000 years. The previous status of surface of the 1.24 m level of the massive slip stone is not entirely visible.

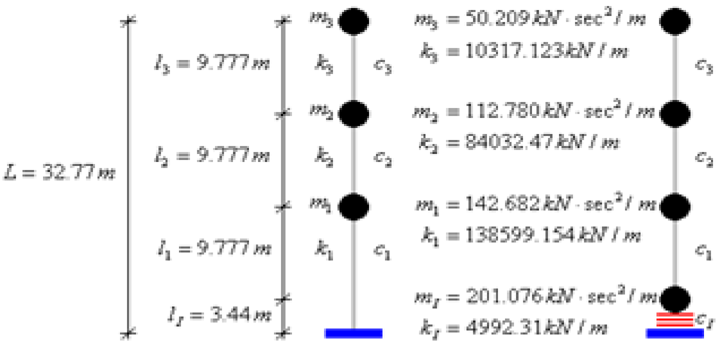



**Figure 4.** Schematic representation of the fixed-base and isolated structure's mass, rigidity, and damping distributions

## 2. MODAL SUPERPOSITION METHOD: ANALYSIS RESULTS

For the discussed mathematical models, calculation of the "natural seismic isolator" and "upper structure's" rigidities depend upon masses and fundamental periods (Figure 5).

$$\omega = \omega_0 \sqrt{1-\xi^2} \; ; \tag{4a}$$

$$\omega_I = \frac{2\pi}{T_i} = \frac{2\pi}{2} = 3.141 \sec^{-1} ; \tag{4b}$$

$$k_I = \omega_I^2 \left(m_1 + m_2 + m_3 + m_I\right) = \left(3.141\right)^2 200.103 = 4992.31 kN/m ; \tag{4c}$$

$$\left[\bar{d}\right]\{a\} - \bar{\lambda}\{a\} = \{0\} ; \tag{4d}$$

$$\bar{\lambda} = \frac{\lambda}{\delta_0 m_0},\ \lambda = \frac{1}{\omega_0^2}, \left[\bar{d}\right] = \frac{[\delta][m]}{\delta_0 m_0} ; \tag{4e, f, g}$$

$$\{\phi\}_i = \frac{a_{ij}}{\sqrt{\{a\}_j^T [m_b]\{a\}_j}} \; ; \tag{4h}$$

$$M_i = \{\phi\}_i^T [m_b]\{\phi\}_i ; \tag{4i}$$

$$F^{st} = \alpha [m]\{\phi\}_i ; \tag{4j}$$

$$\alpha = \frac{1}{\{\phi\}_i^T [m]} ; \tag{4k}$$

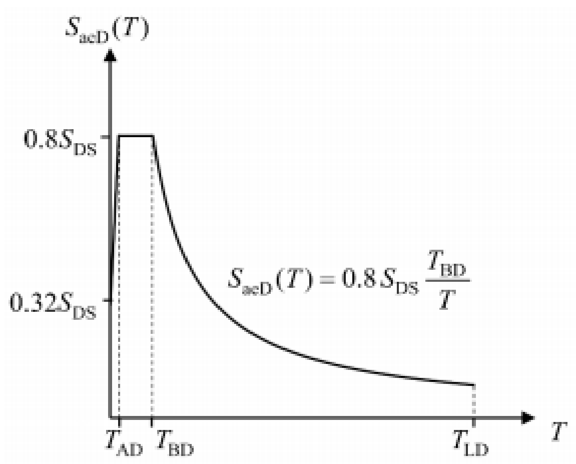



**Figure 5.** Elastic Design Spectrum Used in Multi-Degree Freedom System Sample

$$S_{aeD}(T) = \left(0.32 + 0.48\frac{T}{T_{AD}}\right) S_{DS} \qquad \left(0 \le T \le T_{AD}\right) \; ; \tag{5a}$$

$$S_{aeD}(T) = 0.8 S_{DS} \qquad (T_{AD} \le T \le T_{BD}) ; \tag{5b}$$

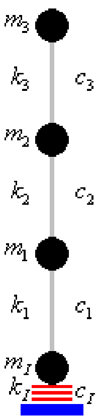
$$S_{aeD}(T) = 0.8 S_{DS} \frac{T_{BD}}{T} \qquad (T_{BD} \le T \le T_{LD}) ; \tag{5c}$$

$$T_{AD} = \frac{T_A}{3} ; \tag{5d}$$

$$T_{BD} = \frac{T_B}{3} ; \tag{5e}$$

$$T_{LD} = \frac{T_L}{2} ; \tag{5f}$$

**Table 1.** Analysis Results from the Modal Superposition Method

|  | **DOF Number** | $F_b / m_s g$ $(m_s g)$ **on the order of** $m_s = 305.671$ $(kN \sec^2 / m)$ | $u_b$ $(mm)$ | $F_I / m_s g$ $(m_s g)$ **on the order of** $m_s = 506.747$ $(kN \sec^2 / m)$ | $u_I$ $(mm)$ |
|---|---|---|---|---|---|
| | **3** | **0.2471 (0.2471)** | **87.39 (87.39)** | **0.0253 (0.0200)** | **256.42 (315.90)** |
| | **2** | **0.3612 (0.3612)** | **27.77 (27.77)** | **0.0533 (0.0420)** | **244.26 (306.53)** |
| | **1** | **0.2925 (0.2925)** | **13.36 (13.36)** | **0.0660 (0.0530)** | **239.62 (302.88)** |
| | **0** | | | **0.2357 (0.1855)** | **234.437 (298.8)** |
| | | $\sum F_b = 0.09008$ **(0.09008)** | | $\sum F_I = 0.3806$ **(0.3008)** | |

**Note:** The values in brackets represent results from building $[m],[c],[k]$ matrices according to the second method, using formulae (3a, b, c, d).

## 3. TIME HISTORY METHOD: ANALYSIS RESULTS

Conventionally, the equations for motion of a system under earthquake action use the matrix notations:

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{F_*\} ; \tag{6a}$$

$$\{F_*\} = (F_{*1} F_{*2} \ldots F_{*i} \ldots F_{*n})^T \text{ , } F_{*i} = -m_i \ddot{x}_g(t)\text{;} \tag{6b, c}$$

In the current study, mass, damping, and rigidity matrices of the isolated structure only use the first method, based on formulae (2a, b, c, d).

Ground acceleration, $\ddot{x}_g(t)$, depending on the appropriate regulations (Turkish Earthquake Code, 2018 and UBC-1997) are changed according to specific design equivalent acceleration spectra (Figure 6) for the first seismic zone and local site class, $ZC$. The solution of the investigated equation of motion by the numerical method (Wilson-$\theta$), appears in Table 2, which includes the numerical analysis steps and transformation of the ground acceleration to the appropriate spectrum. For transitions from acceleration to the spectrum, for the focal–point, the referenced effect (Turkish Earthquake Code, 2018) does not appear, because reference (UBC-1997) was used. The top ground layer, (h1) h1$\leq$ 15 m, of the local site class, $ZC$, is the (C) group ground as referenced in Turkish Earthquake Code, 2018. The (C) group ground is equivalent to the $S_d$ ground in reference in UBC-1997. The reference (UBC-1997) accounts for large to moderate earthquakes' excitations (M>7), and "Seismic Source Type A" is selected. With a horizontal distance of 10 km from the seismic source, and using seismic source Type A with the horizontal distance, produces values from Table 16-S and 16-T for factor of proximity to the source (UBC-1997).

$N_a = 1; N_v = 1.2$.

In Seismic Zone Factor, $Z = 0.4$, $C_a, C_v$ seismic coefficients (UBC-1997) in Table 16-I were used depending on the ground class, $S_d$, and the Seismic Zone Factor ($Z = 0.4$) according to:

$$C_a = 0.44N_a = 0.44$$
$$C_v = 0.64N_v = 0.768$$

Calculation of special design acceleration spectrum values were dependent on the $C_a$ and $C_v$ values (based on UBC-1997) and formulation of acceleration spectrum arises from using SAP 2000 software. Solution of the equation for motion uses the obtained acceleration spectrum by numerical method and includes the steps:

$$[m]_i \{\Delta\ddot{u}\}_i + [c]_i \{\Delta\dot{u}\}_i + [k]_i \{\Delta u\}_i = [m]\{\Delta F\}_i\text{;} \tag{6d}$$

$$\{u\}_{i+1} = \{u\}_i + \{\Delta u\}_i\text{;} \tag{6e}$$

$$\{\dot{u}\}_{i+1} = \{\dot{u}\}_i + \{\Delta\dot{u}\}_i\text{;} \tag{6f}$$

$$\{\ddot{u}\}_{i+1} = [m]^{-1}\left(\{F_*\}_{i+1} - [c]\{\dot{u}\}_{i+1} - [k]\{u\}_{i+1}\right)\text{;} \tag{6g}$$

$$\{\Delta u\}_i = \{u(t_i + \Delta t)\} - \{u(t_i)\}\text{;} \tag{6h}$$

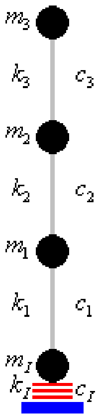
$$\{\Delta\dot{u}\}_i = \{\dot{u}(t_i + \Delta t)\} - \{\dot{u}(t_i)\}\text{;} \tag{6i}$$

$$\{\Delta\ddot{u}\}_i = \{\ddot{u}(t_i+\Delta t)\} - \{\ddot{u}(t_i)\}; \qquad (6j)$$

$$\{\Delta F\}_i = \{F(t_i+\Delta t)\} - \{F(t_i)\}; \qquad (6k)$$

Results appear in Table 2.

**Table 2.** Analysis Results with Time-History Method

|   | **DOF Number** | $F_b / m_s g$ $(m_s g)$ **on the order of** $m_s = 305.671$ $(kN \sec^2 / m)$ | $u_b$ $(mm)$ | $F_I / m_s g$ $(m_s g)$ **on the order of** $m_s = 506.747$ $(kN \sec^2 / m)$ | $u_I$ $(mm)$ |
|---|---|---|---|---|---|
| | **3** | **0.0527** | **22.4** | **0.0141** | **248.6** |
| | **2** | **0.0744** | **7.9** | **0.0306** | **237** |
| | **1** | **0.0829** | **4** | **0.0384** | **232.5** |
| | **0** | | | **0.2275** | **227.5** |
| | | $\sum F_b = 0.21$ | | $\sum F_I = 0.3106$ | |

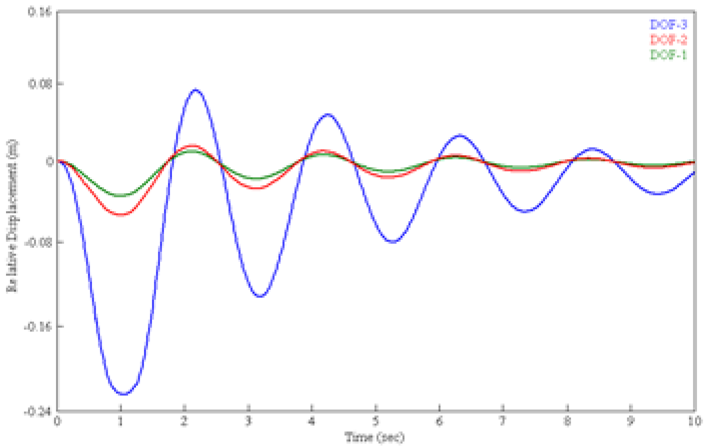



**Figure 6.** Isolated Structure Relative Displacement-Time Diagram

## 4. CONCLUSIONS

Analysis result using from this study's mathematical model of the Örme Dikilitaş structure clarifies the factors contributing to the structure's survival from its initial construction to the presents, spanning ten centuries:

-The comparison shows a fixed base model of the monument could not reciprocate and recover from earthquake actions over the structure's lifetime (1000 years).

-Maximum displacement of the "natural seismic isolator" has an approximate average of 22 cm (Figure 7). The four sliding surfaces of the "natural seismic isolator" produces 22/4 = 5.5 cm displacement and provides confidence that seismic design allows the existing structure's survival of seismic events.

-"Natural seismic isolator" shows fully plastic behavior differing from LRB and maintains the monument's upper part at an equilibrium during earthquake excitation which also varies near zero-equilibrium.

-As seen in Figure 3, at the 1.24 m height above the base surface the massive marble, slip stone could not completely achieve its original position. This suggests a reduction in the special plaster's thickness and viscosity between steps and the massive marble stone.

The determined and presented model of the "natural seismic isolation," according to historical experience (in this monument's 1000–year existence), represents a simplified construction method that requires no special maintenance and is applicable to modern structures existing in regions of seismic activity.

Arguably, the "upper structure," constructed of rough-cut limestone and bonded with Horasan mortar may have contributed to the damping effect. Future study may include analysis that accounts for the upper structure's characteristics other finite elements' factors.

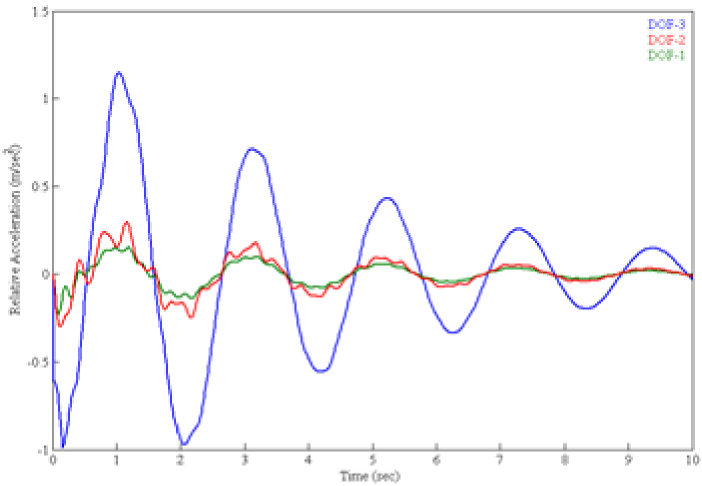



**Figure 7.** Isolated Structure Relative Acceleration-Time Diagram

## 5. SYMBOLS

| | | | |
|---|---|---|---|
| $\{a\}_i$ : | the i-th mode shape of the structure | $[k]$: | the stiffness matrix of the structure |
| $\alpha$ : | the modal participation factor | $[k_b]$: | the stiffness matrix of the "upper structure" |
| $[c_b]$: | damping matrix of the "upper struc-ture" | $[c_I]$: | damping of the of the base isolator |
| $[k_I]$: | stiffness matrix of the base isolator | $\xi_I$ : | damping ratio of the base isolator |
| $[m_b]$: | mass matrix of the "upper structure" | $\xi_i$ : | damping ratio of i-th mod |
| $[m_i]$: | i-th level mass of the structure | $F_b$ : | base force of the fixed-base structure model |
| $[m_I]$: | the mass of the isolator | $F^{st}$ : | modal static reaction force |
| $[m_s]$: | total mass of the structure with isolator | $F_I$ : | base force of the base isolated structure model |
| $\omega_i$ : | angular frequency for i-th mode | $M_i$ : | the modal mass for i-th mode |

| | | | |
|---|---|---|---|
| $\{\phi\}_i$ : | normalized (by mass) eigenvector | $S_a(j)$: | spectral acceleration for j-th mode |
| $\omega_{0i}$ : | angular frequency for i-th mode of the fixed-base structure | $T_i$ : | natural vibration period for i-th vibration mode of structure |
| $u_b$ : | displacement of the fixed-base structure | $T_I$ : | period of the isolator |
| $u_I$ : | displacement of the base isolated structure | | |

## 6. ACKNOWLEDGEMENT

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