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EXAMINING A PRESERVICE SECONDARY TEACHER'S GROWTH: IMPLICATIONS FOR TEACHING[1]

(BİR ORTAÖĞRETİM ÖĞRETMEN ADAYININ GELİŞİMİNİN İNCELENMESİ: ÖĞRETİM İÇİN ÇIKARIMLARI)

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ABSTRACT

The purpose of this study was to examine a preservice mathematics teacher's weaknesses in her content knowledge and pedagogical content knowledge of the concept of function when she taught an episode on exponential functions to a group of high school students. During her planning and teaching of a lesson to the group of high school students, I observed the results of her lack of teaching experience and incomplete content knowledge on her pedagogical content knowledge. The results of her teaching experience showed the preservice secondary teacher had misconceptions and during her teaching she passed on these misconceptions to her students.

Keywords: Preservice Teacher, Content Knowledge, Pedagogical Content Knowledge, Function.

ÖZ

Bu çalışmada bir matematik öğretmen adayının fonksiyon kavramı ile ilgili alan bilgisindeki eksiklikleri incelemeyi ve bu eksikliklerin öğretmen adayının üstel fonksiyon konusunu, bir grup lise öğrencisine anlatırken nasıl etkilediğini belirlemek amaçlanmıştır. Dersi planlama ve planladığı dersi öğretme sürecinde, öğretmen adayının alan bilgisindeki ve pedagojik alan bilgisindeki eksiklikler belirlendi. Elde edilen bulgular, öğretmen adayının kavram yanılgılarının olduğunu ve öğretme sürecinde bu kavram yanılgılarını öğrencilerine aktardığını gösterdi.

Anahtar Sözcükler: Öğretmen Adayı, Alan Bilgisi, Pedagojik Alan Bilgisi, Fonksiyon.

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INTRODUCTION

The challenge of preparing future mathematics teachers is acknowledged and what types of experience that preservice mathematics teachers need should be the central focus for the improvement of teacher education in order to train effective teachers (Cooney, 1994). It is essential to engage preservice teachers in cycles of planning, practicing, and reflecting on lessons. As they engage in preparation of a lesson plan, preservice teachers improve their knowledge of effective instruction beyond the particular lesson and content (Fernandez, 2005). Over the past decade, researchers have turned their attention to examining preservice teachers' content and pedagogical content knowledge. Even (1993) found that many preservice secondary teachers had a limited conception of function, influencing their pedagogical thinking. Research has also shown that it is difficult to change preservice teachers' perspectives about the teaching and learning of functions (Sanchez & Llinares, 2003). When prospective teachers have misconceptions or limited content knowledge, they may pass on these misconceptions to their students. They may fail to challenge them (Ball & McDiarmid, 1990). Their conceptions might limit their ability to present subject matter in appropriate ways, give helpful explanations and conduct discussions (Even & Tirosh, 1995). Wilson (1994) examined a preservice teacher's understanding of the concept of function with extensive materials including graphical, algebraic representations of functions, non-functions, and real-life applications of functions. However, her perspective of mathematics and mathematics teaching remained relatively narrow at the end of the study. Since Shulman (1986) introduced these two concepts, preservice teachers' content knowledge and pedagogical content knowledge have been investigated using questionnaires, preparation and analysis of lesson plans, and hypothetical teaching situations. Even though research studies (Even, 1989; Sanchez & Llinares, 2003;) have investigated preservice secondary teachers' content and pedagogical content knowledge of the concept of function using questionnaires, these research studies have not examined preservice secondary teachers' classroom instructions.

The current study goes beyond administering questionnaires and conducting interviews, and examines a preservice secondary teacher's developing content and pedagogical content knowledge of exponential functions as she participated in activities, preparation and analysis of lesson plans on exponential functions and video teaching episodes, and reflection on her teaching.

In this study, I used Wilson, Shulman and Richert's (1987) model to investigate the participant's growth in content knowledge and pedagogical content knowledge as well as the improvement in her understanding of the

concept for teaching as a result of these tasks. In particular, I focused on the preparation and analysis of the lesson plan on exponential functions and video teaching episode. I also organized the tasks under six aspects (comprehension, transformation, instruction, evaluation, reflection, and new comprehension) suggested by Wilson, Shulman and Richert (1987).

METHODOLOGY

Qualitative research is an approach where extensive description can be written in natural settings (Creswell, 2003). A qualitative research design is also flexible and less structured and it may be a developing design as the research progresses. I selected the qualitative research design because it was appropriate for the purpose of this study.

Case studies are used to explore a program, an event, an activity, a process, or one or more individuals in depth (Creswell, 2003). Case study design was utilized to gain an in-depth understanding (Merriam, 1988) of the prospective secondary teacher's knowledge of the concept of functions and its meaning for the researcher. In this research study, the purposeful sampling was used to identify unique variations that have emerged in adapting to different conditions (Lincoln & Guba, 1985). Wiersma (2000) states that the logic of purposeful sampling is based on a sample of information-rich cases that are studied in depth. Purposeful sampling was used to select the participant for this study.

The purpose of this study was to examine a preservice secondary mathematics teacher's content knowledge and pedagogical content knowledge of the concept of function as well as to identify the strengths and weaknesses of her knowledge for teaching the concept of function as she participated in tasks during the six weeks of data collection from a university in the south eastern United States. I also analyzed the preservice secondary mathematics teacher's content knowledge and pedagogical content knowledge of the concept of function when she taught a lesson on exponential functions to a group of high school students. The data were collected through three stages: (1) the function questionnaire, (2) the card sorting activity, (3) preparation and analysis of lesson plan on exponential functions and video teaching episode.

The function questionnaire adapted from the studies of Even (1989) and Wilson (1992) included fourteen items addressing different aspects of content knowledge (i.e., examples of functions and non-functions, different representations of functions) and five items focusing on analysis of students' incorrect solutions. In the Fall 2005 semester, the function questionnaire consisting of nineteen mathematics problems was administered to preservice secondary mathematics teachers. They were asked to show their method of

solution and explain their answers so that I could examine the nature of their content knowledge and pedagogical content knowledge of the concept of function. The function questionnaire provided information about the preservice teachers' general knowledge of the concept of function and possible approaches for teaching the topic.

The card sorting activity (Cooney, 1996) with a different function or representation of a function on each card included twenty-eight different examples of seven types of functions. These types of functions were linear, quadratic, polynomial, exponential, logarithmic, trigonometric, and rational functions, given in four different representations; tables, graphs, equations, and verbal descriptions. In the card sorting activity, preservice teachers were asked seven questions and organized their knowledge about seven different categories of functions using different representations. The card sorting activity helped me to determine how they utilized different representations and how they translated a function from one representation to another.

Sara, who presented robust mathematical understanding in two of the components (i.e., the function questionnaire and the card sorting activity), was selected for the study. After analysis of each task, I conducted two sixty-minute interviews with the participant. I asked her to elaborate on her responses. Sara was a senior teacher candidate and had a 3.8 overall college G.P.A. She was completing her education to receive middle and high school certification. Sara took advanced mathematics courses such as Calculus I and II, but she had not completed the mathematics education courses at the time of the study.

Prior to preparation and analysis of the lesson plan on exponential functions and the video teaching episode, I wanted the participant to analyze two lesson plans and teaching videos of these lessons because of her lack of experience in designing a lesson. I wanted the participant to gain some experience in preparing lesson plans as well as analyzing video teaching episodes of those lessons before she wrote a lesson plan and taught it herself. Therefore, I asked Mr. A at Southern High School to participate in this study. Mr. A, who had a masters degree in mathematics teaching and was pursuing his Ph.D. in mathematics education agreed to participate in this study. Mr. A agreed that he would prepare two lesson plans. The first lesson was on *Theoretical and Experimental Probability*. Instead of preparing the lesson plan before implementing it, Mr. A taught the lesson unprepared, and then wrote the lesson plan. In addition, teaching of the lesson was teacher-centered by design. The second plan was on *Fundamental Counting Principles* and was written before he taught the lesson.

I first gave the participant the lesson plans, then the videos of the teaching of those lesson plans. The preservice teacher analyzed the lesson plans, and then watched the videos of the teaching of those lesson plans. She analyzed the lesson plans as well as the teaching of the lessons. Later, I met

with Mr. A to talk about the classes he taught for the study. I video-recorded the interview with Mr. A so that I could have the participant watch this video as well as describe her teaching from our point of view. After her analysis, a sixty minute interview was conducted with the participant to discuss what she thought about the lesson plans as well as which lesson plan she found better and why. The participant was asked questions about what she thought about Mr. A's implementation of these two lessons.

In the task of preparation and analysis of the lesson plan on exponential functions and the video teaching episode, the participant was given a lesson plan guideline and objectives, and asked to write a lesson plan on exponential functions. The objectives of this lesson were to have students graph exponential functions and identify data that displays exponential behavior. She tried to choose the most appropriate examples and activities in her lesson plan and tried to implement examples and activities that would be appropriate for the students' needs. I analyzed her lesson plan and then conducted a sixty-minute interview with the participant. My goal was to examine how the participant prepared and selected the questions for her lesson. I also looked at how she used different forms of representations, ideas, examples, activities and explanations in her lesson plan. After analyzing her lesson plan, I videotaped the participant's classes while she was teaching her lesson to a group of high school seniors. Her teaching of the lesson took 40 minutes. When she taught her lesson on exponential functions, I looked at how she made it comprehensible to the group of high school seniors. I examined whether or not the participant recognized what made the learning of exponential functions easy or difficult for the students. Then, I conducted a sixty-minute interview with the participant. I asked the participant to watch the video of her teaching, and evaluate and reflect on her teaching of the lesson. I asked her what she thought about her teaching and whether any change occurred in her thinking about how she should teach exponential functions. I also examined whether or not the participant considered the group of high school seniors' current understanding to make instructional decisions.

RESULTS

The following section focuses on the results of an analysis of the participant's responses about exponential and logarithmic functions as well as her teaching of exponential functions.

The Function Questionnaire

In the function questionnaire, the participant was asked to respond to the question given in Figure 1. She was unable to determine whether or not

logarithmic function and root function were inverses of $f(x)=10^x$. The procedure in Figure 2 shows how to find the inverse function of $f(x)=10^x$.

A student said that there are 2 different inverse functions for the function $f(x)=10^x$. One is the root function and the other is the log function. Is the student right? Explain.

Figure 1. Question

$$\log f(x)=\log 10^x = x$$
$$\log f(x)= x$$

Figure 2. Solution

Similarly, Sara was unable to determine whether or not a logarithmic function and a root function were inverses of $f(x)=10^x$. Sara did not remember what inverse function was or how it should be presented. When she was asked to explain whether or not the function, $f(x)=10^x$ had two inverses, she said, "I don't know that's what I am trying to do. I don't remember what inverse function is and how to find it. I don't really know what I am looking for." Sara took the logarithm of both sides, but she failed to replace 'x' and 'y' values to find the inverse function (see Figure 2).

The Card Sorting Activity

In the card sorting activity, on which the functions were written on the cards, the participant was able to sort functions according to their representations; it was easier for her to recognize functions presented in graphical representations, tables or equations. For example, in the card sorting activity, the participant was asked to sort the cards 1, 6, 7, 16, 18, 20, 21, 26 and 28 into two piles and describe the criterion she used to sort the cards in Figures 3–11.

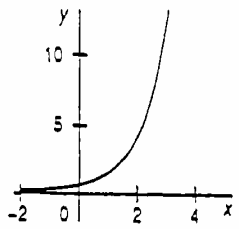

Figure 3. Card 1

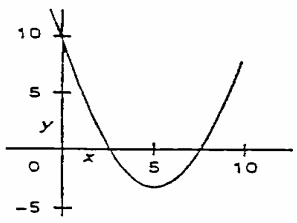

Figure 4. Card 6

$$y=\frac{1}{3}x^2-x+1$$

Figure 5. Card 7

x	1/100	1/10	1	10	100
y	-2	-1	0	1	2

Figure 6. Card 16

$$y = (\tfrac{1}{2})(4^x)$$

Figure 7. Card 18

$$y = \log_4(x+4)$$

Figure 8. Card 20

x	-2	-1	0	1	2
y	1	0	1	4	9

Figure 9. Card 21

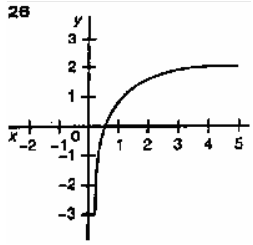

Figure 10. Card 26

x	-2	-1	0	1	2
y	4	2	1	1/2	1/4

Figure 11. Card 28

Sara grouped the function cards using two different ways. Firstly, she grouped the cards according to their representations. She grouped the cards 1, 6, 26 as graphical representations, 7, 18, 20 as formulas, and 16, 21, 28 as tables. Secondly, she grouped the function cards according to their types. She put the cards 16, 26, 20 as logarithmic, 1, 18, 28 as exponential, and 6, 7, 21 as parabolic. When Sara was asked whether or not the function cards could be sorted differently, she said, "I don't see another way. I don't see anything." Sara sorted the functions cards into two piles using two different ways. In her response, she grouped the function cards using their types (i.e., exponential, quadratic, logarithmic) and different representations (graph, equation, table).

As can be seen from Sara's solution, she had a good understanding of the three categories of functions and the use of their representations. In the function questionnaire, Sara failed to find the inverses. She knew exponential and logarithmic functions were related, but she failed to notice that these two functions were inverses of each other. She also did not know what inverse function is or how she could find it. This shows that Sara does not have robust understanding of logarithmic and exponential functions and the relationship between these two functions.

Preparation and Analysis of Lesson Plan on Exponential Functions and Video Teaching Episode

Sara recognized the good and bad aspects of these two lessons. Sara thought the first lesson; *Theoretical and Experimental Probability* was less

organized and structured. She saw that the first lesson did not help students to learn what probability was. Sara recognized that the teacher (Mr. A) gave the definitions, formulas and solution of the problems. She thought the second lesson was designed in a way where the students saw a need for generating a rule for *Fundamental Counting Principles*. The lesson was built on problems in such a way that Mr. A was able to make the transition from one problem to another. Mr. A did not directly tell the students the answers to the problems. He let students investigate the questions and discuss their answers.

In my interview with Sara prior to her teaching, she said that she wanted to use the activity called "money example" in which students would be given three exponential functions (i.e., $f(x)=10^x, f(x)=(2)^x$), $f(x)=\left(\frac{4}{5}\right)^x$) and they would be asked to graph them. Then, she would tell the students if the x-axis measures time in months and the y-axis measures dollars, then ask them to graph these functions. After graphing, the students would be asked to determine which of these functions they would choose if they would receive their salary based on these exponential growths. Her goal was to introduce increasing and decreasing exponential functions with this activity, even though Sara also did not include the definition of an exponential function and properties of exponential functions in her lesson plan and did not talk about discussing these in her teaching.

During her teaching, Sara decided not to use the money example. Instead she distributed worksheets that included functions (i.e., $y=3^x$, $y=4^x$, $y=5^x$, $y=-(5^x)$ and $y=(-5)^x$). She told the students, "My hint is that if you just plug in a number for x, any number. Solve to see what y would equal. Do not think of using the point-wise approach." She asked students to substitute any values for x and she told them that they did not need to think of using the point-wise approach. Sara was unable to see that substituting any number was not appropriate for graphing the equation. The students may choose very large or small values that would make graphing difficult because the graph would be very close to the origin or the points would be very far from each other. During her teaching, Sara recognized that the students had difficulty graphing exponential functions. However, Sara was unable to think of how she could help the students to overcome this difficulty. Sara described what she noticed during the teaching of her lesson in the following statement.

> **Sara:** When they were working on it, I was looking at their papers. They would do like two points. Because I guess they're used to graphing lines. You just graph, you just plot two points and then connect the dots and draw lines. I was like try another one. Do a couple points to figure out what the graph looks like…expected them to try a bunch of points, they were just sitting there looking up to thinking that they had to graph from that.

She also observed that the students had difficulty computing values of negative exponents. She gave an example of negative exponents ($4^{-1} = 1/4$). She ended up graphing exponential function. She utilized the point-wise approach to create a table and then plotted those points and drew the graph. For example, during her teaching, Sara created table values for the graph of $y = (-5)^x$ and $y = -(5^x)$ assuming that ($y = (-5)^x$) was an exponential function. A representative response is as follows:

> **Sara:** This one is a special case [*graph of* $y = (-5)^x$] but otherwise they always go really up. They go for a long time, they just go close to zero. All of a sudden they go really fast. Go down really fast, but that's always how the exponential function is.

After graphing these equations, she was still thinking that the graph of $y = (-5)^x$ was a special case of exponential functions. She did not see that exponential function is defined as f given by $f(x) = a^x$ for every x in R, where $a > 0$ and $a \neq 0$. This clearly indicates that Sara did not know the definition and basic properties of exponential functions.

During her teaching, instead of the money example, she decided to utilize the activity called "bunny rabbit". This activity is a puzzle that Fibonacci posed. The original problem that Fibonacci investigated was about how fast rabbits could breed in ideal circumstances. In this activity, a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits, assuming the rabbits never die. She utilized this activity to show data that would represent exponential behavior. She wanted to show the students how the exponential functions were used and represented in real life. However, she had difficulty creating a table of values for the number of pairs of rabbits in the field at the start of each month. She was unable to show how the series is formed and how it continues. Therefore, she failed to illustrate that the relationship between the number of pairs of rabbits and start of each month was an exponential function. Due to her lack of experience as a classroom teacher, she was unable to ask students to find the relationship between these two.

I also asked Sara if she would change anything in her teaching of the lesson and why she would change it. She thought of changing the examples she used in this lesson. She thought basic examples would be better then using different types of exponential functions. She said,

> **Sara:** I would not give them as many types of functions. Maybe it would be just move function to the left or to the right or either up or down or flip it... then discuss a few different other kinds. Maybe, use those three different types of transitions.

As can be seen from the excerpt above, Sara recognized that she should choose basic examples of exponential functions and give a better explanation to the students. However, she did not mention what these examples would be. She did not recognize that a class discussion is as important as explanations. It is also important to have students share and represent their findings.

I asked Sara whether she considered the students might have difficulty learning exponential functions prior to the teaching of her lesson. She did not think students would have difficulty calculating negative exponents. She said, "I mean, I didn't think about the fact that they couldn't do the negative exponents. I just assumed that they would be able to since I'm teaching them about exponential functions. I figured they must have already known the stuff they needed to do exponential functions." During her teaching, Sara noticed students' difficulties with negative exponents. She was not expecting the students would have difficulty with negative exponents. She gave the rule for finding the value of negative exponents. After evaluating her teaching, she felt that it would have been better if she had focused on the negative exponents in this lesson rather then graphing exponential functions. She said, "They just kind of threw me off because they didn't understand negative powers. It probably would have been appropriate for me to teach them about that [*negative exponents*] instead of exponential function."

For the bunny rabbit activity, she realized that she was unable to display the data that represents exponential growth. She was unable to create the table that displayed the number of rabbits in the first couple of months (see Figure 12). She told the students that the relationship between the number of rabbits and the months was an exponential function without using the table values.

Months x	# Rabbits y
0	4
1	8
2	

Figure 12. Number of Rabbits in the First Couple of Months

When she evaluated her teaching, she said either she would explain the bunny rabbit activity better or she would use a different example of representation of exponential functions to keep the students' attention during

the lesson. She thought that if the activity is explained properly, this activity is a good example of exponential functions. She said,

> I would want either to do a better job of explaining that example [*bunny rabbits*] or use a different example that was more interesting. I think if I had done a better job of explaining it, then it would have been more interesting. I would have kept their attention with that. It does turn out to be a nice example, if you do it properly.

During her teaching, she did not discuss the definitions and properties of exponential functions – increasing or decreasing functions, domain, range, horizontal asymptote – or how to sketch exponential graphs based on the value of the base. This indicates that Sara was unable to introduce the necessary step (i.e., definitions and properties of exponential functions) due to her weak content knowledge. Throughout the lesson, Sara did not give enough time for the students to think about her questions. She asked questions, but she answered most of her questions herself. When the students had difficulty understanding, such as graphing exponential functions and finding the value of negative exponents, she did not know how to help the students or choose appropriate methods to help the students. Due to her limited content knowledge of functions, she failed to ask appropriate initial questions or offer explanations that would make the concept easier for the students. The impact of weak content knowledge and pedagogical content knowledge on her instruction was apparent in her teaching.

DISCUSSION

Sara did not have any teaching experience until she taught an episode for this study. Basically, this was her first time teaching in actual classroom. I believe that one of the biggest factors that prevented her from communicating with students or getting students engaged in the class was her lack of experience of teaching. Sara was a novice teacher, and she would learn and improve her teaching skills by teaching.

The participant was unable to choose appropriate questions and organize a lesson plan due to her weak content knowledge, the concept of functions. When she was describing her instructional strategies to facilitate students' learning during her analyses of the lesson plan, she failed to select an appropriate activity and examples of exponential functions (e.g., $y = (-5)^x$) due to her incomplete content knowledge. This teaching episode also showed that when the preservice secondary teacher has misconceptions; she passed on these misconceptions to her students. During the teaching episode, the participant failed to ask appropriate questions to make the concept easier for students and frequently seemed unaware of students' thinking. She failed to

challenge students and ask critical questions due to her weak pedagogical content knowledge. She also could not determine the fundamental concepts in exponential functions that students needed to know in order to draw and understand the graph of exponential functions. Furthermore, she had difficulty explaining and discussing the definition and properties of exponential functions that would help students, because her pedagogical content knowledge for teaching was weak. When she was evaluating and reflecting on her teaching, she noticed students' difficulties and lack of understanding of the asymptote and graphing exponential functions but she could not think of using any instructional strategy to help them during her teaching. After her teaching, she suggested using alternative approaches that may help students understand the concept or that initiate classroom discussion. However, her instructional strategy for teaching exponential functions changed insignificantly due to her limited content knowledge and pedagogical content knowledge.

The teaching episode experience was valuable for the participant. The teaching episode also provided a relevant context to enhance her content knowledge and pedagogical content knowledge of exponential function. This experience provided an opportunity for a preservice secondary teacher to begin linking theory and practice to teach and analyze her teaching. The teaching experience was also helpful to identify how the weaknesses and strengths of her knowledge of concept of function had influenced her lesson plan and her teaching.

Ball (1990) stated that content knowledge should be a central focus of teacher education in order to teach mathematics effectively. Accordingly, preservice teachers should develop their pedagogical content knowledge as they plan to teach as well as during actual teaching.

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