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Araştırma / Research

EFFECTS OF DIFFERENT OPTIMIZATION METHODS ON THE PREDICTIONS OF YLD2000 YIELD CRITERION COEFFICIENTS

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ABSTRACT

The improved yield criteria are generally used in the finite element simulations of plastic deformation processes. Calculation accuracies of these criteria coefficients result successful simulation outcomes. In this study, the coefficients of the YLD2000 yield criterion are calculated by three most widely used optimization methods in literature, namely the least squares, nonlinear conditional optimization, and genetic algorithm methods. Two different aluminum alloys, AA7003-T6 and AA6063-T6 are selected to verify the prediction results. Results reveal that the nonlinear conditional optimization and genetic algorithm methods are very dependent on the initial values. Therefore, different result is determined for each different case. For this reason, it has been concluded that the least squares method should be preferred to calculate the coefficients of the yield criterion by using optimizing method.

Keywords: YLD2000 yield criteria, coefficients of yield criterion, optimization, AA7003-T6, AA6063-T6

FARKLI OPTİMİZASYON YÖNTEMLERİNİN YLD2000 AKMA KRİTERİ KATSAYILARININ TAHMİNLERİNE ETKİLERİ

ÖZ

Plastik deformasyon proseslerinin sonlu elemanlar simülasyonlarında genellikle gelişmiş akma kriterleri kullanılmaktadır. Bu kriterlerin katsayılarının doğru hesaplanması simülasyonun sonuçlarının başarısına etki etmektedir. Bu çalışmada literatürde en çok kullanılan üç optimizasyon yöntemlerinden en küçük kareler, nonlineer şartlı optimizasyon ve genetik algoritma kullanılarak, YLD2000 akma kriterinin katsayıları hesaplanmıştır. Tahmin edilen sonuçları doğrulamak için iki farklı alüminyum alaşımı seçilmiştir. Elde edilen sonuçlara göre nonlineer şartlı optimizasyon ve genetik algoritma yöntemlerinin girilen başlangıç değerlerine çok bağlı olduğu ve her farklı durum için farklı sonuçlar verdiği tespit edilmiştir. Bu nedenle akma kriterlerinin katsayılarının optimizasyon medodu ile hesaplanması işlemlerinde en küçük kareler yönteminin tercih edilmesi gerektiği sonucuna varılmıştır.

Anahtar kelimeler: YLD2000 akma kriteri, akma kriteri katsayıları, optimizasyon, AA7003-T6, AA6063-T6

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1. INTRODUCTION

Sheet metals are produced by rolling process applying on blooms. The rolling process generally causes significant anisotropy, orthogonal anisotropy particularly. Sheet metals axis orientations are as follows: rolling direction (RD), transverse direction (TD), normal direction (ND) as described in Figure 1.

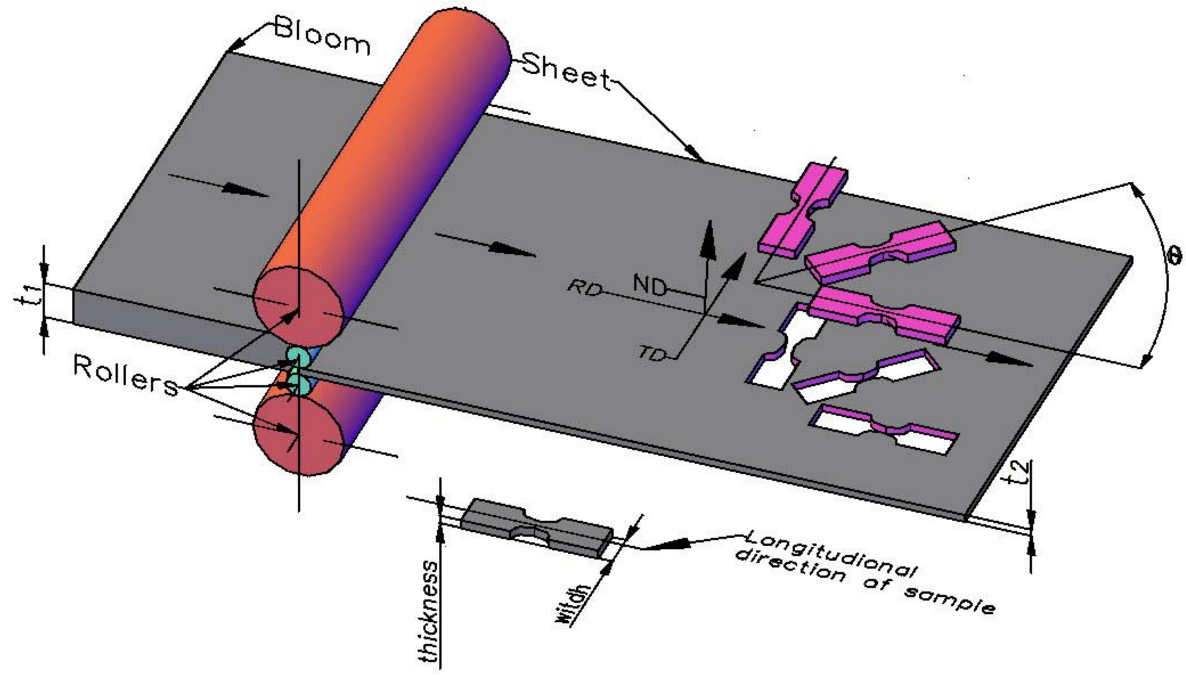


Figure 1. Sheet metal anisotropy and test samples for uniaxial tensile tests.

A yield criterion is an important part of a plasticity model. It has a crucial role to provide knowledge of whether or not yield starts in a plastic deformation simulation performed by a finite element software. Although there are lots of criteria to predict yield condition, efforts on their development are still an ongoing process due to more complexity of a plastic deformation process. A plastic deformation process includes not only yield but also continuous material flow and strain hardening (or softening) based on dislocation movements on slip planes according to crystal lattice structure during deformation. These all three phenomena can be incorporated to simulation by a plasticity model. A proper selection of the plasticity model will have a great impact on simulation results.

A yield criterion is a mathematical function in nature. That function consists of several material constants in general and it creates a closed yield surface on the principle stresses diagram. That surface becomes the border whether plastic deformation starts. Generally, more than one stress component occurs in the structural member during combined loading. By a yield function, it is aimed to convert all stress components into one stress term named as equivalent stress to be able to make comparison with the unique stress named as yield stress obtained from uniaxial tensile test. For isotropic and homogeneous materials, one yield point will be enough for comparison because all mechanical properties are the same at all directions and the yield point can be determined by a tensile test easily. But it is not same for anisotropic materials. Yield point and other mechanical properties vary with direction. It leads to highly anisotropic behavior and different yield points for different orientations. It is almost impossible to determine the yield points experimentally at every different angle with respect to rolling direction (RD). Every one of the experiment just results in a point on the principle stresses space. A curve must be fitted between and by passing these points to create a closed yield surface. So, a full function passing through experimental points is fitted by yield function. Prediction performances of these functions affects the accuracy of simulation directly.

Each yield criterion based on a mathematical function produces its own equivalent stress $\bar{\sigma}$. The concept of equivalent stress first began to be used with the maximum distortion energy criterion presented by von Mises. The function of von Mises draws a completely elliptical yield surface in the principal axis space. However, this criterion is only suitable for ductile and isotropic materials. The first anisotropic yield function is known the Hill48 criterion by Hill (1948) [1]. This function is a quadratic equation and includes anisotropy coefficients. It predicts sheet metal anisotropy accurately. Hill (1993) continued to improve this criterion and subsequently proposed the Hill 1993 criterion [2]. Both are useful for materials having anisotropic behavior along three orthogonal symmetry planes. Barlat et al. (2003) have presented a six component anisotropic yield function known YLD91, YLD94, and YLD96 [3-5]. Then this function was developed and presented as YLD2000-2D for plane stress state in aluminum alloy sheet metal by Barlat (2005) [6]. It is suitable for orthotropic anisotropic materials. Later, Aretz (2004) developed YLD2003 criterion based on the Barlat YLD2000-2D [7]. This criterion was also developed by Barlat (2005) using more accurate flow and consolidation curves and presented as anisotropic yield criterion known as YLD2004. The criterion involves two different linear transformations of the deviatoric stress tensor [8].

In this study, the YLD2000 yield criterion and its parameters were studied. Three optimization methods, the least squares method, nonlinearly constrained optimization, and genetic algorithm were used to find coefficients of yield criterion. Newton Raphson method was used to investigate effects of parameters.

2. MATERIAL AND METHOD

In the previous study, yield surfaces were obtained for AA7003-T6 and AA6063-T6 aluminum alloys by using YLD2000 yield criterion [9]. The performances of the yield criteria on the prediction of yield strength and anisotropy coefficient at different angle with respect to RD were compared. Their functions were nestled $Y(\theta)$ and $r(\theta)$ equations where Y is yield strength and r is anisotropy coefficient.

YLD2000 criterion is reduced to plane stress case as YLD2000-2D. General formula of this criterion is in Eq. (1).

$$f = \emptyset - 2(\bar{\sigma})^m = 0 \tag{1}$$

where $\emptyset$ is a mathematical expression to be a function of this yield criterion. Barlat's main idea to obtain an anisotropic function is that two isotropic functions can be added together $\emptyset = \emptyset' + \emptyset''$. $\bar{\sigma}$ is the equivalent stress produced by this yield function. All comparisons are made by using $\bar{\sigma}$ not $\emptyset$. m is an exponent depending on the microstructure (6 for BCC and 8 for FCC crystal lattice). Function $\emptyset'$ and $\emptyset''$ are defined as in Eq. (2).

$$\begin{aligned} \emptyset' &= \left|\tilde{S}_1' - \tilde{S}_2'\right|^m \\ \emptyset'' &= \left|2\tilde{S}_1'' + \tilde{S}_2''\right|^m + \left|\tilde{S}_1'' + 2\tilde{S}_2''\right|^m \\ \emptyset(\tilde{S}', \tilde{S}'') &= \left|\tilde{S}_1' - \tilde{S}_2'\right|^m + \left|2\tilde{S}_2'' + \tilde{S}_1''\right|^m + \left|2\tilde{S}_1'' + \tilde{S}_2''\right|^m \end{aligned} \tag{2}$$

where $\tilde{S}_i'$ and $\tilde{S}_i''$, i =1, 2, 3 are the eigenvalues of two linear transformed stress tensors $\tilde{S}'$and $\tilde{S}''$ respectively in the context of mathematics. The eigenvalues correspond principle stresses of deviatoric stress tensor in the context of mechanics.

$$\mathbf{S} = \sigma - \frac{1}{3}\mathbf{I_1}\delta_{ij} \tag{3}$$

where $\mathbf{I_1}$ is the first stress invariant and δ_{ij} is the kronecker delta. This criterion includes two linear transformations of deviatoric stress tensor, **S** which is a part of Cauchy stress tensor [10]. When a yield criterion is written in terms of deviatoric component of a stress state, it fulfills the pressure independence condition. While deviatoric stress causes to plastic deformation, hydrostatic stress causes to just volumetric change. The transformations are in the general form of $\tilde{\boldsymbol{S}} = \mathbf{CS}$ where $\mathbf{S}$ is the deviatoric stress tensor and $\tilde{\boldsymbol{S}}$ is the deviatoric stress tensor after linear transformation and $\mathbf{C}$ is linear transformation matrix. To obtain two different functions, two different linear transformations can be set as,

$$\tilde{\boldsymbol{S}}' = \mathbf{C}'\mathbf{S} \text{ and } \tilde{\boldsymbol{S}}'' = \mathbf{C}''\mathbf{S} \tag{4}$$

[.]' and [.]'' superscripts indicate two different linear transformations. Transformation matrices **C'** and **C''** are fully defined in Eq. (5-*a*, *b*).

$$\begin{Bmatrix} \tilde{S}'_{xx} \\ \tilde{S}'_{yy} \\ \tilde{S}'_{xy} \end{Bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & 0 \\ C'_{21} & C'_{22} & 0 \\ 0 & 0 & C'_{66} \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \end{Bmatrix} \tag{5-a}$$

$$\begin{Bmatrix} \tilde{S}''_{xx} \\ \tilde{S}''_{yy} \\ \tilde{S}''_{xy} \end{Bmatrix} = \begin{bmatrix} C''_{11} & C''_{12} & 0 \\ C''_{21} & C''_{22} & 0 \\ 0 & 0 & C''_{66} \end{bmatrix} \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \end{Bmatrix} \tag{5-b}$$

where rolling direction is indicated by *xx*, and transvers direction is *yy*, normal direction is *zz*. those relations can also be written with respect to Cauchy stress as in Eq. (6).

$$\tilde{\boldsymbol{S}}' = \mathbf{C}'\mathbf{S} = \mathbf{C}'\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}'\boldsymbol{\sigma} \text{ and similarly } \tilde{\boldsymbol{S}}'' = \mathbf{C}''\mathbf{S} = \mathbf{C}''\mathbf{T}\boldsymbol{\sigma} = \mathbf{L}''\boldsymbol{\sigma} \tag{6}$$

where **T** is a matrix relating the deviator of the Cauchy stress tensor.

$$T = \begin{bmatrix} 2/3 & -1/3 & 0 \\ -1/3 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$\begin{Bmatrix} \tilde{S}'_{xx} \\ \tilde{S}'_{yy} \\ \tilde{S}'_{xy} \end{Bmatrix} = \begin{bmatrix} L'_{11} & L'_{12} & 0 \\ L'_{21} & L'_{22} & 0 \\ 0 & 0 & L'_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \tag{8-a}$$

where,

$$L'_{11} = \frac{2\propto_1}{3},\ L'_{12} = -\frac{\propto_1}{3},\ L'_{21} = -\frac{\propto_2}{3},\ L'_{22} = -\frac{2\propto_2}{3},\ L'_{66} = \propto_7 \tag{8-b}$$

So, a plane stress state can be described by two principal values.

$$\tilde{s}'_{1,2} = \frac{\tilde{s}'_{xx}+\tilde{s}'_{yy}}{2} \mp \sqrt{\left(\frac{\tilde{s}'_{xx}-\tilde{s}'_{yy}}{2}\right)^2 + \tilde{s}'^{\,2}_{xy}} \tag{8-c}$$

Similarly,

$$\begin{Bmatrix} \tilde{s}''_{xx} \\ \tilde{s}''_{yy} \\ \tilde{s}''_{xy} \end{Bmatrix} = \begin{bmatrix} L''_{11} & L''_{12} & 0 \\ L''_{21} & L''_{22} & 0 \\ 0 & 0 & L''_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} \tag{8-d}$$

where,

$$\begin{aligned} L''_{11} &= \frac{8\propto_5-2\propto_3-2\propto_6+2\propto_4}{9} \\ L''_{12} &= \frac{4\propto_6-4\propto_4-4\propto_5+\propto_3}{9} \\ L''_{21} &= \frac{4\propto_3-4\propto_5-4\propto_4+\propto_6}{9} \\ L''_{22} &= \frac{8\propto_4-2\propto_6-2\propto_3+2\propto_5}{9} \\ L''_{66} &= \propto_8 \end{aligned} \tag{8-e}$$

And similarly, the plane stress state can be described by two principal values.

$$\tilde{s}''_{1,2} = \frac{\tilde{s}''_{xx}+\tilde{s}''_{yy}}{2} \mp \sqrt{\left(\frac{\tilde{s}''_{xx}-\tilde{s}''_{yy}}{2}\right)^2 + \tilde{s}''^{\,2}_{xy}} \tag{8-f}$$

As seen from Eq. (8-*f*), there are totally 8 unknown coefficient, $\alpha_1 - \alpha_8$ of this criterion. Since these coefficients vary from material to material, they must be determined individually for each material. It is necessary to find these eight unknowns.

If at least 8 yield strengths are available, by writing the yield strength values for the equivalent stress $\bar{\sigma}$ in the YLD2000 equation, a total of 8 equations will be obtained mathematically. 7 of them can be obtained from the tests performed at different angles like $\sigma_0, \sigma_{15}, \sigma_{30}, \sigma_{45}, \sigma_{60}, \sigma_{75}, \sigma_{90}$ and the last one can be σ_b where σ_b is the out-of-plane shear stress value obtained on the basis of the test called biaxial stretching.

However, since the anisotropy equation includes YLD2000 yield function by means of the associated flow rule, 4 experiments are sufficient. Because four yield strengths $\sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b$ and four experimental anisotropy values r_0, r_{45}, r_{90}, r_b will be obtained from just 4 tests, where the value indicated by r_b is the anisotropy value obtained from the experiment called biaxial stretching. How the anisotropy equation was obtained? was explained in the following sections. It leads to a set of 8 equations with 8 unknowns from 8 experimental data as seen in Eq. (9). Equations are nonlinear. So, the problem becomes finding parameters, $\alpha_1 - \alpha_8$.

$$\begin{aligned}
\bar{\sigma}(\alpha_1, \ldots, \alpha_8)|_{\theta=0°} &= \sigma_{0°}^{experimental} \\
\bar{\sigma}(\alpha_1, \ldots, \alpha_8)|_{\theta=45°} &= \sigma_{45°}^{experimental} \\
\bar{\sigma}(\alpha_1, \ldots, \alpha_8)|_{\theta=90°} &= \sigma_{90°}^{experimental} \\
\bar{\sigma}(\alpha_1, \ldots, \alpha_8)|_{biaxial} &= \sigma_{b}^{experimental} \\
r(\alpha_1, \ldots, \alpha_8)|_{\theta=0°} &= r_{0°}^{experimental} \\
r(\alpha_1, \ldots, \alpha_8)|_{\theta=45°} &= r_{45°}^{experimental} \\
r(\alpha_1, \ldots, \alpha_8)|_{\theta=90°} &= r_{90°}^{experimental} \\
r(\alpha_1, \ldots, \alpha_8)|_{biaxial} &= r_{b}^{experimental}
\end{aligned} \tag{9}$$

A numerical optimization method was used in order to get the solutions. In literature, the Newton Raphson method is generally used for simultaneous solution. Tensile and biaxial tensile test data for AA7003-T6 and AA6063-T6 sheet metal alloys were used as experimental data in order to have a stress for comparison with the equivalent stress value. The rolling direction was selected as the reference direction and was indicated by the $[.]_0$ subscript.

The data are normally normalized by dividing σ_0 which is the yield stress in the rolling direction so that we can see the difference much better. Since both aluminium alloys face-centered cubic, *m* was taken as 8.

Table 1. Experimental data of yield strength and anisotropy [11].

Parameter	AA7003-T6	AA6063-T6
σ_0/σ_0	*1.000*	*1.000*
σ_{15}/σ_0	*0.970*	*0.917*
σ_{30}/σ_0	*0.980*	*0.923*
σ_{45}/σ_0	*0.840*	*0.990*
σ_{60}/σ_0	*0.863*	*0.983*
σ_{75}/σ_0	*0.967*	*1.027*
σ_{90}/σ_0	*1.037*	*0.957*
σ_b/σ_0	*1.000 (*)*	*1.000 (*)*
r_0	*0.270*	*0.567*
r_{15}	*0.427*	*0.333*
r_{30}	*1.017*	*0.227*
r_{45}	*2.073*	*0.340*
r_{60}	*1.780*	*0.707*
r_{75}	*1.310*	*1.227*
r_{90}	*1.283*	*2.857*
r_b	*0.570*	*0.480*
m	*8*	*8*

(*) Assumed values due to lack of data in cited references

2.1. Numerical Optimizations

Numerical methods are generally used to predict and search for a point of a function by using methods of finding root location or optimization. In this study, the optimization method was used. Optimization method involves searching for either a minimum or a maximum or a target. Main idea for our optimization procedure is to create an objective function first (an error function in our case, Eq.(10)), and later, try to minimize it up to zero as the most desired case. The process is also called "root finding" because right hand side of the equations are equal to zero. When the error is minimized enough, predicted values are accepted as the searching points.

The general forms of the objective function that can used for the coefficients of yield criterion are given in Eq. (10). These are all nonlinear equations.

$$Error = \sigma_\theta^{predicted}(\alpha_1, \ldots, \alpha_8) - \sigma_\theta^{experimental} \qquad \theta = 0°, 45°, 90° \text{ and } \sigma_b$$
$$Error = r_\theta^{predicted}(\alpha_1, \ldots, \alpha_8) - r_\theta^{experimental} \qquad \theta = 0°, 45°, 90° \text{ and } r_b \tag{10}$$

where $\sigma^{experimental}$ is taken from Table 1 obtained from tensile tests. Actually, $\sigma^{predicted}$ can be threaten as equivalent stress $\bar{\sigma}|_\theta$ of the yield functions defined in Eq. (1) where equivalent stress formula have already included unknown coefficients of the yield function inherently. "θ" may become 0°, 15°, 30°, 45°, 60°, 75°, 90°, and *biaxial.* It depicts the number of term in the objective function. In optimization process, more or less terms (input variables) can be used to define an objective function. The number of input variables may affect the accuracy of prediction. For all objective functions above, the minimization was applied for all terms as a goal of the optimization.

Unknown coefficients of YLD2000, $\alpha_1, \ldots, \alpha_8$ were obtained after optimization. Depending on optimization method, there are some differences between predicted values. So, for comparison, three optimization methods were investigated, the least squares method, nonlinearly constrained optimization, and genetic algorithm.

2.1.1. Nonlinear Least Squares

As an optimization method, a nonlinear algorithm has to be used due to nonlinear nature of YLD2000's equivalent stress term $\bar{\sigma}|_\theta$ and anisotropy coefficient r_θ. This method is based on minimization of summation of error square, iteratively [12]. Iterative formula is given in Eq. (11) for our function.

$$Error = \left(\sigma_\theta^{predicted}(\alpha_1, \ldots, \alpha_8)\right)_i - \left(\sigma_\theta^{experimental}\right)_i$$
$$Error = \left(r_\theta^{predicted}(\alpha_1, \ldots, \alpha_8)\right)_i - \left(r_\theta^{experimental}\right)_i \qquad \theta = 0°, 45°, 90° \text{ and } r_b \tag{11}$$

where *i* is iteration number and *N* is total iterations. Optimization functions are stated in Eq. (12).

$$\sum_{i=0}^{N} Error_i^2 = \sum_{i=0}^{N} \left[\left(\sigma_\theta^{predicted}(\alpha_1, \ldots, \alpha_8)\right)_i - \left(\sigma_\theta^{experimental}\right)_i\right]^2$$
$$\sum_{i=0}^{N} Error_i^2 = \sum_{i=0}^{N} \left[\left(r_\theta^{predicted}(\alpha_1, \ldots, \alpha_8)\right)_i - \left(r_\theta^{experimental}\right)_i\right]^2 \tag{12}$$

This method applies its own algorithm to find the unknown coefficient vector $\{\alpha_1, \ldots, \alpha_8\}^T$ to do $\sum_{i=0}^{N} Error_i^2 = 0$. Initial value(s) must be defined in this method. Any initial value can be selected technically. But, it is recommended that initial values are set as closer as to experimental data (like Table 1). Otherwise, bigger difference may lead to more deviation on iteration results of optimization algorithm. Iterative solution is finished when iteration number or error tolerance value achieve a certain value. This method takes longer solution time than others. A package program is usually used to find the minimums [13].

2.1.2. Nonlinearly Constrained Optimization

This method is suitable to find the minimum point of a nonlinear multivariable function subjected to constrains which can be linear inequality constraints, linear equality constraints, lower bounds, upper bounds, and nonlinear constraints. Iterative formula is given in Eq. (13) for our function.

$$\underset{Error}{minimum}\left[Error(\sigma, \alpha) = \bar{\boldsymbol{\sigma}}_i\left(\sigma_\theta^{predicted}, \alpha_1, \ldots, \alpha_8\right) - \left(\sigma_\theta^{experimental}\right)_i\right] \tag{13}$$

This method finds minimum of *f*(*Error*). It is very fast optimization method. Method allows not only scalar variables, but also, vector and matrices inside function and constrain equations. Function and/or constrains may also be nonlinear. Neither constrains nor bounds and conditions are applied. Similarly, in this method, initial values must be chosen for the parameters.

2.1.3. Genetic algorithm

In the nature, individuals are randomly selected from a population and used as parents to form future generations in the cycle of biological evolution. That natural selection process is mimed by genetic algorithm (GA) method. The algorithm iteratively modifies a population of points at each iteration. At each iteration, the algorithm selects points from the current population randomly and uses them as parents to produce the child points for the next iteration. At the end of successive iterations, the population "evolves" toward an optimal solution. The best point in the population is selected as an optimal solution. So, the algorithm is suitable for optimization problems [14]. It doesn’t matter whether objection function is constrained or unconstrained. No initial value is required for solution, and defining a range for roots is not compulsory.

Especially, this method is suitable to solve optimization problems in which the objective or constraint function is continuous, discontinuous, stochastic, does not possess derivatives, or includes simulations or black-box functions. Therefore, it can give very good results in cases where solution cannot be obtained with other optimization methods. Our flow chart of the genetic algorithm is given in Figure 2.

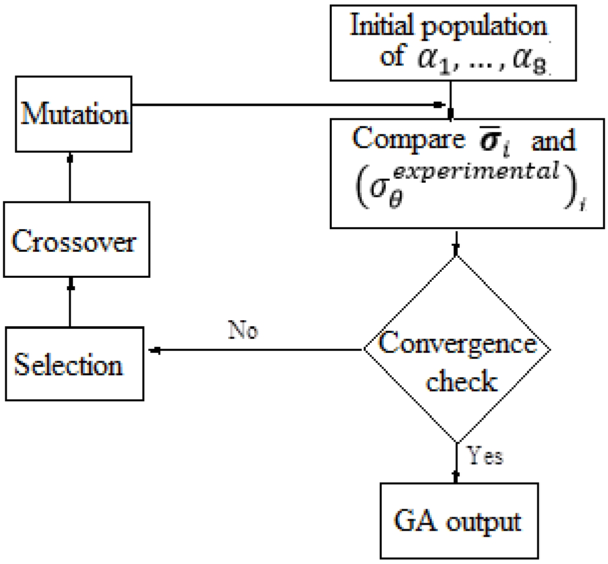


Figure 2. Flow chart of a genetic algorithm (adapted from [15]).

The following parameters were selected as optimization parameters.

- Population size was taken as 100000.
- Number of elite children was taken as %5 of population size
- Crossover fraction was taken as 0,8
- Migration among subpopulations was taken as 0,2

Four different cases were investigated. No initial value was defined for all cases. Root ranges were defined as [-2, 2] for case 1, [-10, 10] for case 2, [0, 1] for case 3, [0, 2] for case 4. Following steps were also selected as our genetic algorithm options.

- GA has uniform creation.
- Fitness scaling is rank-based.
- Members inside population are selected by using stochastic uniform method selection. Members are crossing over in scattered way.
- Mutation is uniform in GA.

3. RESULTS AND DISCUSSIONS

3.1. Yield Surfaces

First, the nonlinear *least squares method* was applied. For this method, one initial value set must be defined first. Two sets of initial values were used in optimization method to compare the effects of different initial values, as seen in Table 2 to find (optimize) totally 8 unknowns, $\alpha_1 - \alpha_8$.

Table 2. Initial value sets for the nonlinear least squares method.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-2	-2	-2	-2	-2	-2	-2	-2
Case 2	2	2	2	2	2	2	2	2

For AA7003-T6, unknowns (coefficients of YLD2000 criterion) were optimized up to error tolerance, 1e-24 and given in Table 3.

As seen, in both cases, all calculated unknowns were less than initial value, 2 and -2 absolutely. Each one of the α, has almost the same magnitude absolutely but sense differs with respect to initial values. While, all parameters possess negative sign in the case 1, all has positive sign in the case 2. Solution time became between 60-150 minutes at the same computer and processing conditions.

Table 3. Optimized values of unknowns in the YLD2000 criterion by the nonlinear least squares method for AA7003-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-0.7142	-1.1472	-1.050	-0.945	-1.0437	-0.968	-1.2237	-1.2928
Case 2	0.7141	1.1472	1.047	0.9446	1.0437	0.968	1.2237	1.2929

By writing the unknown coefficients $\alpha_1 - \alpha_8$ inside the function of YLD2000 criterion, yield surfaces can be drawn on the principle stresses space as displayed in Figure 3 for AA7003-T6. Data are generally given as normalized by dividing to the yield stress σ_0 on RD to show the difference between each other. It is seen that the curves passes just on experimental points as expected. The same yield surfaces were obtained for both case 1 and 2 unexpectedly. Both curves have a little bit distorted elliptical shape. While von Mises's criterion draws a fully elliptical shape as an isotropic yield criterion, the YLD2000 criterion draws a distorted elliptical shape due to its anisotropic ability as expected.

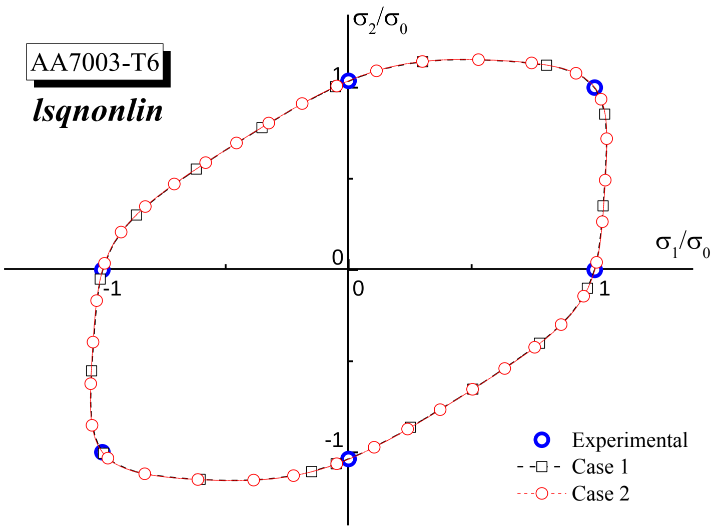


Figure 3. Yield surfaces obtained from the YLD2000 criterion for different initial value sets for AA7003-T6.

The nonlinear least squares method was also applied for the unknowns, $\alpha_1 - \alpha_8$ by using the same initial values as seen in Table 2 for AA6063-T6. Approximate solutions were found by the nonlinear least squares method up to error tolerance, 1e-24 are given in Table 4. Similarly, as seen, in both cases, all calculated unknowns were less than initial value, 2 and -2 absolutely. Each one of the α, has almost the same magnitude absolutely but sense differs with respect to initial values. While, all parameters possess negative sign in the case 1, all has positive sign in the case 2.

Table 4. Optimized values of unknowns in the YLD2000 criterion by the nonlinear least squares method for AA6063-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-0.7422	-1.3178	-1.0906	-0.9061	-1.0356	-1.0079	-0.8981	-1.1436
Case 2	0.7422	1.3179	1.0907	0.9061	1.0356	1.0079	0.8981	1.1436

For AA6063-T6, optimized values of unknowns in the YLD2000 criterion by the nonlinear least squares method were used inside Eq. (1) and yield surfaces were drawn as in Figure 4. For both cases, curves were overlapped as that of AA7003-T6.

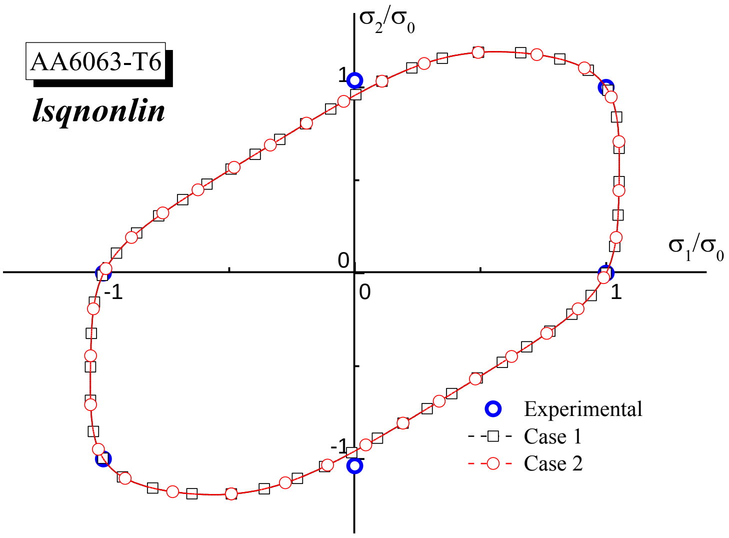


Figure 4. Yield surfaces obtained from the YLD2000 criterion for different initial value sets for AA6063-T6.

For another optimization method, the *nonlinearly constrained optimization* was used to find (optimize) those 8 unknowns, $\alpha_1 - \alpha_8$ of the YLD2000 criterion. For this method, one initial value set must be defined first. Three sets of initial values were used in optimization method to compare the effects of different initial values as seen in Table 5. Error tolerance was set to 1e-24, and the number of iterations was set to 1e8. Lower or upper bounds were not used.

Table 5. Initial value sets for the nonlinearly constrained optimization method.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5	-2.5
Case 2	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5	-0.5
Case 3	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5

For AA7003-T6, unknowns (coefficients of YLD2000 criterion) were optimized as in Table 6. As seen, in each cases, all calculated unknowns were less than initial value, 2 and -2 absolutely. Each one of α, did not have the same magnitude, it depends on initial values. Sense of α did not have the same sign with its initial value.

Solution time became between 5-10 minutes at the same computer and processing conditions. A consequence of this is that estimation of initial parameter should be as close as practicable to their (unknown!) optimal values.

Table 6. Optimized values of unknowns in the YLD2000 criterion by the nonlinearly constrained optimization method for AA7003-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-1.9538	0.6413	-1.626	-0.7881	0.0704	-1.5064	-1.1937	-1.272
Case 2	-0.7141	-1.1472	-1.047	-0.9446	-1.0437	-0.968	-1.2237	-1.2929
Case 3	1.9538	-0.6413	1.626	0.7881	-0.0704	1.5064	1.1937	1.272

Using those found unknown coefficients $\alpha_1 - \alpha_8$, function of the YLD2000 criterion can be drawn on the principle stresses space as shown in Figure 5 for AA7003-T6. It is seen that curve of function passes just on experimental points as expected. While case 1 and 3 gave the same yield surface, that of case 2 was different.

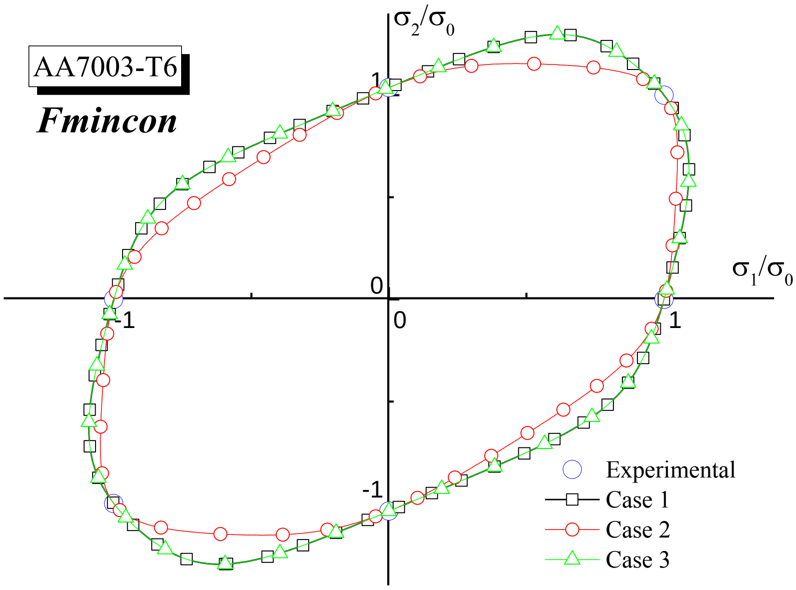


Figure 5. Yield surfaces obtained from the YLD2000 criterion for different initial value sets for AA7003-T6.

For optimization of unknowns of the YLD2000 criterion for AA6063-T6, the same initial value sets were used as in Table 5. Error tolerance, iteration values were taken the same as 1e-24 and 1e8 respectively. Approximate solutions found are given in Table 7. The calculated coefficients had different magnitude and sense in all three cases. When compared case 1 and 3 where magnitudes were the same but senses were different, no similarity was seen.

Table 7. Optimized values of unknowns in the YLD2000 criterion by the nonlinearly constrained optimization method for AA6063-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-0.7422	-1.3179	-1.0907	-0.9061	-1.0356	-1.0079	-0.8981	-1.1436
Case 2	-1.8869	0.5662	0.55	-0.7168	-0.8724	-1.5066	-0.4898	-1.4435
Case 3	0.5506	1.4339	1.7233	0.7515	0.9461	0.5767	-0.8751	1.1940

In the yield surface diagram in Figure 6, while case 2 and 3 gave the overlapped curves, that of case 1 was different.

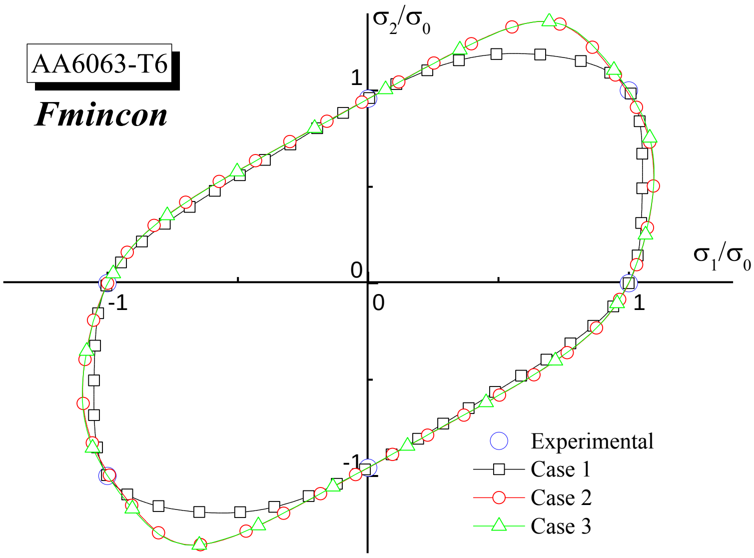


Figure 6. Yield surfaces obtained from the YLD2000 criterion for different initial value sets for AA6063-T6.

Another optimization method, the *genetic algorithm*, was used to find (optimize) those 8 unknowns. Four different cases were investigated. No initial value was defined for all cases. Error tolerance was set to 1e-24. Roots ranges were defined as [-2, 2] for case 1, [-10, 10] for case 2, [0, 1] for case 3, [0, 2] for case 4 as seen in Table 8. Computer processing time was approximately 5-10 minutes at the same computer and processing conditions.

Table 8. Root ranges for genetic algorithm method.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	-2	-2	-2	-2	-2	-2	-2	-2
	+2	+2	+2	+2	+2	+2	+2	+2
Case 2	-10	-10	-10	-10	-10	-10	-10	-10
	+10	+10	+10	+10	+10	+10	+10	+10
Case 3	0	0	0	0	0	0	0	0
	+1	+1	+1	+1	+1	+1	+1	+1
Case 4	0	0	0	0	0	0	0	0
	+2	+2	+2	+2	+2	+2	+2	+2

For AA7003-T6, results were summurized in Table 9. As seen, the genetic algorithm abides by interval defined as range. Interval band is getting wider; absolute distribution of results is also getting wider as long as staying within range.

Table 9. Optimized values of unknowns in the YLD2000 criterion by the genetic algorithm method for AA7003-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	1.5823	-1.5832	1.9691	0.3321	0.0721	-1.5046	-1.1958	0.0007
Case 2	1.1732	-1.8023	2.0793	0.4312	-0.1802	1.3722	-1.192	1.1188
Case 3	0.8311	1	0.9813	0.9133	1	0.9253	1	0.8577
Case 4	0.1426	1.5051	1.9607	0.326	0.8071	1.5822	1.2359	1.3319

Yield functions of YLD2000 criterion were drawn on the principle stresses space as seen in Figure 7 for AA7003-T6. Case 1 and 4 were overlapped and curves of functions passed on experimental points as expected. Case 2's curve was not overlapped but passed on experimental points. But Case 3's curve exhibited totally different behavior, by neither passing on experimental point nor overlapping with any other curves.

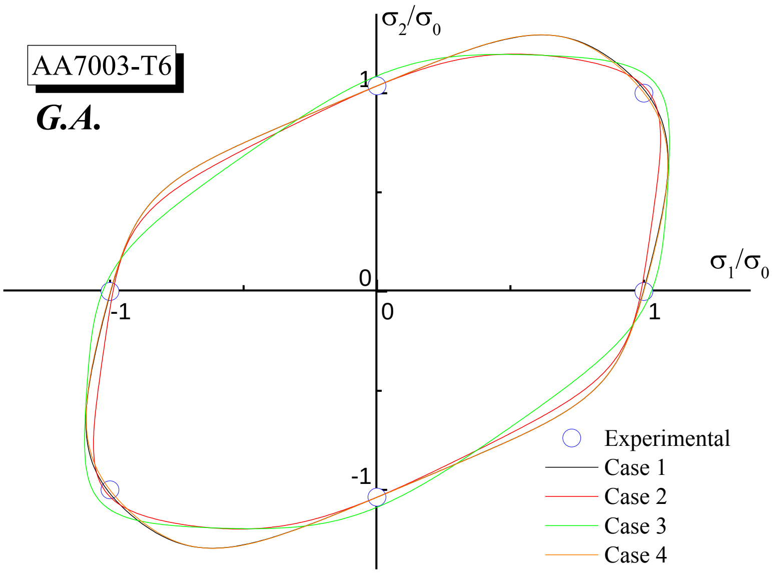


Figure 7. Yield surfaces obtained from the YLD2000 criterion for different ranges for AA7003-T6.

For AA6063-T6, the same initial value sets were used as in Table 8. The same genetic algorithm was used as for that of AA7003-T6. Approximate solutions found were given in Table 10. Magnitude of α may take various values greater than 2 depending range, but within range.

Table 10. Optimized values of unknowns in the YLD2000 criterion by the genetic algorithm method for AA6063-T6.

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
Case 1	1.1156	-1.8073	0.7193	-0.6737	-1.0398	-1.0542	0.6165	1.4568
Case 2	1.1138	1.1924	0.4401	0.5623	1.5099	2.8535	-1.0434	-0.0043
Case 3	0.9435	1	1	0.8804	1	0.981	0.8686	1
Case 4	0.7575	1.3035	1.0851	0.9099	1.0385	1.0114	0.8991	1.1415

As seen in Figure 8, curves of Case 1 and 4 were similar and the best fit with experimental points. But curve of case 2 was unsuccessful.

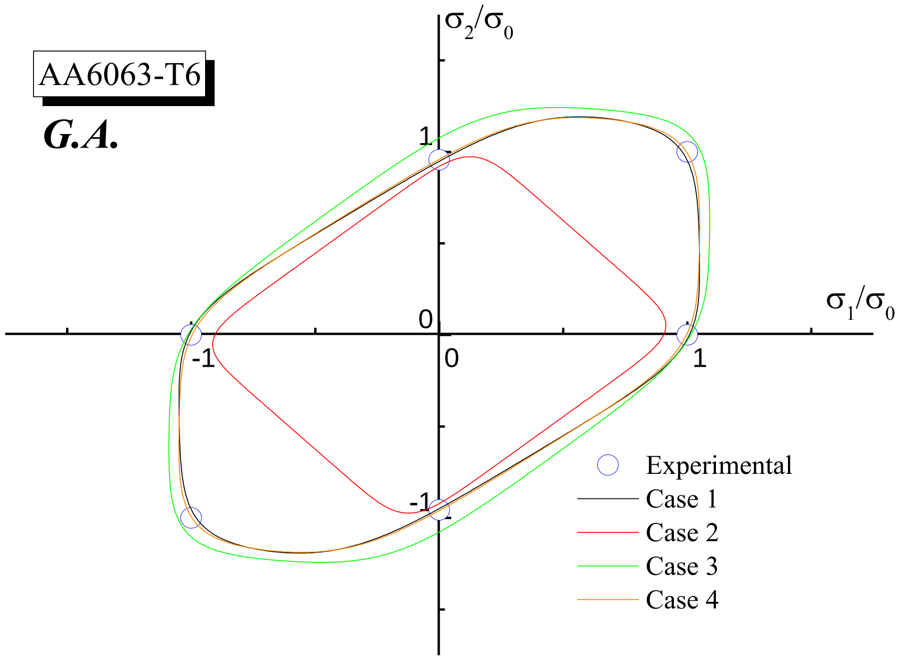


Figure 8. Yield surfaces obtained from the YLD2000 criterion for different initial value sets for AA6063-T6.

3.2. Prediction performances

While any isotropic material has one yield strength σ_y and one anisotropy coefficient *r*, rolling process causes to orthotropic anisotropy on sheet metals and experiments show that yield strength $\sigma_y|_\theta$ and anisotropy coefficients r_θ vary on plane directions. So, it is expected that an anisotropic yield criterion should provide two things mainly as appropriate as possible;

- yield point at any desired direction $\sigma_y|_\theta$,
- anisotropy coefficient at any desired direction, r_θ.

If a series of tensile specimens are cut as long as its longitudinal direction has any inclined angle between 0-90° with respect to RD, experimental r_θ and $\sigma_y|_\theta$ are obtained from uniaxial tensile tests. These experimental points are compared with predicted points (or curves) obtained from function of yield criterion. So, a function's performance is evaluated by comparisons.

A relation between yield point and yield criterion is set as $\sigma_y|_\theta = \sigma_0/F_\theta$ and similarly, a relation between anisotropy coefficient and yield criterion is set as $r_\theta = \frac{\bar{\sigma}_\theta}{\sigma_y|_\theta\left(\frac{\partial\bar{\sigma}}{\partial\sigma_{11}}+\frac{\partial\bar{\sigma}}{\partial\sigma_{22}}\right)_\theta} - 1$ by Banabic at all (2016) [16]. $\sigma_y|_\theta$means yield strength at inclined angle θ according to rolling direction. σ_0 is yield strength at rolling direction and taken as reference point. F_θ is a function depending on yield criterion. It is strongly recommended to read Ref [16].

Predictions from the YLD2000 yield function having coefficients that were given in Figure 9-15. In Figures, experimental points between angles 0-90° and curves of functions obtained from cases were compared. In the least squares method, the prediction curves of the case 1 and case 2 for AA7003-T6 and AA6063-T6 were overlapped as in Figure 9 and 10. The sense of initial value sets did not affect the curve shapes.

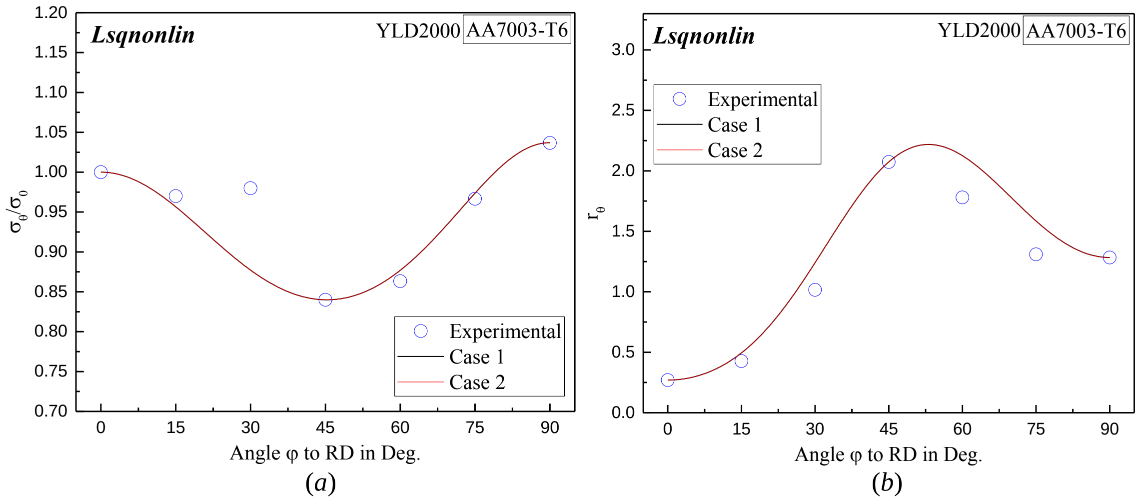


Figure 9. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of the YLD2000 function having coefficients obtained by the least squares method for AA7003-T6.

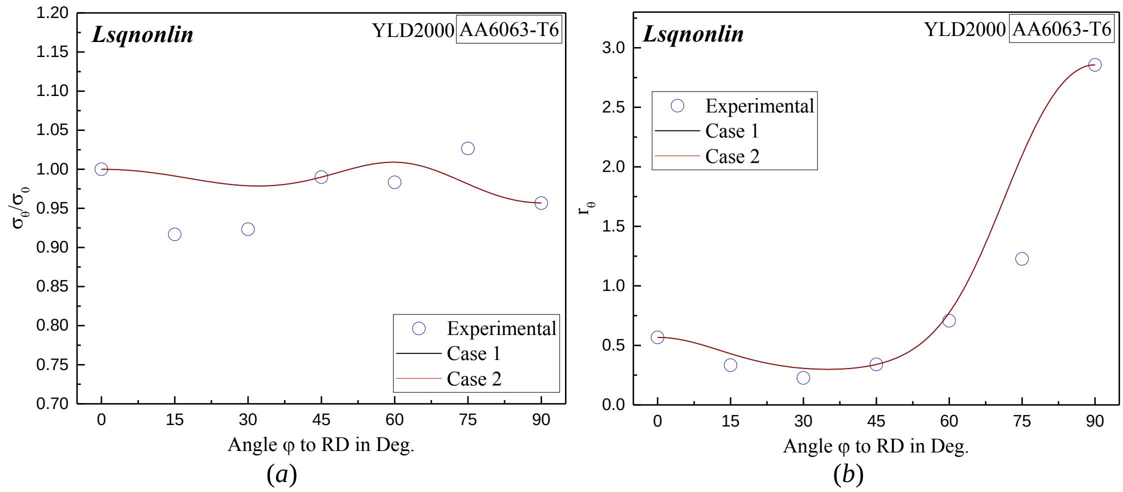
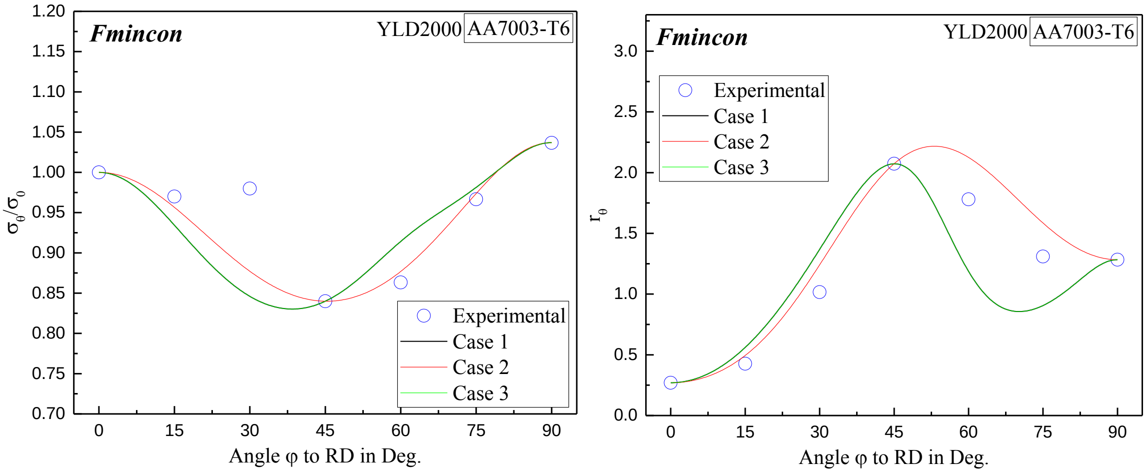


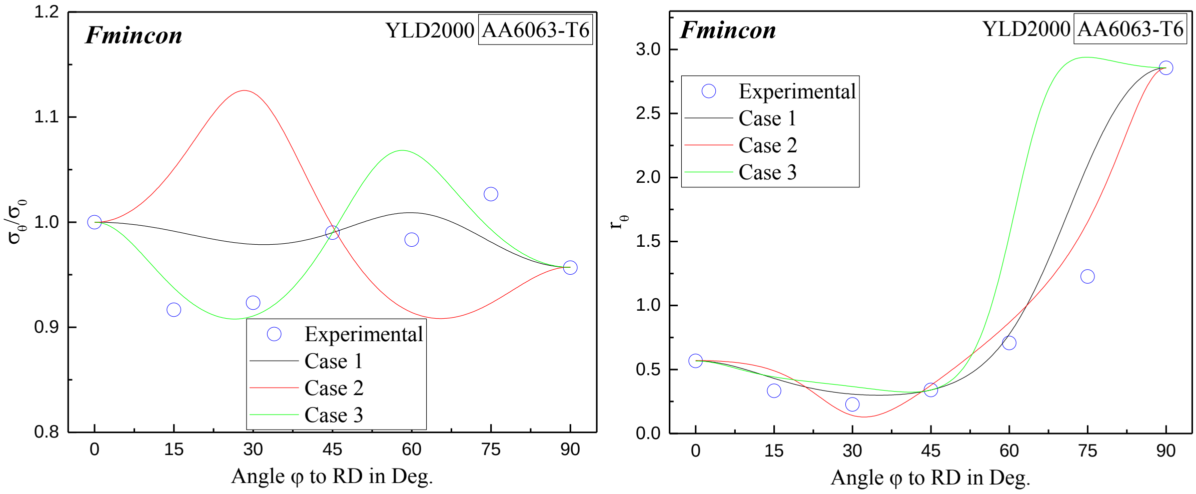
Figure 10. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of YLD2000 function having coefficients obtained by the least squares method for AA6063-T6.

In the nonlinearly constrained optimization method, prediction curves of the case 1 and case 2 for AA7003-T6 were overlapped as seen in Figure 11. But case 3 was completely different from case 1 and 2. Case 1 and case 2 were the best fit with experimental points. For AA6063-T6, while case 1 was agreeable and case 3 was the best fit, case 2 differed significantly in yield strength predictions as in Figure 12. Case 2 exhibited the best fit in anisotropy predictions. Case 1 and 2 had the closest curves to experimental points.



(*a*) (*b*)

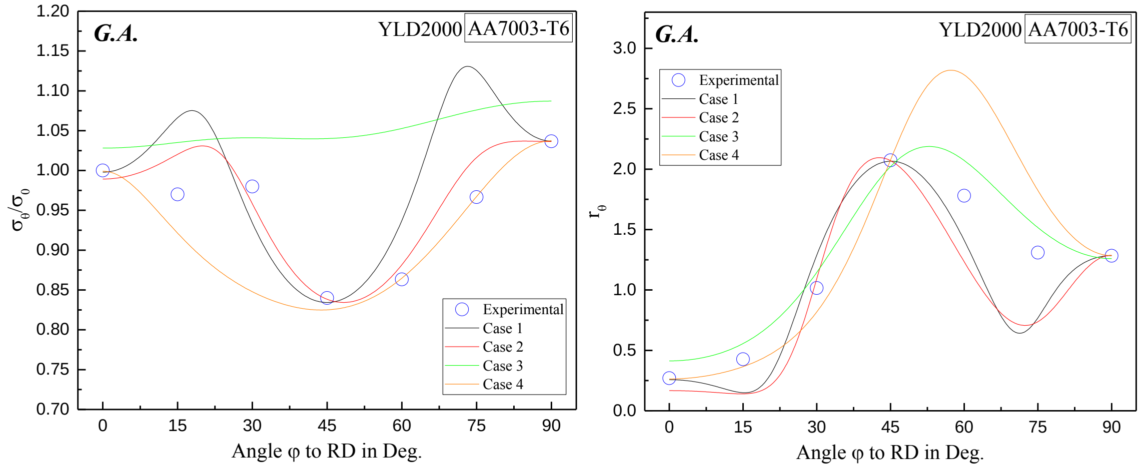
Figure 11. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of the YLD2000 function having coefficients obtained by the nonlinearly constrained optimization method for AA7003-T6.



(*a*) (*b*)

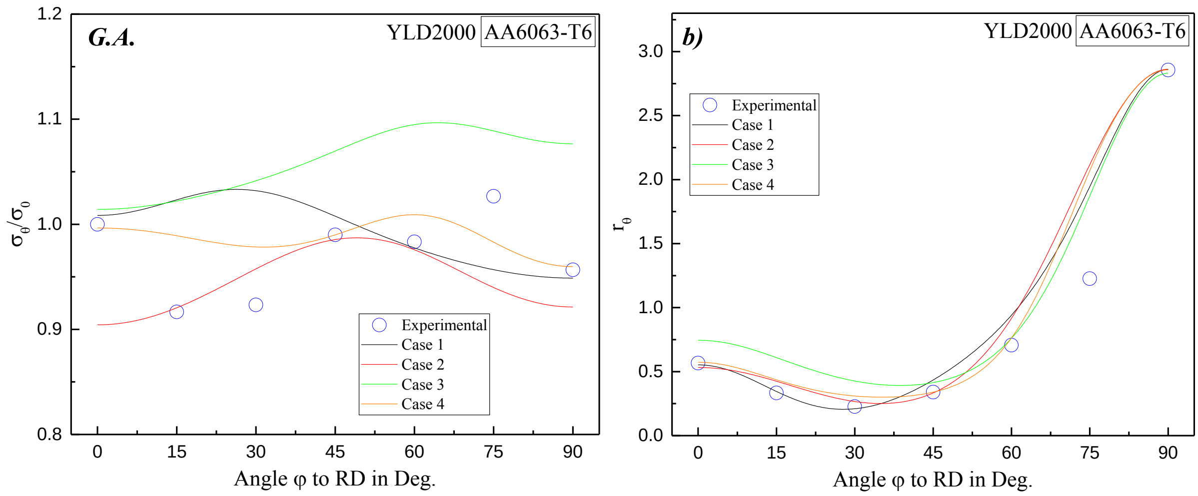
Figure 12. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of the YLD2000 function having coefficients obtained by the nonlinearly constrained optimization method for AA6063-T6.

In the genetic algorithm method, case 2 had the closest curve in yield strength predictions, just case 3 presented the closest curve in anisotropy predictions as in Figure 13 for AA7003-T6. In the genetic algorithm method, while almost all curves presented the best fit in anisotropy predictions, just case 4 presented the closest curve in yield strength predictions as in Figure 14 for AA6063-T6. Selected ranges had great effect on curve fitting.



(*a*) (*b*)

Figure 13. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of the YLD2000 function having coefficients obtained by the genetic algorithm method for AA7003-T6.



(*a*) (*b*)

Figure 14. (*a*) Yield strength and (*b*) anisotropy comparisons of experimental points in 0-90° with prediction curves of the YLD2000 function having coefficients obtained by the genetic algorithm method for AA6063-T6.

4. CONCLUSIONS

YLD2000 yield criterion is one of the most favorite anisotropic yield criterion used in plastic deformation simulations due to its easy use. In this study, the coefficients of the YLD2000 yield criterion were calculated for two aluminum alloys by three different optimization methods: the least squares, nonlinearly constrained optimization, and genetic algorithm. Different optimization conditions were investigated with different initial value sets and ranges. The main findings obtained from investigations were summarized as follow:

- Although the most computing time was consumed by the least squares method, the most consistent results were obtained without any dependency on initial values for both alloys.
- The success of the nonlinearly constrained optimization and genetic algorithm strongly depends on initial values or ranges. Theoretically, any initial value can be selected. But, it is recommended that initial values should be selected as close as possible to the provided experimental data. Otherwise, a bigger difference may cause more deviation on iteration results of optimization algorithm.

EFFECT OF DIFFERENT OPTIMIZATION METHODS ON THE ESTIMATIONS OF YLD2000 YIELD CRITERION COEFFICIENTS

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