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**Araştırma Makalesi**

# USING GLOBAL CRITERION METHOD TO DEFINE PRIORITIES IN LEXICOGRAPHIC GOAL PROGRAMMING AND AN APPLICATION FOR OPTIMAL SYSTEM DESIGN

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**Abstract**

While there is no certain method which provides solutions of Multiple Objective De Novo Programming problems, Multiple Objective Decision Making techniques can be applied for them. Therefore, goals have to be weighted and priorities have to be ranked for many methods. When the number of goal functions exceeds three, it is possible to get many different solution results. This is the first study to use Lexicographic Goal Programming for solutions of a Multi Objective De Novo Programming problem with positive ideal solutions. Additionally, the same problem was solved with Global Criteria Method, and the results were compared. The comparison concluded that Global Criteria Method could be used for priority ranking among the goals in Lexicographic Goal Programming.

**Keywords:** De Novo Programming, Global Criterion Method, Lexicographic Goal Programming.

## ÖNCELİKLİ HEDEF PROGRAMLAMADA ÖNCELİKLERİN BELİRLENMESİNDE GLOBAL KRİTER YÖNTEM KULLANIMI VE OPTİMAL SİSTEM TASARIMI İÇİN BİR UYGULAMA

**Öz**

Çok Amaçlı De Novo Programlama problemlerinin çözümünü gerçekleştiren kesin bir yöntem olmamasına rağmen Çok Amaçlı Karar Verme teknikleri de novo için çözümde kullanılabilmektedir. Bu durum sebebiyle birçok yöntem için amaçlar arasında bir ağırlıklandırma veya öncelik sıralamasının yapılması gerekmektedir. Özellikle amaç fonksiyonu sayısının 3'ten fazla olması durumunda çok farklı çözüm sonucu elde etmek mümkündür. Bu çalışmada pozitif ideal çözümler kullanılarak Çok Amaçlı De Novo Programlama probleminin çözümü için ilk kez Öncelikli Hedef Programlama kullanılmıştır. Ayrıca aynı problem Global Kriter yönteme göre çözülerek elde edilen sonuçlar karşılaştırılmıştır. Bu karşılaştırma sonucunda Öncelikli Hedef Programlamada hedefler arasındaki öncelik sıralamasının yapılmasında Global Kriter Yöntemin kullanılabileceği ortaya çıkmıştır.

**Anahtar Kelimeler:** De Novo Programlama, Global Kriter Yöntem, Lexicographic Hedef Programlama.

## 1. INTRODUCTION

Decision making process in the systems which focus on a goal starts with the goal oriented targets and the constraints which shape them. Systematic decision making process was set up by Simon (1960: 10-13) and includes the three phases of intelligence, design, and selection. Intelligence phase is where the problem is identified and described. The relation between decision variables are clearly identified in the design phase, and the mathematical model is formed with assumptions which simplify the real problem. The selection phase investigates the applicability of the solution provided by the model. If the acquired solution is logical, then the real problem is solved with it (Lu et al., 2007: 5). Decision making is prudential and places responsibility on the decision maker. It is not a moment in time for the businesses and decision makers as it involves several activities which are supposed to happen in a future time period (Yaralıoğlu, 2010: 2). The mathematical models which are used in decision making processes aim to reach a certain goal under the present constraints. When the models are evaluated in terms of how much resource the constraints use, it is seen that resource amounts are either excessive or deficient. On the other hand, the use of each resource constraint at full capacity is almost always impossible. Therefore, the constraint functions which directly effect and limit the goal should be considered and formed carefully in mathematical models instead of the value at which they are realized. When the constraint functions of the resource amounts are used efficiently, it also means that wastefulness is avoided.

Businesses usually define their production plans in short terms and make it their principle to use their resources based on the defined constraints and targets. Resources should be modified or restructured in the long run or during the next planning phase even if some of the available resources are constant in the short term. A determination of resource quantities inappropriately to the system capacity leads to inadequate optimization and insufficient use of scarce resources (Zeleny, 1984: 310). Instead of optimizing an existing system, De Novo assumption, which is also known as optimal system design to enable the restructuring of resources with flexible resource constraints, states how an optimal system should be organized with the highest value for goals and the full capacity use of constraints. In other words, an optimal system is possible where the resource amounts supplied optimally (Babić and Pavić, 1996). An optimal system not only determines the best mixture of all outputs but also that of the inputs (Tabucanon, 1988: 102). A system design, redesign, and optimization must include the reformation of system limits and constraints based on goals. System design is not a selection of alternatives but a creation of alternatives (Zeleny, 1986).

This study uses to Global Criterion Method and Lexicographic Goal Programming to create the optimal design of a production process. First, a production problem which was set in Multiobjective Linear Programming (MOLP) model was reorganized with De Novo assumption and solved with Global Criteria Method. Later, the same problem was solved with Lexicographic Goal Programming, and the results were compared with each other.

## 2. MULTIOCJECTIVE DE NOVO PROGRAMMING

Instead of optimizing a system, Zeleny (1976) conducted the first study on De Novo Programming proposing to design the optimal system. According to Zeleny (1984) De Novo Programming enables optimal design thanks to the long-term restructuring of resources, more efficient use of scarce resources, and prevention of wastefulness. While de novo hypothesis was applied only to classical linear programming problems in the beginning, it can easily be applied to Multiobjective Linear Programming problems. Multi Criteria De Novo Programming problem proposed by Zeleny (1990) is given mathematically below.

$$Max\ Z_k = C^k x_j$$

$$Min\ W_s = W^s x_j$$

Subject to (1)

$$Ax - b \leq 0$$

$$pb \leq B$$

$$x_j \geq 0$$

where, $Z_k = C^k x_j = \sum_{j=1}^{n} C_{kj} x_j, k = 1,2, \dots, l$ are objective functions $Z_k$ to be maximized simultaneously. $W_s = C^s x_j = \sum_{j=1}^{n} C_{sj} x_j, s = 1,2, \dots, r$ objective functions $W_s$ to be minimized simultaneously. $C^k \in R^{lxn}, C^s \in R^{rxn}$ and $A \in R^{mxn}$ are matrices of dimensions $lxn, rxn$ and $mxn$ respectively. $b \in R^m$ is m-dimensional unknown resources vector, $p \in R^m$ is vector of unit prices of m resource vector, and B is the given total budget. (1) can be rewritten as seen below based on budget constraint.

$$Max\ Z_k = C^k x_j$$

$$Min\ W_s = W^s x_j$$

Subject to (2)

$$Vx \leq B$$

$$x_j \geq 0$$

Here $V = (V_1, V_2, \dots, V_n) = pA \in R^n$.

Zeleny (1986) used “meta-optimality” concept based on positive ideal solutions to solve Multicriteria De Novo Programming problems. Positive ideal solutions are acquired from the solution of each objective function based on their given direction. Positive ideal solutions are also named as the best performance of each objective function in (1) or (2). The set of positive ideal solutions is expressed as follows.

$$I^* = \{Z_1^*, Z_2^*, \dots, Z_l^*; W_1^*, W_2^*, \dots, W_r^*\} \tag{3}$$

Meta-optimal problem is formed as seen below.

$Min\ B = Vx$

Subject to; (4)

$C^k x_j = Z_k^*$

$W^s x_j = W_s^*$

$Vx \leq B$

$x_j \geq 0$

with the solution of (4), one can obtain $x^*$, $B^* = Vx^*$. $B^*$value is named as meta-optimum budget. Solving (M1.3) identifies the minimum budget $B^* = Vx^*$. at which the metaoptimum performance $Z_k^*$ and $W_s^*$ can be realized through $x^*$ and $b^*$. Solving (M1.3) must exceed any given budget *B*. Optimum-path ration “r” can be used with a pre-defined budget “B”, $r = \frac{B}{B^*}$. Using “r”, final solution formulations can be defined as: $x = rx^*$, $b = rb^*$, $Z_k = rZ_k^*$ and $W_s = rW_s^*$. Additionally**,** Shi (1995) developed a new approach to solve De Novo Programming problems and defined six different types of optimum-path ratios

Although there is no general method to solve Multicriteria De Novo Programming problems, there are various methods which are used for the solutions of Multicriteria/Multiobjective De Novo Programming problems. A literature summary on the methods used for De Novo Programming problems can be provided as follows. Lai and Hwang (1992) used and analysed single criterion de Novo Programming problem for the first time in fuzzy environments. Later, Li and Lee (1990) developed a two-phase fuzzy approach based on ideal solutions for Multi Criteria De Novo Programming. Lee and Li (1993) proposed fuzzy goals and fuzzy coefficients simultaneously and proposed a different approach. Umarusman (2013) and Umarusman and Türkmen (2013) proposed Minmax Goal programming method and Global Criteria Method, respectively, to solve Multi Criteria De Novo Programming problems. Zhuang and Hocine (2018) used Meta-goal programming in solution of Multiobjective De Novo Programming problems. Banik and Bhattacharya (2018) proposed weighted goal programming technique for solving General De Novo programming problem. Umarusman (2018) proposed how an optimal design can be reached based on Minmax approach.Bhattacharya and Chakraborty (2018) developed an alternative approach for the solution of the general multiobjective De-Novo

Programming Problem under fuzzy environment.

### 2.1. Lexicographic Goal Programming

Goal programming studies were started by Charnes et al. (1955). Later, Charnes and Cooper (1961) formulated Goal Programming. Goal Programming aims to minimize deviation from aspired levels set by the decision maker and carries that minimization process with various methods. There are three fundamental Goal Programming methods (Romero, 1991:3-4): The first study on Archimedean Goal Programming was carried out by Ijiri (1965) who considered priority and weight factors together. Later Charnes and Cooper (1977) formulated Archimedean Goal Programming Model. Archimedean goal programming considers all goals simultaneously as they are embodied in a composite objective function. This composite function tries to minimize the sum of all the deviations between the goals and their aspirational levels. The deviations are weighted according to the relative importance for the DM of each goal. Lexicographic Goal Programming was developed by Lee (1972), and Charnes and Cooper (1977) proposed the model which only ranks priorities among goals but excludes weights. Minmax Goal Programming which was developed by Flavell (1976) minimizes maximum deviation instead of the sum of deviating variables, which is different from the weighted and prioritized structures of Goal Programming.

As goals may have different units, it is inevitable to normalize them in Goal Programming. There are a few normalisation techniques in the literature such as percentage normalisation, Euclidean normalisation, Summation normalisation, Zero-one normalisation (Tamiz et al.,1998). This study uses positive ideal solutions of each goal function as the normalization constant for Lexicographic Goal Programming. Lexicographic Goal Programming can be stated mathematically as follows.

$$Min\, Z = \sum_{i=1}^{m} P_i(d_i^- + d_i^+)$$

Kısıtlar; (5)

$$\sum_{j=1}^{n} a_{ij}x_j + d_i^- - d_i^+ = b_i$$

$$x_j, d_i^-, d_i^+ \geq 0$$

$$i = 1,2, \dots, m \text{ ve } j = 1,2, \dots, n$$

$$P_1 \gg P_2 \gg \cdots \gg P_i$$

The solution starts with the primary goal and goes on towards lower priority goals.

The optimum values of high-priority goals should not be degraded by lower-priority goals. Prioritized Goal Programming for ideal solutions can be organized as follows.

$$Min\ Z = P_k\left(\frac{d_k^-}{Z_k^*} + \frac{d_k^+}{Z_k^*}\right), P_s\left(\frac{d_s^-}{W_s^*} + \frac{d_s^+}{W_s^*}\right)$$

Kısıtlar; (6)

$$Z_k(x) + d_k^- - d_k^+ = Z_L^*$$

$$W_s(x) + d_s^- - d_s^+ = W_s^*$$

$$x_j, Z_L^*, W_s^*, d_k^-, d_k^+, d_s^-, d_s^+ \geq 0$$

$Z_k^*$: is the normalization constant for maximization-directed goal, (positive ideal solution),

$W_s^*$ is the normalization constant for minimization-directed goal, (positive ideal solution),

$P_k$: The priority for $k$th maximization goal,

$P_s$: The priority for $i$th minimization goal,

There can be different solution results for a given problem based on the priority ranking by the decision-maker. Although there is no set rule for priority ranking among given goals in Lexicographic Goal Programming, a number of methods can be utilized based on the number of decision-maker. These can be named as the paired comparison method, Kendall Array method, Thurstone procedure, etc. (Ignizio, 1976). There are two significant situations if the solution of Multiobjective Linear Programming problem is to be done based on goal programming by using positive ideal solutions. It should be kept in mind that as maximization goals cannot exceed their own positive ideal solutions, it is $d_k^+ = 0$, and similarly, as minimization goals cannot be lower than their own positive ideal solutions, it is $d_s^- = 0$.

### 2.2. Global Criterion Method

Global Criterion Method fall under the class of MCDM methods that do not require any preference information from the DM (Hwang and Masud, 1979: 21). Namely, weights and priority ranking are not used for goal functions in Global Criteria Method. Buna karşın Arora (2004: 673) offered a weighted model for Global Critera Method. The Global Criterion Method measures the distance by using Minkowski's Lp metric. In this method, the aim is to minimize a function which defines a global criterion which is a measure of how close the decision maker can get to the ideal solution. Mathematical formulation is as follows:

$$\left(\sum_{k=1}^{l}\left[\frac{Z_k(x^*)-Z_k(x)}{Z_k(x^*)}\right]^p\right)^{1/p} \quad (7)$$

Where $Z_k(x^*)$ is the value of objective function *l* at its individual optimum $x^*$, $Z_k(x)$ is the function itself, p $(1 \leq p \leq \infty)$ is integer valued exponent that serves to reflect the importance objectives. Setting p=1 implies that equal importance is given to all deviations (Boychuk and Ovchinnikov, 1973), while p=2 implies that these deviations are weighted proportionately with the largest deviations having the largest weight (Salukvadze,1974). Setting p>2 means that more and more weight is given to the largest of deviations. In addition, where p=1 (1.1) function is linear, whereas p=2 makes it a non-linear function (Tabucanon, 1988:37). In order to keep the function linear, p value is taken 1. Global Criterion Method for minimization objectives can be composed as following (Umarusman and Türkmen, 2013).

$$\left(\sum_{s=1}^{r}\left[\frac{W_s(x)-W_s(x^*)}{W_s(x^*)}\right]^p\right)^{1/p} \quad (8)$$

Taking (7) and (8) under consideration together, maximization and minimization objectives can be written as following:

$$F(x)=\left(\sum_{k=1}^{l}\left[\frac{Z_k(x^*)-Z_k(x)}{Z_k(x^*)}\right]^p\right)^{1/p}+\left(\sum_{s=1}^{r}\left[\frac{W_s(x)-W_s(x^*)}{W_s(x^*)}\right]^p\right)^{1/p} \quad (9)$$

where $W_s(x^*)$ is the value of objective function r at its individual optimum $x^*$.

$$\text{Min } G=\sum_{k=1}^{l}\left[\alpha_k^p\frac{Z_k(x^*)-Z_k(x)}{Z_k(x^*)}\right]^p+\sum_{s=1}^{r}\left[\beta_s^p\frac{W_s(x)-W_s(x^*)}{W_s(x^*)}\right]^p$$

Subject to (10)

$Ax \leq b$

$x \geq 0$,

$Z_k(x)$: maximization-directed objectivefunction,

$Z_k(x^*)$: $x^*$ the value of *l*th objective function at the optimum point,

$W_s(x)$: Minimization-directed goal function,

$W_s(x^*)$: $x^*$ the value of *r*th objectivefunction at the optimum point.

*p*: $(1 \leq p \leq \infty)$

where, $Z_k = (Z_1, \dots, Z_l) \in \mathbb{R}^l$, $W_s = (W_1, \dots, W_r) \in \mathbb{R}^r$ and $V = (V_1, , , , . V_n) = pA \in \mathbb{R}^n, A \in \mathbb{R}^{mxn}$ is matrices of dimension mxn,$b \in \mathbb{R}^m$ is the m-dimensional unknown resource vector, $Z_k(x^*)$ is the value of objective function l at its individual optimum $x^*$ and $W_s(x^*)$ is the value of objective function r at its individual optimum $x^*$.

## 3. APPLICATION

Table 1 provides the amounts of the raw materials planned to be used monthly by a business which produces plastic balls with four different weights (Umarusman and Türkmen, 2013).

**Table 1.** Resource Use Amounts and Unit Prices

| Resources | $PT_1$ $(x_1)$ (75 gr) | $PT_2$ $(x_2)$ (90 gr) | $PT_3$ $(x_3)$ (125 gr) | $PT_4$ $(x_4)$ (150 gr) | Use Amount | Unit Price (Dollar/kg) |
|---|---|---|---|---|---|---|
| PVC (kg) | 33 | 40 | 58 | 66 | 850 | 1.2 |
| DOP (kg) | 32 | 38 | 56 | 64 | 870 | 0.9 |
| Powder Paint(kg) | 6 | 7 | 9 | 11 | 72 | 0.5 |
| Varnish (kg) | 4 | 5 | 7 | 9 | 83 | 0.3 |

The business management decided on certain conditions considering the demand for products in light of their market. They can be stated as the monthly production capacity of 9500 units as minimum and 11000 unit as maximum. Based on the previous knowledge, the difference between the production of the first and second types of plastic balls and the production of the third and fourth types of plastic balls can be 700 units at most. The minimum production units are 275 for the first type, 150 for the second, 100 for the third, and 500 for the fourth. Based on the data above, the business management focused on profit and cost goals. The unit profits for profit goal were defined as \$1.5, \$1.6, \$1.95, and \$1.87, and the unit costs for cost profit were defined as \$0.8, \$0.92, \$1.65, and \$1.87. The Multiobjective Linear Programming model for this problem is organized as follows based on the aforementioned information.

$$Max\ Z_1: 1.5x_1 + 1.6x_2 + 1.95x_3 + 2.3x_4$$

$$Max\ W_1: 0.8x_1 + 0.92x_2 + 1.65x_3 + 1.87x_4$$

Subject to (P1)

$$0.033x_1 + 0.04x_2 + 0.058x_3 + 0.066x_4 \leq 850$$

$$0.032x_1 + 0.038x_2 + 0.056x_3 + 0.064x_4 \leq 870$$

$$0.006x_1 + 0.007x_2 + 0.009x_3 + 0.011x_4 \leq 72$$

$$0.004x_1 + 0.005x_2 + 0.007x_3 + 0.009x_4 \leq 83$$

$x_1 + x_2 + x_3 + x_4 \leq 11000$

$x_1 + x_2 + x_3 + x_4 \geq 9500$

$x_1 + x_2 - x_3 - x_4 \leq 7000$

$x_1 \geq 275; x_2 \geq 150$

$x_3 \geq 100; x_4 \leq 500$

$x_1, x_2, x_3, x_4 \geq 0$ *and integer*.

### 3.1. The Solution of (P1) in terms of MOLP

Before (P1) is organized based on De Novo assumption, solutions were done for each goal function. The main goal here is to analyse the changes in goal functions from MOLP problem to the solution of Multiobjective De Novo problem. Table 1 presents the variable values and goal function values acquired from the solution of (P1) as MOLP based on each goal function.

**Table 2.** Solution for MOLP

| Variables | Objective Functions $Z_1$ | $W_1$ |
|---|---|---|
| $x_1$ | 8840 | 8100 |
| $x_2$ | 150 | 150 |
| $x_3$ | 1990 | 1250 |
| $x_4$ | 0 | 0 |
| Objective Function Value | 17380.50 | 8680.5 |

Based on Table 2, each goal function is realized at different values of the variables. Therefore, the acquired solution is an "unfeasible solution". It is an expected result. It is an expected result because the realization of goal functions at the same value of the variables in MOLP problems is hardly possible. The positive ideal solutions of (P1) are $I^* = \{17380.5; 8680.5\}$. In this study, the satisficing and compromise solution of (P1) based on De novo assumption were not investigated.

### 3.2. Solution of (P1) based on Multiobjective De Novo Programming Model

(P1) is organized below based on Multiobjective De Novo Programming Model:

$Max\ Z_1 : 1.5x_1 + 1.6x_2 + 1.95x_3 + 2.3x_4$

$Min\ W_1 : 0.8x_1 + 0.92x_2 + 1.65x_3 + 1.87x_4$

Subject to (P2)

$0.033x_1 + 0.04x_2 + 0.058x_3 + 0.066x_4 - b_1 = 0$

$0.032x_1 + 0.038x_2 + 0.056x_3 + 0.064x_4 - b_2 = 0$

$0.006x_1 + 0.007x_2 + 0.009x_3 + 0.011x_4 - b_3 = 0$

$0.004x_1 + 0.005x_2 + 0.007x_3 + 0.009x_4 - b_4 = 0$

$1.2b_1 + 0.90b_2 + 0.5b_3 + 0.3b_4 <= 2441.9$

$x_1 + x_2 + x_3 + x_4 \leq 11000$

$x_1 + x_2 + x_3 + x_4 \geq 95000$

$x_1 + x_2 - x_3 - x_4 \leq 7000$

$x_1 \geq 275; x_2 \geq 150$

$x_3 \geq 100; x_4 \leq 500$

$x_1, x_2, x_3, x_4 \geq 0$ *and integer.*

Table 3 shows the solution of the problem named as (P2) and organized in Multiobjective De Novo Programming model.

**Table 3.** Multiobjective De Novo Programming Solution

| Variables | Objective Functions | |
|---|---|---|
| | $Z_1$ | $W_1$ |
| $x_1$ | 275 | 8100 |
| $x_2$ | 150 | 150 |
| $x_3$ | 10075 | 1250 |
| $x_4$ | 500 | 0 |
| Objective Function Value | 21448.75 | 8680.5 |

As a result of the solution according to each goal function, the goal functions were realized at different variable values. Therefore, the solution acquired for (P2) is an unfeasible solution as well. The positive ideal solutions of (P2) are $I^* = \{21448.75; 8680.5\}$. On the other hand, Table 3 provides the optimal resource amounts for each goal acquired from the solutions based on each goal function.

**Table 4.** Optimal Resource Amount for Each Goal

| Resources | Amount | $Z_1$ | $W_1$ |
|---|---|---|---|
| $b_1$ | 850 | 632.4 | 345.8 |
| $b_2$ | 870 | 610.7 | 334.9 |
| $b_3$ | 72 | 98,874 | 60.9 |
| $b_4$ | 83 | 76,875 | 41.9 |

If (P2) had one single goal, the values in the column $Z_1$ ve $W_1$ in Table 4 would show the optimal resource amounts based on De Novo assumption. However, as (P2) has more than one goal function, the resource amounts fit for both goal functions must be determined to acquire a “satisficing” or “compromise” solution.

### 3.3. Compromise Solution for (P2)

The compromise solution of (P2) is concluded according to Global Criteria Method. The Global Model of (P2) is formed as seen below: When the global goal function is organized first,

$$Min\ G: \left[\frac{21448.75 - Z_1}{21448.75} + \frac{W_1 - 8680.5}{8680.5}\right]$$

acquired. The final form of the global goal function is given below.

$$Min\ G: \left[-\frac{Z_1}{21448.75} + \frac{W_1}{8680.5}\right].$$

The Global model which is reorganized for (P2) is given below.

$$Min\ G: 0.00059x_1 + 0.000692x_2 + 0.000134x_3 + 0.00152x_4$$

Subject to (P3)

$$0.033x_1 + 0.04x_2 + 0.058x_3 + 0.066x_4 - b_1 = 0$$

$$0.032x_1 + 0.038x_2 + 0.056x_3 + 0.064x_4 - b_2 = 0$$

$$0.006x_1 + 0.007x_2 + 0.009x_3 + 0.011x_4 - b_3 = 0$$

$$0.004x_1 + 0.005x_2 + 0.007x_3 + 0.009x_4 - b_4 = 0$$

$$1.2b_1 + 0.90b_2 + 0.5b_3 + 0.3b_4 <= 2441.9$$

$$x_1 + x_2 + x_3 + x_4 \leq 11000$$

$$x_1 + x_2 + x_3 + x_4 \geq 95000$$

$$x_1 + x_2 - x_3 - x_4 \leq 7000$$

$$x_1 \geq 275;\ x_2 \geq 150$$

$$x_3 \geq 100;\ x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0\ ve\ integer.$$

The results acquired from the solution of (P3) are given in Table 5. Based on this information, the decision variables for both objective functions are realized at the same value, which provides a global compromise solution.

**Table 5**. Global Solution

| Variables | Objective Functions | |
|---|---|---|
| | $Z_1$ | $W_1$ |
| $x_1$ | 8100 | 8100 |
| $x_2$ | 150 | 150 |
| $x_3$ | 1250 | 1250 |
| $x_4$ | 0 | 0 |
| Objective Function Value | 14827.5 | 8680.5 |

$G = 6.568408$ is defined as the objective function of (P3) which is solved based on the global criterion method. On the other hand, the resource amounts of the optimal system model for (P1) in terms of the global model should be $b_1$=345.799988, $b_2 = 334.900024$, $b_3 = 60.900002$, and $b_4 = 41.900002$. The distance of profit and cost objectives to their respective positive ideal solutions are defined from the information in Table 5.

$$\text{The distance degree of profit objective; } d_1 = \frac{21448.75 - 14827.5}{21448.75} = 0.308$$

$$\text{The distance degree of cost objective ; } d_1 = \frac{8680.5 - 8680.5}{8680.5} = 0$$

in Table 5 The distance degree of cost objective to its positive ideal solution is zero. It shows that the global solution is realized on the cost goal.

### 3.4. Lexicographic Goal Programming Solution for (P2)

According to Table 2, the positive ideal solutions of profit and cost goals are $21448.75$ and $8680.5$ , respectively. Based on this data, (P2) is reorganized according to Lexicographic Goal Programming, as seen below. First, each goal function is organized in its goal programming model, and

$$G_1: Max\ Z_1: 1.5x_1 + 1.6x_2 + 1.95x_3 + 2.3x_4 + n_1 - p_1 = 21448.75$$

$$G_2: Min\ W_1: 0.8x_1 + 0.92x_2 + 1.65x_3 + 1.87x_4 + n_2 - p_2 = 8680.5$$

acquired. Additionally, the profit goal $Z_1$ must be $p_1 = 0$ as it cannot exceed its own positive ideal solution. As the cost goal $W_1$ cannot be lower than its positive ideal solution, it must be $n_2 = 0$ . Therefore, each goal function is reorganized as seen below.

$$G_1: Max\ Z_1: 1.5x_1 + 1.6x_2 + 1.95x_3 + 2.3x_4 + n_1 = 21448.75$$

$$G_2: Min\ W_1: 0.8x_1 + 0.92x_2 + 1.65x_3 + 1.87x_4 - p_2 = 8680.5$$

Apart from these organizations, a priority ranking must exist among the targets. As there are two goals, the possible priority rankings can be $G_1 > G_2$, $G_1 < G_2$, or $G_1 = G_2$ . Accordingly,

$$Min \left(\frac{n_1}{21448.75}\right), \left(\frac{p_2}{8680.5}\right)$$

Subject to (P4)

$$1.5x_1 + 1.6x_2 + 1.95x_3 + 2.3x_4 + n_1 = 21448.75$$

$$0.8x_1 + 0.92x_2 + 1.65x_3 + 1.87x_4 - p_2 = 8680.5$$

$$0.033x_1 + 0.04x_2 + 0.058x_3 + 0.066x_4 - b_1 = 0$$

$$0.032x_1 + 0.038x_2 + 0.056x_3 + 0.064x_4 - b_2 = 0$$

$$0.006x_1 + 0.007x_2 + 0.009x_3 + 0.011x_4 - b_3 = 0$$

$$0.004x_1 + 0.005x_2 + 0.007x_3 + 0.009x_4 - b_4 = 0$$

$$1.2b_1 + 0.90b_2 + 0.5b_3 + 0.3b_4 <= 2441.9$$

$$x_1 + x_2 + x_3 + x_4 \leq 11000$$

$$x_1 + x_2 + x_3 + x_4 \geq 9500$$

$$x_1 + x_2 - x_3 - x_4 \leq 7000$$

$$x_1 \geq 275; x_2 \geq 150$$

$$x_3 \geq 100; x_4 \leq 500$$

$$x_1, x_2, x_3, x_4 \geq 0 \; ve \; integer.$$

acquired. The results of the solutions based on the aforementioned priority rankings are provided in Table 6.

**Table 6.** Lexicographic Solution

| Variables | $G_1 > G_2$ | | $G_1 < G_2$ | | $G_1 = G_2$ | |
|---|---|---|---|---|---|---|
| | $Z_1$ | $W_1$ | $Z_1$ | $W_1$ | $Z_1$ | $W_1$ |
| $x_1$ | 8100 | 8100 | 8100 | 8100 | 8100 | 8100 |
| $x_2$ | 150 | 150 | 150 | 150 | 150 | 150 |
| $x_3$ | 750 | 750 | 1250 | 1250 | 1250 | 1250 |
| $x_4$ | 500 | 500 | 0 | 0 | 0 | 0 |
| Objective Function Value | 15002.5 | 8790.5 | 14827.5 | 8680.5 | 14827.5 | 8680.5 |
| Deviations from Goals | 0.932008 | 126720 | 0.990425 | 0 | 0.990425 | |

The variable values for the priority situations "$G_1 > G_2$", "$G_1 < G_2$" and "$G_1 = G_2$" among the goals are given in Table 6. It is concluded the priority rankings"$G_1 < G_2$" and "$G_1 = G_2$" among the goals provide the same result.

**Table 7.** Resource Amounts Based on Priority Ranking

| Resources | Amount | $G_1 > G_2$ | $G_1 < G_2$ | $G_1 = G_2$ |
|---|---|---|---|---|
| $b_1$ | 850 | 349.8 | 345.8 | 345.8 |
| $b_2$ | 870 | 338,851 | 334.9 | 334.9 |
| $b_3$ | 72 | 61.8999 | 60.8999 | 60.8999 |
| $b_4$ | 83 | 42.9 | 41.9 | 41.9 |

The proposal by Lexicographic Goal Programming for resource amounts based on these decision variables are given in Table 7. The resource use amounts "$G_1 < G_2$" " among the goals and "$G_1 = G_2$" priority ranking are the same. All the results in the application section are provided in Table 8.

**Table 8**. From MOLP to Optimal System Design

| | $b_i$ | $p_i$ | $b_i.p_i$ | MOLP | | Global Solution for Multiobjective De Novo Prog. | Lex. Goal Prog. Soluiton for Multiobjective De Novo Prog. ($G_1 < G_2$) |
|---|---|---|---|---|---|---|---|
| PVC (kg) | 850 | 1.5 | 1275 | | | 345.799988 | 345.8 |
| DOP (kg) | 870 | 1.2 | 1044 | unfeasible | | 334.900024 | 334.9 |
| Powder Paint(kg) | 72 | 0.9 | 68.8 | solution | | 60.900002 | 60.8999 |
| Varnish (kg) | 83 | 0.7 | 58.1 | | | 41.900002 | 41.9 |
| Budget | $\sum_{i=1}^{4} b_i.p_i$ | | 24441.9 | 24441.9 | | 1004.719982 | 1004.719982 |
| Variables | | | | $Z_1$ | $W_1$ | $Z_1$ $W_1$ | $Z_1$ $W_1$ |
| $x_1$ | | | | 8840 | 8100 | 8100 | 8100 |
| $x_2$ | | | | 150 | 150 | 150 | 150 |
| $x_3$ | | | | 1990 | 1250 | 1250 | 1250 |
| $x_4$ | | | | 0 | 0 | 0 | 0 |
| Profit | | | | 17380.5 | | 14827.5 | 14827.5 |
| Cost | | | | 8680.5 | | 8680.5 | 8680.5 |

The phases of the satisficing optimal system design from MOLP is given in Table 8. As the decision variable of each goal function for MOLP were realized at different values, a solution could not be reached. Additionally, the budget for the resources of MOLP problem named (P1) is $24441.90. Afterwards, the solution of De Novo Programming problem which was reorganized based on De Novo assumption and named (P2) was carried out. Because the acquired solution was unfeasible, a compromise solution (P3) was done according to Global Criteria Method. According to De Novo assumption, the budget constraint can be “ $\leq$ "or”=” type. “$\leq$ "” was used as the budget constraint in this study. Based on global solution, the budget for the required resources must be $1004.719982. Finally, (P2) was solved according to Lexicographic Goal Programming, and the “satisficing” solution was determined. Three different priority rankings were done among the goals in this solution, and the solution values for “$G_1 < G_2$” and “$G_1 = G_2$” were concluded as the same. The target deviations for the first priority goal $G_2$ and the second priority goal $G_1$ are 0 and 0.990425, respectively, in the solution based on “$G_1 < G_2$” ranking. These results and those provided by the global solution are the same. Namely, the cost goal is dominant over the profit goal, which provides information about each goal which were transformed into targets. In addition to that, it is possible to observe the changes in goal functions in each phase.

## 4. CONCLUSION

Priority ranking among goals, relative significances among goals, or both could be used at the same time while investigating compromise or satisficing solutions in mathematical models used to solve Multiobjective Linear Programming problems. There is no precise method to determine priorities or weights for goals/targets. Therefore, results from models could vary depending on the information provided by decision makers. While the priority ranking among goals are determined by decision maker in Lexicographic Goal Programming, solution in Global Criteria Method are conducted based on positive ideal solutions without weights or priorities.

Table 8 shows the results which were given by the methods used for the solution of Multi Objective De Novo Programming problem. In this regard, the comments below are possible. If the priority ranking among the goals in Lexicographic Goal Programming is doubtful, Global Criterion Method may be used. It is because a goal function with 0 distance to its positive ideal solution means that it is realized on its own positive ideal solution. In other words, that goal dominates others. Therefore, the goals which are realized at its own positive ideal value must be the first priority in Goal Programming. The rest of the goals which are distanced from their ideal solutions must be prioritized based on their proximity to their positive ideal values. Consequently, Global Criteria Method may be applied in determining the priority ranking among the goals in Goal Programming.

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