Invariant Magnetic Vector Potential in the Mercury Based High-Temperature Superconductor and Its Prospective Technological Application

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Received: 21.11.2018 Accepted: 17.12.2018

Abstract: Magnetic vector potential, A has great importance in various superconducting phenomena such as Aharonov-Bohm effect, Josephson effect, SQUID applications etc. due to the fact that A is directly related to the phase difference in the superconducting system. In this work, magnetic vector potential value has been calculated numerically for the optimally oxygen doped $HgBa_2Ca_2Cu_3O_{8+x}$ mercury cuprate superconductor via interlayer theory at low temperatures by using magnetization versus applied magnetic field data. It has been surprisingly determined that regardless of the temperature variation; the quantity of magnetic vector potential remains unchanged. In this context, it has been determined that magnetic vector potential is an invariant parameter of the system investigated. Moreover, momentum conserving interlayer tunneling i.e. coupling between superconducting copper oxide layers for a low-temperature interval of 3K-5K has been proved for the first time. Hence, invariant magnetic vector potential corresponds to the concept of the constant phase difference. Ultimately, this work gives a reliable method for deciding about working temperature interval for technologists who want to design an intrinsic phase detector which has the property of constant phase difference.

Keywords: Invariant magnetic vector potential, mercury cuprates, momentum conserving interlayer theory, phase.

1. INTRODUCTION

From a theoretical point of view, magnetic vector potential, A has great importance as a physical quantity in the unified field theories of fundamental forces which combine the electromagnetic force, weak and strong nuclear forces and gravity. As is known, vector potential is the most known gauge field of electromagnetism. Observation of the Aharanov-Bohm (AB) effect also enables us to comprehend the derivation of a gauge field physically [1]. The motion of a charged particle can sometimes be affected by electromagnetic fields in regions that the particle never enters. This phenomenon is known as the AB effect which is a non-local effect. In this effect, a physical object moves along a closed loop through a gauge field free region and hence it experiences some physical changes [2]. Due to the existence of a magnetic vector potential, in AB effect a charged particle exhibits a phase shift even if there is no magnetic field [3]. Moreover, quantization of magnetic flux is closely related to AB effect and superconductors constitute a natural frame of reference for observation of magnetic flux quantum [4, 5]. In this context, there have been various superconducting applications whose working principle is based on magnetic vector potential i.e. phase such as SQUID etc. Furthermore, achieving non-varying magnetic vector potential i.e. constant phase difference has a crucial role in various technological applications which are based on phase locking. As is known, there have been several works on cryogenic phase detector which is based on Josephson junction and superconducting flux flow oscillator for phase locking with Nb [6-9].

In addition to this technological importance of magnetic vector potential, achieving non-varying (invariant) magnetic vector potential has a crucial role for theoretical works focused on the explanation of electrical conductivity mechanism in cuprate superconductors. So that verification of the invariant magnetic vector potential experimentally in cuprate superconductors enables us to understand the momentum conserving interlayer coupling which is responsible for electrical conductivity in cuprate superconductors that was proposed by P. W. Anderson et al. in the mid-1990s [10,11].

In this context, due to reasons mentioned above, determining of magnetic vector potential has vital importance for advanced technological applications as well as theoretical works.

In this work, a numeric calculation of magnetic vector potential of the optimal oxygen doped mercury-based copper oxide layered high-temperature superconductor has been realized by using magnetization versus magnetic field hysteresis curves obtained by Superconducting Quantum Interference Device (SQUID).

Mercury cuprates have the highest Meissner critical transition temperature of 140K at normal atmospheric pressure, high critical current densities in the order of 10^{12} [A/m^2]. Moreover, in our previous works, it has been determined that the mercury-based high-temperature superconducting system is a reliable frame of reference for observing some symmetry breakings. As was previously reported that one-dimensional global gauge symmetry U (1), time reversal symmetry, T and electro-weak symmetry are broken in a mercury cuprate superconductor. Moreover, mercury cuprate exhibits some relativistic effects such as shifting of plasma frequency from microwave to infrared that corresponds to a gravitational redshift in general relativity and Paramagnetic Meissner effect which origins from spin-orbit coupling process in the system [12-15]. From this point of view, determination of magnetic vector potential, A of the mercury cuprate has crucial importance.

2. THEORETICAL

Magnetic vector potential has been investigated in the context of interlayer theory for the optimal oxygen doped $HgBa_2Ca_2Cu_3O_{8+x}$ (Hg-1223) superconductors since the interlayer theory had already been confirmed in three dimensions at low temperatures in our previous works [16-18].

The interlayer theory explains the superconductivity in the copper oxide layered superconductors in terms of the occurrence of crossover from two-dimensional to three-dimensional coherent electron pair transport [18]. The realization of the three-dimensional coherent electron pair transport is achieved by the Josephson or Lawrence-Doniach–like coupling between the superconducting copper oxide layers [19,20]. Accordingly, the Josephson coupling energy is to be equal to superconducting condensation energy so that the superconducting system exhibits perfect coupling along the c-axis [19]. For low temperatures ($T \ll T_c$) free energy of superconductor, F can be taken as superconducting condensation energy, E_b. In this respect, critical current density, J_c is written as

$$J_c = c\frac{\partial F}{\partial A} \approx c\frac{\partial E_b^0}{\partial A} \tag{1}$$

Since the superconducting condensation energy almost comes from Josephson coupling between copper oxide layers, E_b is written as Josephson coupling energy form

$$E_b = -E_b^0 \cos\theta \tag{2}$$

where E_b^0 and θ are the amplitude of superconducting condensation energy and the phase difference between superconducting layers, respectively [19].

The phase difference is defined in terms of the magnetic vector potential, A by Eq. (3)

$$\theta = \frac{2es}{\hbar c}A \tag{3}$$

where s is the average distance between copper oxide layers, e is the charge of an electron, c is the velocity of light and $\hbar$ is the Planck's constant divided by 2π. For relatively small phase differences, the critical current density is determined as Eq. (4) by using Eqs. (1), (2) and (3).

$$A = \frac{J_c}{4cE_b^0}\frac{\hbar^2 c^2}{e^2 s^2} \tag{4}$$

The c-axis penetration depth i.e Josephson penetration depth is also written by London equation [19]. The Josephson penetration depth represents the penetration of the magnetic field induced by the supercurrent flow in the superconductor.

$$\lambda_c = \sqrt{\frac{c}{4\pi}\frac{A}{J_c}} = \frac{\hbar c}{4es}\frac{1}{\sqrt{\pi E_b^0}} \tag{5}$$

3. RESULTS

In our one of the previous works, the interlayer theory had already been confirmed for the optimal oxygen doped $HgBa_2Ca_2Cu_3O_{8+x}$ (Hg-1223) mercury cuprates by determining the equality of Josephson coupling energy and superconducting condensation energy at the temperature interval of 3K-7K [16]. Due to this reason, this work has been focused on this low-temperature interval.

Critical current densities and c-axis penetration depths of the optimally oxygen doped Hg-1223 superconductors, which are accepted as the key parameters for calculation of magnetic vector potential, had already been calculated for 4,2K, 27K and 77K temperatures by Bean critical state and Lawrence - Doniach model, respectively [16-18]. The critical current density has been calculated according to Eq. (6).

$$J_c = 120\pi\frac{\Delta M}{t} \tag{6}$$

Where ΔM is the magnetization difference between the increasing and decreasing field branches and t is the average grain size of the sample.

These parameters had been determined via

ISSN: 2651-5199

magnetization versus applied magnetic field (*M-H*) dynamic hysteresis curves obtained by very sensitive quantum design SQUID magnetometer, model MPMS-5S. Related M-H dynamic hysteresis curves are given in Fig. 1. According to related data, critical current densities and *c*-axis penetration depths of the optimally oxygen doped Hg-1223 superconductors at the temperature interval of 3K-7K has been determined by fitting functions which are given Eqs. (7) and (8), respectively. The results are also listed in Table 1.

$$J_c = 1.398\times10^{12}\exp\left(-\frac{T}{12.262}\right) + 7.379\times10^{9} \quad (\text{with } R^2 = 1) \qquad (7)$$

$$\lambda_J = 0.499 + 0.016T + 6.717\times10^{-4}T^2 \quad (\text{with } R^2 = 1) \qquad (8)$$

According to Eq. (5), the amplitude of the superconducting condensation energy,   has been calculated. By using Eq. (4) and values in Table 1, magnetic vector potential *A* has been determined.

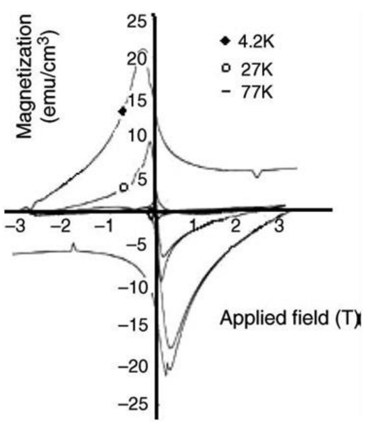

Figure 1. Magnetization versus applied magnetic field (*M–H*) for the optimally oxygen doped Hg-1223 superconductors at 4.2, 27 and 77 K [18].

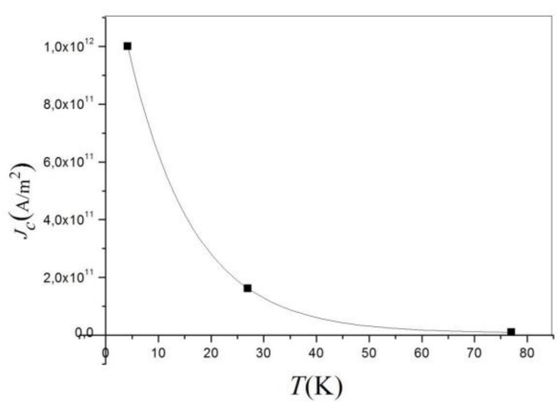

Figure 2. Critical current density versus temperature for the optimally O_2 doped Hg-1223 superconductor.

As shown in Table 1, regardless of the temperature variation, magnetic vector potential has the same value for 3K-5K temperature interval. From this point of view, magnetic vector potential is considered as an invariant parameter of the system for this temperature interval. So that the phase difference between superconducting layers in the system is also constant. The constant i.e. invariant phase difference for temperature interval of 3K-5K has been calculated by Eq. (3) in terms of magnetic vector potential:

$$\theta = 1{,}125.10^{-10}(m/s)^{-1}$$

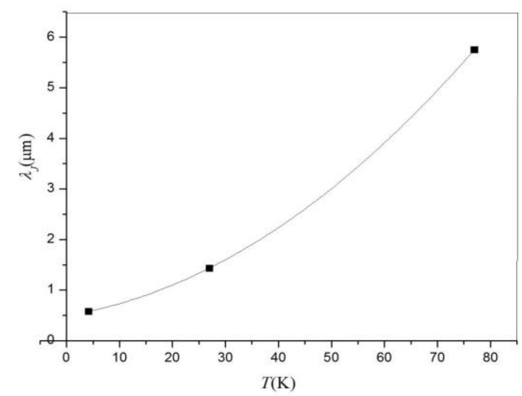

Figure 3. Josephson penetration depth versus temperature for the optimally O2 doped Hg-1223 superconductor.

Table 1. Josephson penetration depth, critical current density, the amplitude of superconducting condensation energy and magnetic vector potential values for the optimally oxygen doped Hg-1223 superconductor at low temperatures.

***T*(K)**	**J_c(A/m^2)**	**λ_J(μm)**	**E_b^0 (J/m^3)**	***A*(Vs/m)**
3	1.10×10^{12}	0.55	4.05×10^{15}	1.41×10^{-8}
4	1.02×10^{12}	0.58	3.77×10^{15}	1.41×10^{-8}
5	0.94×10^{12}	0.59	3.48×10^{15}	1.41×10^{-8}
6	0.86×10^{12}	0.62	3.22×10^{15}	1.39×10^{-8}
7	0.80×10^{12}	0.65	2.97×10^{15}	1.40×10^{-8}

According to Eq. (3), the unit of the phase difference between superconducting layers in the system corresponds to the inverse of the unit of velocity. Since magnetic vector potential is related to the phase difference, which is interpreted as an inverse of velocity in the system, the magnetic vector potential is considered as "electromagnetic momentum" per charge. Traditionally, in order to obtain an "invariant momentum" by means of "conserved momentum", the electromagnetic momentum of *qA* is added to linear momentum mv as shown in Eq. (6)

$$p = mv + qA \qquad (6)$$

where *m* is mass, *v* is the velocity of the charged particle, and *q* is the charge. So that the different choices for vector potential *A* cause different generalized momentums and some choices of A result in non-conserved generalized momentum [21]. In this context, choices for A have great importance to get conserved momentum which is related to homogeneous space that is invariant under translational symmetry.

On the other hand, since invariant magnetic vector potential leads to conserved momentum; it is natural to conclude that superconducting copper oxide layers in mercury cuprates are coupled by Josephson tunneling mechanism with interlayer momentum conservation. In this context, the concept of momentum conserving interlayer coupling has been numerically proved for the first time by using some electrodynamics parameters that are derived from a magnetization versus applied magnetic field experimental data.

Moreover, achieving invariant magnetic vector potential i.e. constant phase difference has also great importance for superconducting technological applications related to phase locking. In this point of view, if one works with the optimally oxygen doped mercury cuprates at the temperature interval of 3K-5K, since the phase difference is constant, any kind of minor effect that makes a change of the phase of the system could be detected by the suitable measurement system.

4. CONCLUSIONS

In this work, it has been numerically proved that magnetic vector potential acts as an invariant gauge field in the mercury-based high-temperature superconductors at low temperatures. This result has important theoretical and technological consequences. From a theoretical point of view, it has been understood that mercury cuprates realize Josephson coupling with the conserved momentum by introducing constant magnetic vector potential intrinsically. According to the technological point of view, this work gives a reliable working temperature interval of 3K-5K to technologists who want to design a phase detector with mercury cuprates. Since both bulk mercury cuprates consist of intrinsic Josephson junction array that displays three dimensional Josephson plasma resonance and exhibits invariant phase difference for a particular temperature interval, the optimal oxygen doped $HgBa_2Ca_2Cu_3O_{8+x}$ mercury cuprate superconductors can be proposed as the promising candidate for development of cryogenic phase detector and superconducting integrated receiver.

ACKNOWLEDGMENT

I would like to thank Prof. Dr. Ü. Onbaşlı for her valuable discussions and experimental contributions on magnetic vector potential.

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ISSN: 2651-5199