

Golden Ratio Discovered in Nonlinear Superconductor

Özden Aslan Çataltepe¹, Zeynep Güven Özdemir² and Ülker Onbaşlı³

¹ Faculty of Engineering, İstanbul Gedik University, Kartal, 34876, İstanbul, Turkey.

² Department of Physics, Yıldız Technical University, Davutpaşa, 34010, İstanbul, Turkey.

³ Department of Physics, Marmara University, Göztepe, 34722, İstanbul, Turkey.

ozden.aslan@gedik.edu.tr, zguven@yildiz.edu.tr

Received: 23.11.2018 Accepted: 19.12.2018

Abstract: Mercury based high temperature superconductor as a nonlinear dynamical system constitutes a natural laboratory for searching quantum chaotic transitions. In our previous works, these chaotic transitions had been expressed in details and mathematically proved by determination of negative Schwarzian derivative. In this work, we have focused on the net effective mass of the quasi particles, m^* at the vicinity of absolute zero temperature and the mass of the double helix quantum wave i.e. topological Segah solitons. The net effective mass of the quasi particles as the function of the phase of the superconducting system had already been determined in our previous works by using an advanced differential method that is based on the magnetic data obtained by SQUID (Superconducting Quantum Interference Device) measurements. The mass of the double helix quantum wave, whose wave length coincides with ultraviolet region of the electromagnetic spectrum, m_w , has also been calculated with the wave length of the solitons in relativistic manner. Since we have already proved the three dimensional resonance at low temperatures for the mercury cuprates, the determination of the ratio of these masses at the vicinity of absolute zero would be a promising method for searching the properties of resonance. Hence, we have determined the resonance state which appears as the golden ratio in m_w/m^* values for the distances $x=0.23\mu\text{m}$ and $x=2,361\mu\text{m}$ at absolute zero temperature for the optimally and over oxygen doped mercury cuprate superconducting nonlinear condensed matter system, respectively.

Keywords: Golden ratio, net effective mass, solitons' mass, mercury cuprate, nonlinear condensed matter system.

1. INTRODUCTION

Everything in nature exists with an intrinsic measure and the evidence of this fact is realized by means of the fundamental called as golden ratio. Golden ratio is a universal irrational mathematical constant and appears in both macroscopic and microscopic scales, such as the design of the Egyptians' Pyramids and architecture, in music, in the physical proportion of the human body, the DNA molecule and many other aspects of life and that of the Universe. Moreover, in order to achieve harmony, balance and beauty, mankind also uses the Nature's golden ratio in their own creations of art, architecture, colors, design, and music [1]. Furthermore, in January 2010, the golden ratio has been discovered in cobalt niobate at the quantum physical level. When a strong magnetic field has been applied to cobalt niobate, it enters a "quantum critical state" and hence the spin dynamics of the system shows a fine structure with two sharp modes at low energies. Golden ratio has been determined in the ratio of the first two

frequencies of the mentioned modes [2]. In this context, it is natural to think that the quantum world has also its own special order which is described by the golden ratio, as well. From this point of view, this work is devoted to investigate the special order to describe the nature of the high temperature superconductors which is mainly based on harmony and resonance. Moreover, superconductors, which have a crucial role in the establishment of nonlinear quantum theory due to observation of some macroscopic quantum effects, also constitute a natural frame of reference to observe the quantum chaotic transitions which emanate themselves as spontaneous symmetry breakings due to nonlinear interactions within the system [3, 4]. The concept of symmetry breakings in superconductors especially in mercury based high temperature superconductors had been discussed by means of chaotic behavior (i.e. chaotic transitions) in details in our previous works [5-8]. Furthermore, solitons, which are accepted as a universal concept in nonlinear science, had also been previously predicted in the mercury based high temperature

superconductors [5]. Due to these reasons mentioned above, determination of the special order (i.e. golden ratio) in the resonance state of the nonlinear condensed matter system may give an insight for both experimentalists and theoreticians.

In this work, the golden ratio has been investigated for both the optimally and over oxygen doped high T_c copper oxide layered mercury cuprates in the context of the net effective mass of the quasi-particles and solitons' mass.

The net effective mass of the quasi-particles has already been calculated by "Ongüas Equation" which we have derived in our previous works [9-11]. The derivation of the Ongüas equation is based on an advanced analogy between the supercurrent density and the phase of the superconducting order parameter. The Ongüas Equation also enables us to determine the net effective mass of the quasi-particles by using the magnetization versus applied magnetic field experimental data obtained by high sensitive superconducting quantum interference device (SQUID). Moreover, the net effective mass of the quasi-particles is related to the occurrence of the Higgs mechanism in the superconducting state [12, 13].

Mass of solitons i.e. topological solitons so called Segâh solitons in the mercury cuprates has been calculated for the first time in this work. Topological Segâh soliton has already been defined in three dimensional quantum mechanical unit cell of the d-wave $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+x}$ (Hg-1223) superconductor in our previous works [5,14].

According to our calculations, we have theoretically discovered that m_{uv}/m^* values approach Golden ratio for both the optimally and over oxygen doped mercury cuprate superconductors for different distances in micrometer at the vicinity of absolute zero temperature. As is accepted that absolute zero temperature is the most appropriate temperature for searching the golden ratio since it represents the lowest quantum state. Moreover, it is a remarkable point that golden ratio, which is the signature of intrinsic special order appeared in the nonlinear system by means of the superconductor, is determined within micrometer scale, that is comparable with Josephson penetration depth. However, the special order appeared in micrometer interval for the optimally oxygen doped Hg-1223 superconductor has been determined ten times smaller than that of the over doped sample.

2. THEORETICAL

The derivation of the net effective mass of the quasi-particles with respect to temperature has already been expressed in our previous works [9-11]. Since calculation of the net effective mass is one of the essential steps of this work, the derivation process is expressed as follows.

The term of "net effective mass" represents the total output of the effective masses in the layered structured superconducting system since the mercury cuprate superconductors exhibit spatial (three dimensional)

resonance due to occurrence of the intrinsic Josephson effects between superconducting copper oxide layers [15,16]. The net effective mass of the quasi-particles, m^* , has been calculated by the Ongüas Equation which constitutes a relationship between the net effective mass and the phase, φ of the superconducting order parameter, Ψ . The only variable phase, φ is accepted as the most important parameter of this quantum system. Moreover, due to the intrinsic Josephson effect in mercury cuprates, for application of low magnetic fields (lower than H_{c1} at which all of the magnetic flux is totally expelled from the superconductor), the phase in the junctions is written as a function of distance x ,

$$\varphi(x) = \varphi_o \exp\left(-\frac{x}{\lambda_j}\right) \quad (1)$$

where φ_o is phase value at $x=0$ and λ_j is Josephson penetration depth [17,18].

The Josephson penetration depth, λ_j describes the penetration of the magnetic field induced by the super current flow in the superconductor [15, 19].

The magnetic field at any points of the intrinsic Josephson junction mentioned, $H(x)$ and the supercurrent density, J_s are proportional to the first and second derivation of the phase with respect to distance, respectively [17,18]. The first derivative of the velocity with respect to wave vector is proportional to the reverse of the effective mass which is well known in condensed matter physics. Due to the fact that supercurrent density depends on the velocity of the quasi-particles, in order to obtain the net effective mass of the quasi-particles, the first derivative of the J_s with respect to distance has been taken. In this context, the net effective mass formula "Ongüas Equation" has been achieved [9-11].

$$\frac{dJ_s}{dx} = \frac{1}{\varphi_o m^*} = \frac{c\Phi_o}{8\pi^2 d} \left(-\frac{1}{\lambda_j}\right)^3 \exp\left(-\frac{x}{\lambda_j}\right) \quad (2)$$

where c is the speed of light, Φ_o is the magnetic flux quantum and d is the average distance between the superconducting copper oxide layers.

"Ongüas Equation" also confirms the P.W. Anderson's suggestion that the effective mass is expected to scale like the reverse of the supercurrent density [20].

3. EXPERIMENTAL WORKS and RESULTS

In this section, the variation of the net effective mass of the quasi-particles (Cooper pairs) with distance and calculation of solitons' i.e. double helix quantum wave's mass appeared in the quantum mechanical unit cell of the superconducting system have been investigated at the

vicinity of the absolute zero temperature for both the optimally and over oxygen doped mercury cuprate superconductors, Hg-1223.

3.1. Net Effective Mass of the Quasi-Particles

The net effective mass values of the quasi-particles at zero Kelvin have been calculated by Ongüas Equation for various distance values varying from zero to λ_j . The required electrodynamics parameter of the Josephson penetration depths have already been calculated in our previous works [15,16]. In the calculation process of λ_j , the critical current densities have been calculated by Bean critical state model via the magnetization versus magnetic field data performed by SQUID [15]. All calculations have been made at or below the lower critical field magnetic field of H_{c1} . So that there is no need to consider any vortex contribution since all magnetic field applied is totally expelled from the superconductor. The related $\lambda_j = f(T)$ functions for the optimally and over oxygen doped Hg-1223 superconductors are given in Figure 1 (a) and (b), respectively.

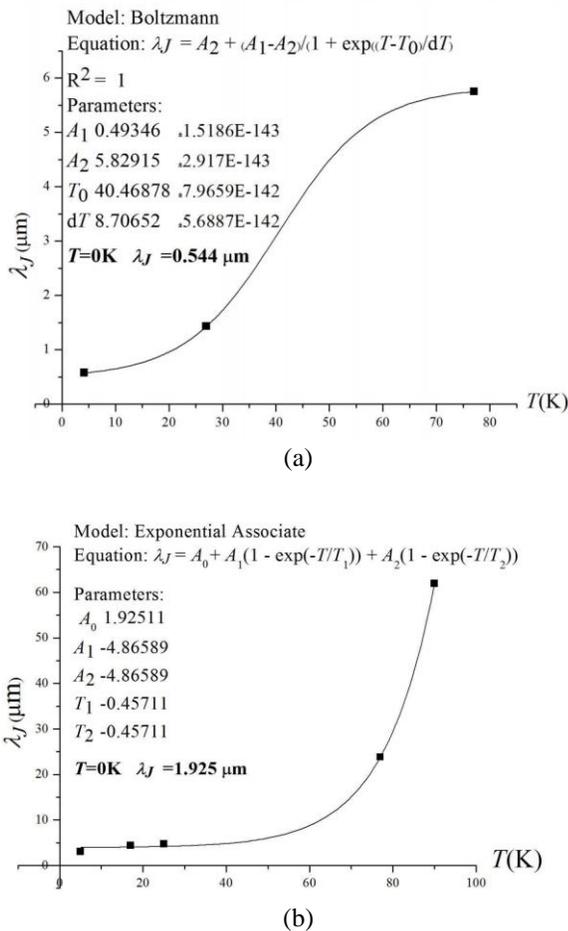


Figure 1. The Josephson penetration depth versus temperature graphics and their fitting functions for (a) the

optimally and (b) over oxygen doped Hg-1223 superconductor.

The Josephson penetration depth values at zero Kelvin have been determined by the fitting functions of λ_j in Figure 1 for the optimally and over oxygen doped Hg-1223 superconductors, respectively. The related Josephson penetration depths are given in Table 1.

Table 1. The Josephson penetration depth values at the absolute zero temperature for the optimally and over oxygen doped Hg-1223 superconductors.

Hg-1223 mercury cuprate material	λ_j (μm) at $T=0\text{K}$
Optimally O ₂ doped	0.544
Over O ₂ doped	1.925

According to Table 1 it has been determined that the magnitude of the Josephson penetration depth value of over doped sample at absolute zero increases four times with respect to that of the optimally doped samples.

The net effective mass of the quasi-particles at zero Kelvin for different distance values have been calculated by Eq. (2). The related data have been given in Table 2.

3.2. Double Helix Quantum Wave's Mass

The double helix quantum wave, DHQW, is a natural and characteristic consequence of the d-wave symmetry in the copper oxide layers of cuprate superconductors, which is represented as a four-leaf clover. As was previously explained in details in our previous works, the three dimensional double helix quantum structure can be attributed to the solitons as the time period of the wave is taken infinite in the phase space. We have called this intrinsic quantum waves as "Segâh Solitons" which refers to the topological solitons [5, 14]. Determination of the topological solitons in the mercury based superconductors enable us to discuss the system as a nonlinear condensed matter media, as well.

The topological solitons are considered as the intrinsic property of the superconducting copper oxide systems. The frequency of the DHQW that corresponds to ultraviolet (u.v.) region of electromagnetic spectrum has been calculated from Figure 2.

The average distance between copper oxide layers is $d=7.887 \times 10^{-10}$ m determined by XRD measurements. According to Figure 2, the wave length of DHQW is calculated as follows. d-wave symmetry schematically represented in the copper oxide layers as a four-leaf clover completes 360° within seven layers.

$$\lambda = 7d = 5.5209 \times 10^{-9} \text{ m}$$

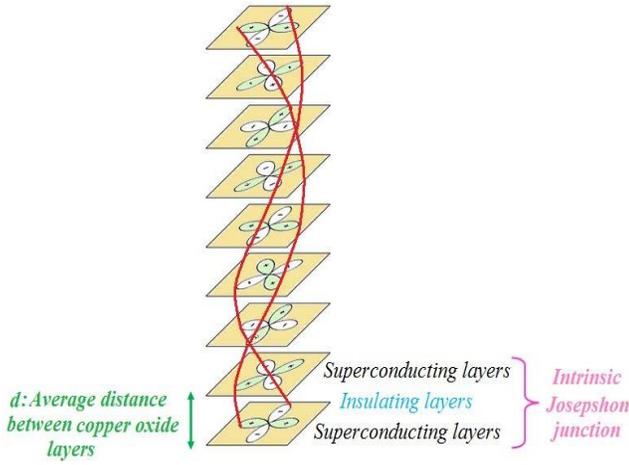


Figure 2. The schematic illustration of the DHQW (red line) as a topological solitons in the mercury cuprate superconductors [5,14]. There is a 45° phase difference between each superconducting copper oxide layer [21].

The wave length of DHQW is independent from the oxygen content, since we have determined that the parameter “ d ” is very slightly affected by oxygen annealing process. In this context, one has the right to take the same wave length value for both the optimally and over oxygen doped samples [7].

In order to calculate topological solitons’ mass, a relativistic approach has been applied. As is known that the superconducting system mentioned is considered as a relativistic frame of reference due to exhibition of two characteristics relativistic effects which are summarized as follows:

Mercury cuprate superconductors can be accepted as the most convenient and reliable candidate frame of reference to extend the comprehension to understand the emerging procedure of mass since they display electroweak symmetry breaking [5, 10, 14]. Moreover, plasma frequency of Hg-1223 system shifts from microwave to infrared. Both the occurrence of the electroweak symmetry breaking and the frequency shifting phenomenon in the mercury cuprate system enable us to deal with solitons’ mass in terms of relativistic manner [10, 14]. Solitons’ mass is abbreviated as m_{uv} since its frequency coincides with ultraviolet region of electromagnetic spectrum. In this context,

$$m_{uv} = h/\lambda c = 3.997 \times 10^{-34} \text{ kg}$$

where h is the Planck constant.

Both m_{uv} and m^* which are the intrinsic masses, define the wave and particle properties of the system, respectively. Since m_{uv} is related to d-wave symmetry character of the Hg-1223, m_{uv} value is independent from the temperature and oxygen content in the superconducting state. In this context, we have taken the same m_{uv} mass value belonging

to DHQW for the optimally and over doped Hg-1223 superconductors. Since we have already proved the three dimensional resonance for low temperatures in our previous works [15, 16, 22], it is convenient to take the ratio of the masses at the vicinity of absolute zero for searching the properties of the resonance in terms of mass.

According to data in Table 2, the ratio of m_{uv}/m^* versus distance x graphics has been given in Figure 3 for the optimally and over oxygen doped Hg-1223 superconducting materials, respectively.

Table 2. The ratios of m_{uv} and m^* at different distances for the optimally and over oxygen doped mercury cuprates at absolute zero temperature. The data written in bold has been obtained by fitting functions given in Figure 3.

The optimally oxygen doped Hg-1223		
$x(\mu\text{m})$	$m^*(\text{kg})$	m_{uv}/m^*
0	$1,618 \times 10^{-20}$	2.469×10^{-14}
0.1	$1,945 \times 10^{-20}$	2.055×10^{-14}
0.2	$2,337 \times 10^{-20}$	1.710×10^{-14}
0.23		1.618×10^{-14}
0.3	$2,808 \times 10^{-20}$	1.423×10^{-14}
0.4	$3,375 \times 10^{-20}$	1.184×10^{-14}
0.5	$4,057 \times 10^{-20}$	0.985×10^{-14}
0.6	$4,880 \times 10^{-20}$	0.819×10^{-14}
The over oxygen doped Hg-1223		
$x(\mu\text{m})$	$m^*(\text{kg})$	m_{uv}/m^*
0	$7,173 \times 10^{-19}$	5.572×10^{-16}
0.5	$9,297 \times 10^{-19}$	4.299×10^{-16}
1	$1,205 \times 10^{-18}$	3.316×10^{-16}
2.379		1.618×10^{-16}
1.5	$1,563 \times 10^{-18}$	2.557×10^{-16}
2	$2,026 \times 10^{-18}$	1.972×10^{-16}
3	$3,410 \times 10^{-18}$	1.172×10^{-16}
4	$5,734 \times 10^{-18}$	0.697×10^{-16}

According to the exponential decay fitting functions obtained by Origin Lab 8.0 in Figure 3, it has been determined that for $x=0.23\mu\text{m}$ and $x=2.361\mu\text{m}$ at $T=0\text{K}$, the magnitude of the ratio of m_{uv}/m^* approaches to the golden ratio for the optimally and over oxygen doped Hg-1223 superconductors, respectively. As is known, the magnitude of the ratios has the great importance to decide about the golden ratio since multiplication factor (10^{-14} and 10^{-16}) depends on the frame of reference investigated. As an

example, by dealing with the superconductivity in neutron stars the multiplication factor is completely different than that of a superconductor investigated in a laboratory.

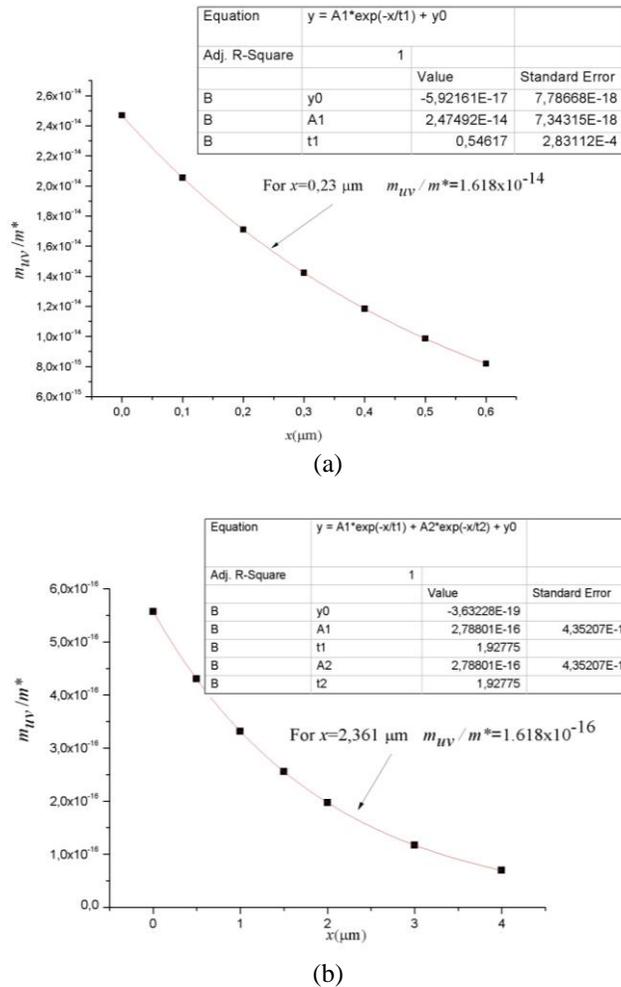


Figure 3. The ratio of m_{HV}/m^* versus distance x for (a) the optimally and (b) over oxygen doped Hg-1223 superconductor. According to the fitting functions determined by Origin Lab, golden ratio emanates itself at $x=0.23 \mu\text{m}$ and $x=2.361 \mu\text{m}$, respectively

4. DISCUSSION and CONCLUSION

In this work, the topological soliton's mass, which is of the intrinsic properties of the superconducting copper oxide layered systems, has been calculated for the first time. It has been determined that the DHQW's mass is 10^{-3} times lighter than electron's mass. Also, we have found that the ratio of solitons' mass to net effective mass of the quasi-particles i.e. the magnitude of m_{HV}/m^* approaches to the universal number so called as the golden ratio for Hg-1223 nonlinear superconducting system. The oxygen content of the system affects the distance parameter of x at which the golden ratio is observed. x values are different from London penetration depth of the samples. While x value is bigger than λ_j value

for over doped sample, it is lower than that for optimum doped. In the study, it has been interpreted that observing of the golden ratio in the ratio of masses for the nonlinear system by means of superconducting sample is an intrinsic property of the system.

Ultimately, determining the golden ratio in superconducting systems reveals the intrinsic quantum order that leads to develop the resonance in a superconducting system which is essential for the construction of various lossless electromagnetic wave sources such as terahertz devices.

REFERENCES

- [1] www.goldennumber.net.
- [2] R Coldea, D A Tennant, E M Wheeler, E Wawrzynska, D Prabhakaran, M Telling, K Habicht, P Smeibidl, K Kiefer, "Quantum criticality in an Ising chain: experimental evidence for emergent E8 symmetry", *Science*, 327, 177, 2010.
- [3] X F Pang, Y PFeng, *Quantum Mechanics in Nonlinear Systems* World Scientific Publishing, Singapore, 2005.
- [4] Physics Survey Committee, *Physics Through the 1990's Condensed Matter Physics*, National Academy Press, Washington DC, 1986.
- [5] Ü. Onbaşlı, Z. Güven Özdemir, and Ö. Aslan, "Symmetry breakings and topological solitons in mercury based d-wave Superconductors", *Chaos Solitons Fractals*, vol. 42, pp. 1980-1989, 2009.
- [6] Ö Aslan, Z. Güven Özdemir, S S Keskin, Ü Onbaşlı, "The chaotic points and XRD analysis of Hg-based superconductors", *Journal of Physics: Conference Series*, 153, 012002, 2009.
- [7] Ö. Aslan Çataltepe, "Some chaotic points in cuprate superconductors", in: *Superconductor*, A.M. Luiz, Ed., India: Sciy Company, 2010, pp. 273-290. (Available from: <http://www.intechopen.com/articles/show/title/some-chaotic-points-in-cuprate-superconductors>)
- [8] Z. Güven Özdemir, Ö. Aslan, and Ü. Onbaşlı, "Terahertz oscillations in mercury cuprate Superconductors", *Pramana-J. Phys.*, vol. 73, pp. 755-763, 2009.
- [9] Ö Aslan, Z. Güven Özdemir, Ü Onbaşlı, Correlation Between The Anisotropy and The Effective Mass of The Quasi-Particles in Oxide Superconductors, *AIP Conference Proceedings* 899 (1), 271-272, 2007.
- [10] Ü Onbaşlı, Z. Güven Özdemir, "Superconductors and Quantum-Gravity" in *Superconductor* edited by A M Luiz, Sciy Company, India, 2010, p.291 (Available from: <http://www.intechopen.com/books/superconductor/superconductors-and-quantum-gravity>)
- [11] Ö. Aslan Çataltepe, "Mercury cuprates bring symmetry breaking of the Universe to laboratory", in: *Lifetime of the Waves From Nano To Solitons In My Life*, Ü. Onbaşlı, Ed., Kerela, India: Transworld Research Network, 2012, pp. 215-243. (Available from: http://www.tnres.com/ebook/uploads/onbaslicontent/T_1350723744_5%20Onbasli.pdf)
- [12] K Kadowaki, I Kakeya, K Kindo, "Observation of the Nambu-Goldstone Mode in the High Temperature Superconductor $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$.", *Europhys.Lett.*, 42, 203, 1998.
- [13] P.W. Anderson, "Plasmons, Gauge Invariance, and Mass", *Phys. Rev.*, 130, 439, 1963.
- [14] Z. Güven Özdemir, A Aslan Çataltepe, Ü Onbaşlı, "The Correlation between the Double Helix Quantum Wave in d-Wave Superconductors and Human DNA", *Acta Physica Polonica A*, 121, 13, 2012.
- [15] Z. Güven Özdemir, Ö. Aslan, and Ü. Onbaşlı, "Determination of c-axis electrodynamic parameters of mercury cuprates", *J. Phys. Chem. Solids*, vo. 67, pp. 453-456, 2006.
- [16] Z. Güven Özdemir, Ö. Aslan, and Ü. Onbaşlı, "Calculation of microwave plasma oscillations in high temperature Superconductors", in: *The 7th International Conference on Vibration Problems (ICOVP 2005)* Springer Proceedings in Physics

- 111, E. İnan, E. Kırış (Eds.), Dordrecht, The Netherlands: Springer, 2007, pp. 377-382.
- [17] R Ferrell, R Prange, "Self-Field Limiting of Josephson Tunneling of Superconducting Electron Pairs", Phys. Rev. Lett., 10, 479, 1963.
- [18] V V Schmidt, in The Physics of Superconductors, Introduction to Fundamentals and Applications edited by P Müller and A V Ustinov Springer, Erlangen, p.81,1997
- [19] M Tinkham, Introduction to Superconductivity, McGraw-Hill, New York, 1996.
- [20] P W Anderson, The Theory of Superconductivity in the High-Tc Cuprate Superconductors, Princeton University Press, New Jersey, 1997.
- [21] M S Li, "Paramagnetic Meissner effect and related dynamical phenomena", Phys. Rep., 376, 133, 2003.
- [22] Z. Güven Özdemir, Ö. Aslan, and Ü. Onbaşlı, "Terahertz oscillations in mercury cuprate Superconductors", Pramana-J. Phys., vol. 73, pp. 755-763, 2009.