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Some properties of soft θ-topology

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Abstract

For dealing with uncertainties researchers introduced the concept of soft sets. Georgiou et al. [10] defined several basic notions on soft θ-topology and they studied many properties of them. This paper continues the study of the theory of soft θ-topological spaces and presents for this theory new definitions, characterizations, and results concerning soft θ-boundary, soft θ-exterior, soft θ-generalized closed sets, soft Λ-sets, and soft strongly pu-θ-continuity.

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1. Introduction

In 1999, Molodtsov [20] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. Also, he applied this theory to several directions (see, for example, [21-23]). The soft set theory has been applied to many different fields (see, for example, [1-2], [4-5], [7-8], [13-18], [24], [26], [28], [30]). Later, few researches (see, for example, [3], [6], [11-12], [19], [25], [27], [29]) introduced and studied the notion of soft topological spaces. Recently, in 2013, D. N. Georgiou, A. C. Megaritis, and V. I. Petropoulos [10] initiated the study of soft θ-topology. They proved that the family of all soft θ-open sets defines a soft topology on X. Consequently, they defined some basic notions of soft θ-topological spaces such as soft θ-interior point, soft θ-closure set, and soft θ-continuity and established some of their properties. This paper continues the study of the theory of soft θ-topology. It is organized as follows . The first section is the introduction. In section 2 known basic notions and results concerning the theory of soft sets, soft topological spaces and soft θ-topological spaces are given. In section 3 the notions of soft θ-boundary and soft θ-exterior are defined and some of their properties are studied. Also, some other characterizations of soft θ-closure and soft θ-interior are

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given. In section 4 the basic properties of soft θ-generalized closed sets, soft θ-generalized open sets, and soft Λ-sets are introduced. Finally, in section 5, the basic properties of soft strongly pu-θ-continuity are introduced and studied.

2. preliminaries

2.1. Definition. [20]. Let X be an initial universe set, $P(X)$ the power set of X, that is the set of all subsets of X, and A a set of parameters. A pair (F, A), where F is a map from A to $P(X)$, is called a soft set over X.
In what follows by $SS(X, A)$ we denote the family of all soft sets (F, A) over X.

2.2. Definition. [20]. Let $(F, A), (G, A) \in SS(X, A)$. We say that the pair (F, A) is a soft subset of (G, A) if $F(p) \subseteq G(p)$, for every $p \in A$. Symbolically, we write $(F, A) \sqsubseteq (G, A)$. Also, we say that the pairs (F, A) and (G, A) are soft equal if $(F, A) \sqsubseteq (G, A)$ and $(G, A) \sqsubseteq (F, A)$. Symbolically, we write $(F, A) = (G, A)$.

2.3. Definition. [20]. Let I be an arbitrary index set and $\{(F_i, A) : i \in I\} \subseteq SS(X, A)$. Then

(1) The soft union of these soft sets is the soft set $(F, A) \in SS(X, A)$, where the map $F : A \to P(X)$ is defined as follows: $F(p) = \cup\{F_i(p) : i \in I\}$, for every $p \in A$. Symbolically, we write $(F, A) = \sqcup\{(F_i, A) : i \in I\}$.

(2) The soft intersection of these soft sets is the soft set $(F, A) \in SS(X, A)$, where the map $F : A \to P(X)$ is defined as follows: $F(p) = \cap\{F_i(p) : i \in I\}$, for every $p \in A$. Symbolically, we write $(F, A) = \sqcap\{(F_i, A) : i \in I\}$.

2.4. Definition. [29]. Let $(F, A) \in SS(X, A)$. The soft complement of (F, A) is the soft set $(H, A) \in SS(X, A)$, where the map $H : A \to P(X)$ defined as follows: $H(p) = X \backslash F(p)$, for every $p \in A$. Symbolically, we write $(H, A) = (F, A)^{c}$. Obviously, $(F, A)^{c} = (F^{c}, A)$ [10]. For two given subsets $(M, A), (N, A) \in SS(X, A)$ [27], we have

(i) $((M, A) \sqcup (N, A))^c = (M, A)^c \sqcap (N, A)^c$;

(ii) $((M, A) \sqcap (N, A))^c = (M, A)^c \sqcup (N, A)^c$.

2.5. Definition. [20]. The soft set $(F, A) \in SS(X, A)$, where $F(p) = \phi$, for every $p \in A$ is called the A-null soft set of $SS(X, A)$ and denoted by $\mathbf{0}_A$. The soft set $(F, A) \in SS(X, A)$, where $F(p) = X$, for every $p \in A$ is called the A-absolute soft set of $SS(X, A)$ and denoted by $\mathbf{1}_A$.

2.6. Definition. [29]. The soft set $(F, A) \in SS(X, A)$ is called a soft point in X, denoted by e_F, if for the element $e \in A, F(e) \neq \mathbf{0}_A$ and $F(e') = \mathbf{0}_A$ for all $e' \in A \backslash \{e\}$. The set of all soft points of X is denoted by $\mathbf{SP}(X)$. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \widetilde{\in} (G, A)$, if for the element $e \in A$ and $F(e) \subseteq G(e)$.

2.7. Definition. [29]. Let $SS(X, A)$ and $SS(Y, B)$ be families of soft sets. Let $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X, A) \to SS(Y, B)$ is defined as:

(1) The image of $(F, A) \in SS(X, A)$ under f_{pu} is the soft set $f_{pu}(F, A) = (f_{pu}(F), B)$ in $SS(Y, B)$ such that

$$f_{pu}(F)(y) = \begin{cases} \bigcup\limits_{x \in p^{-1}(y)} u(F(x)), & p^{-1}(y) \neq \phi \\ \phi, & \text{otherwise} \end{cases}$$

for all $y \in B$.

(2) The inverse image of $(G, B) \in SS(Y, B)$ under f_{pu} is the soft set $f_{pu}^{-1}(G, B) = (f_{pu}^{-1}(G), A)$ in $SS(X, A)$ such that $f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$ for all $x \in A$.

2.8. Proposition. [9]. *Let $(F,A),(F_1,A)\in SS(X,A)$ and $(G,B),(G_1,B)\in SS(Y,B)$. The following statements are true:*

(1) If $(F,A)\sqsubseteq(F_1,A)$, then $f_{pu}(F,A)\sqsubseteq f_{pu}(F_1,A)$.
(2) If $(G,B)\sqsubseteq(G_1,B)$, then $f_{pu}^{-1}(G,B)\sqsubseteq f_{pu}^{-1}(G_1,B)$.
(3) $(F,A)\sqsubseteq f_{pu}^{-1}(f_{pu}(F,A))$.
(4) $f_{pu}(f_{pu}^{-1}(G,B))\sqsubseteq(G,B)$.
(5) $f_{pu}^{-1}((G,B)^{c})=(f_{pu}^{-1}(G,B))^{c}$.
(6) $f_{pu}((F,A)\sqcup(F_1,A))=f_{pu}(F,A)\sqcup f_{pu}(F_1,A)$.
(7) $f_{pu}((F,A)\sqcap(F_1,A))\sqsubseteq f_{pu}(F,A)\sqcap f_{pu}(F_1,A)$.
(8) $f_{pu}^{-1}((G,B)\sqcup(G_1,B))=f_{pu}^{-1}(G,B)\sqcup f_{pu}^{-1}(G_1,B)$.
(9) $f_{pu}^{-1}((G,B)\sqcap(G_1,B))=f_{pu}^{-1}(G,B)\sqcap f_{pu}^{-1}(G_1,B)$.

2.9. Definition. [29]. Let X be an initial universe set, A a set of parameters, and $\widetilde{\tau}\subseteq SS(X,A)$. We say that the family $\widetilde{\tau}$ defines a soft topology on X if the following axioms are true:

(1) $\mathbf{0}_A,\mathbf{1}_A\in\widetilde{\tau}$.
(2) If $(G,A),(H,A)\in\widetilde{\tau}$, then $(G,A)\sqcap(H,A)\in\widetilde{\tau}$.
(3) If $(G_i,A)\in\widetilde{\tau}$ for every $i\in I$, then $\sqcup\{(G_i,A):i\in I\}\in\widetilde{\tau}$.

The triplet $(X,\widetilde{\tau},A)$ is called a soft topological space. The members of $\widetilde{\tau}$ are called soft open sets in X. Also, a soft set (F,A) is called soft closed if the complement $(F,A)^{c}$ belongs to $\widetilde{\tau}$. The family of all soft closed sets is denoted by $\widetilde{\tau}^{c}$.

2.10. Definition. Let $(X,\widetilde{\tau},A)$ be a soft topological space and $(F,A)\in SS(X,A)$.

(1) The soft closure of (F,A) [27] is the soft set

$$Cl_S(F,A)=\sqcap\{(S,A):(S,A)\in\widetilde{\tau}^{c},(F,A)\sqsubseteq(S,A)\}.$$

(2) The soft interior of (F,A) [29] is the soft set

$$Int_S(F,A)=\sqcup\{(S,A):(S,A)\in\widetilde{\tau},(S,A)\sqsubseteq(F,A)\}.$$

2.11. Definition. [29]. A soft set (G,A) in a soft topological space $(X,\widetilde{\tau},A)$ is called a soft neighborhood (briefly: nbd) of a soft point $e_F\in\mathbf{SP}(X)$ if there exists a soft open set (H,A) such that $e_F\widetilde{\in}(H,A)\sqsubseteq(G,A)$. The soft neighborhood system of a soft point e_F, denoted by $N_{\widetilde{\tau}}(e_F)$, is the family of all of its soft neighborhoods.

2.12. Definition. [3]. Let $(X,\widetilde{\tau},A)$ be a soft topological space.

(1) A subcollection B of $\widetilde{\tau}$ is called a base for $\widetilde{\tau}$ if every member of $\widetilde{\tau}$ can be expressed as a union of members of B.

(2) A subcollection S of $\widetilde{\tau}$ is said to be a subbase for $\widetilde{\tau}$ if the family of all finite intersections of members of S forms a base for $\widetilde{\tau}$.

2.13. Definition. [29]. Let $(X,\widetilde{\tau},A)$ and $(Y,\widetilde{\tau^*},B)$ be two soft topological spaces, $u:X\to Y$ and $p:A\to B$ be mappings, and $e_F\in\mathbf{SP}(X)$.

(1) The map $f_{pu}:SS(X,A)\to SS(Y,B)$ is soft pu-continuous at $e_F\in\mathbf{SP}(X)$ if for each $(G,B)\in N_{\widetilde{\tau^*}}(f_{pu}(e_F))$, there exists $(H,A)\in N_{\widetilde{\tau}}(e_F)$ such that $f_{pu}(H,A)\sqsubseteq(G,B)$.

(2) The map $f_{pu}:SS(X,A)\to SS(Y,B)$ is soft pu-continuous on X if f_{pu} is soft pu-continuous at each soft point in X.

2.14. Definition. [12]. Let $(X,\widetilde{\tau},A)$ be a soft topological space and $(F,A)\in SS(X,A)$.

(1) (F,A) is said to be a soft generalized closed set in $(X,\widetilde{\tau},A)$ if $Cl_S(F,A)\sqsubseteq(G,A)$ whenever $(F,A)\sqsubseteq(G,A)$ and $(G,A)\in\widetilde{\tau}$. The set of all soft generalized closed sets of X is denoted by $(GC)_S(X)$.

(2) (F,A) is said to be a soft generalized open set in $(X,\widetilde{\tau},A)$ if $(F,A)^{c}$ is a soft generalized closed set. The set of all soft generalized open sets of X is denoted by $(GO)_S(X)$.

2.15. Definition. [10]. Let $(X,\widetilde{\tau},A)$ be a soft topological space. The soft θ-interior of a soft subset $(F,A)\in SS(X,A)$ is the soft union of all soft open sets over X whose soft closures are soft contained in (F,A), and is denoted by $Int_S^{\theta}(F,A)$. The soft subset (F,A) is called soft θ-open if $Int_S^{\theta}(F,A)=(F,A)$. The complement of a soft θ-open set is called soft θ-closed. Alternatively, a soft set (F,A) of X is called soft θ-closed set if $Cl_S^{\theta}(F,A)=(F,A)$, where $Cl_S^{\theta}(F,A)$ is the soft θ-closure of (F,A) and is defined to be the soft intersection of all soft closed soft subsets of X whose soft interiors contain $(F,A)^c$ [10, Proposition 5.18 (3) and Definitions 5.10 and 5.11]. We observe that $Cl_S^{\theta}(F,A)=(Int_S^{\theta}(F,A)^c)^c$ [10, Corollary 5.17 (1)]. The family of all soft θ-open sets forms a soft topology on X, denoted by $\widetilde{\tau}_{\theta}$, and is called soft θ-topology. The set of all soft θ-closed sets over X is denoted by $\widetilde{\tau}_{\theta}^{c}$.

2.16. Definition. [10]. Let $(X,\widetilde{\tau},A)$ and $(Y,\widetilde{\tau^*},B)$ be two soft topological spaces, $u: X\to Y$ and $p:A\to B$ be mappings, and $e_F\in \mathbf{SP}(X)$.

(1) The map $f_{pu}: SS(X,A)\to SS(Y,B)$ is soft pu-θ-continuous at e_F if for each $(G,B)\in N_{\widetilde{\tau^*}}(f_{pu}(e_F))$, there exists $(H,A)\in N_{\widetilde{\tau}}(e_F)$ such that $f_{pu}(Cl_S(H,A))\sqsubseteq Cl_S(G,B)$.

(2) The map $f_{pu}: SS(X,A)\to SS(Y,B)$ is soft pu-θ-continuous on X if f_{pu} is soft pu-θ-continuous at each soft point in X.

3. Soft θ-boundary and soft θ-exterior

3.1. Definition. Let $(X,\widetilde{\tau},A)$ be a soft topological space and $(F,A)\in SS(X,A)$. The soft θ-boundary of soft set (F,A) over X is denoted by $Bd_S^{\theta}(F,A)$ and is defined as $Bd_S^{\theta}(F,A)=Cl_S^{\theta}(F,A)\sqcap Cl_S^{\theta}(F^{c},A)$.

3.2. Remark. From the above definition it follows directly that the soft sets (F,A) and (F^{c},A) have same soft θ-boundary.

3.3. Proposition. *Let $(X,\widetilde{\tau},A)$ be a soft topological space and $(F,A),(G,A)\in SS(X,A)$. Then:*

(1) $Int_S^{\theta}(\mathbf{0}_A)=\mathbf{0}_A$ *and* $Int_S^{\theta}(\mathbf{1}_A)=\mathbf{1}_A$*;*

(2) $Int_S^{\theta}(F,A)\sqsubseteq (F,A)$*;*

(3) $Int_S^{\theta}(Int_S^{\theta}(F,A))\sqsubseteq Int_S^{\theta}(F,A)$*;*

(4) $(F,A)\sqsubseteq (G,A)$ *implies* $Int_S^{\theta}(F,A)\sqsubseteq Int_S^{\theta}(G,A)$*;*

(5) $Int_S^{\theta}(F,A)\sqcap Int_S^{\theta}(G,A)=Int_S^{\theta}((F,A)\sqcap (G,A))$*;*

(6) $Int_S^{\theta}(F,A)\sqcup Int_S^{\theta}(G,A)\sqsubseteq Int_S^{\theta}((F,A)\sqcup (G,A))$*.*

Proof. Obvious. ∎

The following example shows that the equalities do not hold in Proposition 3.3 (3) and (6).

3.4. Example. (1) Let $X=\{h_1,h_2\}$, $A=\{e_1,e_2\}$ and $\widetilde{\tau}=\{\mathbf{0}_A,\mathbf{1}_A,(F_1,A),(F_2,A),(F_3,A),(F_4,A)\}$, where $(F_1,A)=\{(e_1,X),(e_2,\{h_2\})\}$, $(F_2,A)=\{(e_1,\{h_1\}),(e_2,\phi)\}$, $(F_3,A)=\{(e_1,\{h_2\}),(e_2,\phi)\}$, and $(F_4,A)=\{(e_1,X),(e_2,\phi)\}$. Then $\widetilde{\tau}$ defines a soft topology on X. Let $(F,A)=\{(e_1,\{h_1\}),(e_2,X)\}$. One observe that $Int_S^{\theta}(Int_S^{\theta}(F,A))\sqsubseteq Int_S^{\theta}(F,A)$ and $Int_S^{\theta}(F,A)\neq Int_S^{\theta}(Int_S^{\theta}(F,A))$.

(2) Let $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\mathbf{0}_A, \mathbf{1}_A, (F_1, A), (F_2, A)\}$, where $(F_1, A) = \{(e_1, \{h_1\}), (e_2, \{h_1, h_2\})\}$, and $(F_2, A) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_3\})\}$. Then $\widetilde{\tau}$ defines a soft topology on X. Suppose that (F, A) $=\{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_2\})\}$, and $(G, A) = \{(e_1, \{h_2\}), (e_2, \{h_3\})\}$. One can deduce that $Int_S^{\theta}(F, A) \sqcup Int_S^{\theta}(G, A) \sqsubset Int_S^{\theta}((F, A) \sqcup (G, A))$ and $Int_S^{\theta}((F, A) \sqcup (G, A)) \neq Int_S^{\theta}(F, A) \sqcup Int_S^{\theta}(G, A)$.

3.5. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(H, A), (M, A) \in SS(X, A)$. Then:*

(1) $Cl_S^{\theta}(\mathbf{0}_A) = \mathbf{0}_A$ and $Cl_S^{\theta}(\mathbf{1}_A) = \mathbf{1}_A$;
(2) $(H, A) \sqsubseteq Cl_S^{\theta}(H, A)$;
(3) $Cl_S^{\theta}(H, A) \sqsubseteq Cl_S^{\theta}(Cl_S^{\theta}(H, A))$;
(4) $(H, A) \sqsubseteq (M, A)$ implies $Cl_S^{\theta}(H, A) \sqsubseteq Cl_S^{\theta}(M, A)$;
(5) $Cl_S^{\theta}((H, A) \sqcup (M, A)) = Cl_S^{\theta}(H, A) \sqcup Cl_S^{\theta}(M, A)$;
(6) $Cl_S^{\theta}((H, A) \sqcap (M, A)) \sqsubseteq Cl_S^{\theta}(H, A) \sqcap Cl_S^{\theta}(M, A)$.

Proof. (1), (2) and (4) are obvious.
(3) Follows from [10, Proposition 5.13 (3)].
(5) Follows from (2) above and [10, Proposition 5.13 (2)].
(6) Follows from (4) above. ∎

The following example shows that the equalities do not hold in Proposition 3.5 (3) and (6).

3.6. Example. (1) The soft topological space $(X, \widetilde{\tau}, A)$ is the same as in Example 3.4 (1). Let $(R, A) = (F, A)^c$. We have $Cl_S^{\theta}(Cl_S^{\theta}(R, A)) = \{(e_1, X), (e_2, X)\} \neq Cl_S^{\theta}(R, A) = \{(e_1, \{h_2\}), (e_2, X)\}$.
(2) The soft topological space $(X, \widetilde{\tau}, A)$ is the same as in Example 3.4 (2). Suppose that $(H, A) = (F, A)^c$ and $(M, A) = (G, A)^c$. So $Cl_S^{\theta}((H, A) \sqcap (M, A)) = \mathbf{0}_A \sqsubset Cl_S^{\theta}(H, A) \sqcap Cl_S^{\theta}(M, A) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_3\})\}$. Therefore $Cl_S^{\theta}(H, A) \sqcap Cl_S^{\theta}(M, A) \neq Cl_S^{\theta}((H, A) \sqcap (M, A))$.

3.7. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then the following statements are true.*

(1) $Bd_S^{\theta}(F, A) = Cl_S^{\theta}(F, A) \backslash Int_S^{\theta}(F, A)$.
(2) $Bd_S^{\theta}(F, A) \sqcap Int_S^{\theta}(F, A) = \mathbf{0}_A$.
(3) $(F, A) \sqcup Bd_S^{\theta}(F, A) = Cl_S^{\theta}(F, A)$.
(4) $Bd_S^{\theta}(F, A) \notin \widetilde{\tau}_{\theta}^{c}$.

Proof. (1), (2) and (3) are obvious.
(4) Let $(X, \widetilde{\tau}, A)$ be a soft topological space, where $X = \{h_1, h_2, h_3\}$, $A = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\mathbf{0}_A, \mathbf{1}_A, \{(e_1, \{h_1\}), (e_2, \{h_1\})\}, \{(e_1, \{h_2\}), (e_2, \{h_2\})\}, \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\}\}$. Then $Bd_S^{\theta}(\{(e_1, X), (e_2, \{h_1, h_3\})\}) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_2, h_3\})\} \notin \widetilde{\tau}_{\theta}^{c}$. ∎

3.8. Theorem. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then $Bd_S^{\theta}(F, A) = \mathbf{0}_A$ if and only if (F, A) is soft θ-closed and soft θ-open.*

Proof. Obvious. ∎

3.9. Theorem. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then*

(1) (F, A) is soft θ-open if and only if $(F, A) \sqcap Bd_S^{\theta}(F, A) = \mathbf{0}_A$.
(2) (F, A) is soft θ-closed if and only if $Bd_S^{\theta}(F, A) \sqsubseteq (F, A)$.

Proof. (1) *Necessity.* Follows from Proposition 3.7 (2).
Sufficiency. Follows from [29, Proposition 3.6 (1)].
(2) *Necessity.* Obvious.
Sufficiency. Follows from (1) above. ∎

3.10. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then the following statements are true.*

(1) $(F, A) \backslash Bd^{\theta}_{S}(F, A) = Int^{\theta}_{S}(F, A)$.

(2) If (F, A) is soft θ-closed, then $(F, A) \backslash Int^{\theta}_{S}(F, A) = Bd^{\theta}_{S}(F, A)$.

Proof. (1) Obvious.
(2) Follows from Proposition 3.7 (1). ∎

3.11. Definition. Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. The soft θ-exterior of (F, A) over X is denoted by $Ext^{\theta}_{S}(F, A)$ and is defined as $Ext^{\theta}_{S}(F, A) = Int^{\theta}_{S}(F, A)^{c}$.

3.12. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then the following statements are true.*

(1) $Ext^{\theta}_{S}(\mathbf{0}_A) = \mathbf{1}_A$ *and* $Ext^{\theta}_{S}(\mathbf{1}_A) = \mathbf{0}_A$.

(2) $Ext^{\theta}_{S}((F, A) \sqcup (G, A)) = Ext^{\theta}_{S}(F, A) \sqcap Ext^{\theta}_{S}(G, A)$.

(3) $Ext^{\theta}_{S}(F, A) \sqcup Ext^{\theta}_{S}(G, A) \sqsubseteq Ext^{\theta}_{S}((F, A) \sqcap (G, A))$.

(4) $Ext^{\theta}_{S}((Ext^{\theta}_{S}(F, A))^{c}) \sqsubseteq Ext^{\theta}_{S}(F, A)$.

(5) $Ext^{\theta}_{S}(F, A) \notin \widetilde{\tau}_{\theta}$.

Proof. (1), (2), (3) and (4) are obvious.
(5) See Example 3.13. ∎

The following example shows that the equalities do not hold in Proposition 3.12 (3) and (4).

3.13. Example. In Example 3.4 (1), we have $Ext^{\theta}_{S}((Ext^{\theta}_{S}(F_3, A))^{c}) \neq Ext^{\theta}_{S}(F_3, A)$ and $Ext^{\theta}_{S}(F_3, A) \notin \widetilde{\tau}_{\theta}$. In Example 3.4 (2), we obtain $Ext^{\theta}_{S}(F, A) \sqcup Ext^{\theta}_{S}(G, A) \sqsubset Ext^{\theta}_{S}((F, A) \sqcap (G, A))$ and $Ext^{\theta}_{S}((F, A) \sqcap (G, A)) \neq Ext^{\theta}_{S}(F, A) \sqcup Ext^{\theta}_{S}(G, A)$.

4. Basic properties of soft θ-generalized closed sets

4.1. Definition. Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. (F, A) is said to be a soft θ-generalized closed set in $(X, \widetilde{\tau}, A)$ if $Cl^{\theta}_{S}(F, A) \sqsubseteq (G, A)$ whenever $(F, A) \sqsubseteq (G, A)$ and $(G, A) \in \widetilde{\tau}$. The set of all soft θ-generalized closed sets over X is denoted by $(GC)^{\theta}_{S}(X)$.

4.2. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then the following statement are true.*

(1) If $(F, A) \in \widetilde{\tau}^{c}_{\theta}$, then $(F, A) \in (GC)^{\theta}_{S}$;

(2) If $(F, A) \in (GC)^{\theta}_{S}$, then $(F, A) \in (GC)_{S}$.

Proof. (1) Obvious.
(2) Follows from [10, Definition 5.11]. ∎

The converses of (1) and (2) in Proposition 4.2 are not true as illustrated by the following examples.

4.3. Example. Let $(X, \widetilde{\tau}, A)$ be the soft topological space of Example 3.4 (2) and Example 3.6 (2). Since $(F_2, A) \in \widetilde{\tau}$, $(H, A) \sqsubseteq (F_2, A)$ and $Cl_S^{\theta}(H, A) \sqsubseteq (F_2, A)$, we have $(H, A) \in (GC)_S^{\theta}$. But $(H, A) \notin \widetilde{\tau}_{\theta}^{c}$.

4.4. Example. Let $X = \{h_1, h_2\}$, $A = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\mathbf{0}_A, \mathbf{1}_A, (F_1, A), (F_2, A), (F_3, A)\}$ where $(F_1, A) = \{(e_1, X), (e_2, \{h_2\})\}$, $(F_2, A) = \{(e_1, \{h_1\}), (e_2, X)\}$, and $(F_3, A) = \{(e_1, \{h_1\}), (e_2, \{h_2\})\}$. Then $(X, \widetilde{\tau}, A)$ is a soft topological space over X. We have $(H_2, A) = (F_2, A)^c$ is a soft closed set and hence soft generalized-closed. But $(H_2, A) \notin (GC)_S^{\theta}$.

4.5. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F_1, A), (F_2, A) \in SS(X, A)$. If $(F_1, A), (F_2, A) \in (GC)_S^{\theta}$, then $(F_1, A) \sqcup (F_2, A) \in (GC)_S^{\theta}$.*

Proof. Follows from Proposition 3.5 (5). ∎

4.6. Corollary. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A), (G, A) \in SS(X, A)$. Then the following statement are true.*

(1) If $(F, A) \in \widetilde{\tau}_{\theta}^{c}$ and $(G, A) \in (GC)_S^{\theta}$, then $(F, A) \sqcup (G, A) \in (GC)_S^{\theta}$.

(2) If $(F, A) \in (GC)_S^{\theta}$ and $(G, A) \in (GC)_S$, then $(F, A) \sqcup (G, A) \in (GC)_S$.

Proof. (1) Follows from Proposition 4.2 (1) and Proposition 4.5.

(2) Follows from Proposition 4.2 (2) and [12, Theorem 3.5]. ∎

4.7. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then the following statement are true.*

(1) If $(F, A) \in \widetilde{\tau}$ and $(F, A) \in (GC)_S^{\theta}$, then $(F, A) \in \widetilde{\tau}_{\theta}^{c}$.

(2) If $\widetilde{\tau} = \widetilde{\tau}_{\theta}^{c}$, then every soft subset of X is in $(GC)_S^{\theta}$.

Proof. Clear. ∎

4.8. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A) \in SS(X, A)$. Then $(G, A) \in (GC)_S^{\theta}$ if and only if the only soft closed soft subset of $Cl_S^{\theta}(G, A) \backslash (G, A)$ is $\mathbf{0}_A$.*

Proof. Necessity. Let $(F, A) \in \widetilde{\tau}^{c}$ such that $(F, A) \sqsubseteq Cl_S^{\theta}(G, A) \backslash (G, A) = Cl_S^{\theta}(G, A) \sqcap (G, A)^c$ which implies that $(F, A) \sqsubseteq Cl_S^{\theta}(G, A), (F, A) \sqsubseteq (G, A)^c$. Thus $(G, A) \sqsubseteq (F, A)^c$. Since $(G, A) \in (GC)_S^{\theta}$ and $(F, A)^c \in \widetilde{\tau}$, we have $Cl_S^{\theta}(G, A) \sqsubseteq (F, A)^c$ or $(F, A) \sqsubseteq (Cl_S^{\theta}(G, A))^c$. Since $(F, A) \sqsubseteq Cl_S^{\theta}(G, A)$, we have $(F, A) \sqsubseteq (Cl_S^{\theta}(G, A))^c \sqcap Cl_S^{\theta}(G, A) = \mathbf{0}_A$. This shows that $(F, A) = \mathbf{0}_A$.

Sufficiency. Suppose that $(G, A) \sqsubseteq (U, A)$ and that $(U, A) \in \widetilde{\tau}$. If $Cl_S^{\theta}(G, A) \not\sqsubseteq (U, A)$, then $Cl_S^{\theta}(G, A) \sqcap (U, A)^c$ is a non-A-null soft closed soft subset of $Cl_S^{\theta}(G, A) \backslash (G, A)$, a contradiction. Therefore $Cl_S^{\theta}(G, A) \sqsubseteq (U, A)$ and $(G, A) \in (GC)_S^{\theta}$. ∎

4.9. Corollary. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space, $(F, A) \in SS(X, A)$ and $(F, A) \in (GC)_S^{\theta}$. Then $(F, A) \in \widetilde{\tau}_{\theta}^{c}$ if and only if $Cl_S^{\theta}(F, A) \backslash (F, A) \in \widetilde{\tau}^{c}$.*

4.10. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. Then $(F, A) \in (GC)_S^{\theta}$ if and only if $(F, A) \sqcup (Cl_S^{\theta}(F, A))^c \in (GC)_S^{\theta}$.*

Proof. Follows from Proposition 4.8. ∎

4.11. Lemma. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A) \in SS(X, A)$. If $(F, A) \in \widetilde{\tau}_{\theta}^{c}$, then $(F, A) \in \widetilde{\tau}^{c}$.*

The converse of Lemma 4.11 is not true in general as illustrated by the following example.

4.12. Example. Let $X = \{h_1, h_2\}, A = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\mathbf{0}_A, \mathbf{1}_A, (F_1, A)\}$ is a soft topology over X, where $(F_1, A) = \{(e_1, X), (e_2, \{h_2\})\}$. We observe that $(H_1, A) = (F_1, A)^c \in \widetilde{\tau}^c$. But $(H_1, A) \notin \widetilde{\tau}_\theta^c$.

4.13. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A) \in SS(X, A)$. Then $(G, A) \in (GC)_S^\theta$ if and only if $(G, A) = (F, A) \backslash (H, A)$, where $(F, A) \in \widetilde{\tau}_\theta^c$ and the only soft closed soft subset of (H, A) is $\mathbf{0}_A$.*

Proof. Necessity. Follows from Proposition 4.8.
Sufficiency. Follows from Lemma 4.11. ∎

4.14. Definition. Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A) \in SS(X, A)$. (G, A) is said to be a soft θ-generalized open set in $(X, \widetilde{\tau}, A)$ if $(G, A)^c$ is soft θ-generalized closed. The set of all soft θ-generalized open sets over X is denoted by $(GO)_S^\theta(X)$.

4.15. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A), (F, A) \in SS(X, A)$. Then $(G, A) \in (GO)_S^\theta$ if and only if $(F, A) \sqsubseteq Int_S^\theta(G, A)$ whenever $(F, A) \sqsubseteq (G, A)$ and $(F, A) \in \widetilde{\tau}^c$.*

Proof. Obvious. ∎

As a direct consequence of Proposition 4.2 we have

4.16. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A) \in SS(X, A)$. Then*

(1) If $(G, A) \in \widetilde{\tau}_\theta$, then $(G, A) \in (GO)_S^\theta$;
(2) If $(G, A) \in (GO)_S^\theta$, then $(G, A) \in (GO)_S$.

The converses of (1) and (2) in Proposition 4.16 are not true as illustrated by the following examples.

4.17. Example. Let $(X, \widetilde{\tau}, A)$ be the soft topological space of Example 3.4 (2) and Example 3.6 (2). Since $(R_2, A) \in \widetilde{\tau}^c$, $(R_2, A) \sqsubseteq (F, A)$ and $(R_2, A) \sqsubseteq Int_S^\theta(F, A)$, we have $(F, A) \in (GO)_S^\theta$. But $(F, A) \notin \widetilde{\tau}_\theta$.

4.18. Example. The soft topological space $(X, \widetilde{\tau}, A)$ is the same as in Example 4.4. We observe that $(F_2, A) \in (GO)_S$. But $(F_2, A) \notin (GO)_S^\theta$.

4.19. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G_1, A), (G_2, A) \in SS(X, A)$. If $(G_1, A), (G_2, A) \in (GO)_S^\theta$, then $(G_1, A) \sqcap (G_2, A) \in (GO)_S^\theta$.*

Proof. Follows from Proposition 3.3 (5). ∎

4.20. Corollary. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(F, A), (G, A) \in SS(X, A)$.*

(1) If $(F, A) \in \widetilde{\tau}_\theta$ and $(G, A) \in (GO)_S^\theta$, then $(F, A) \sqcap (G, A) \in (GO)_S^\theta$.
(2) If $(F, A) \in (GO)_S^\theta$ and $(G, A) \in (GO)_S$, then $(F, A) \sqcap (G, A) \in (GO)_S$.

Proof. (1) Follows from Proposition 4.16 (1) and Proposition 4.19 .
(2) Follows from Proposition 4.16 (2) and [12, Theorem 4.5]. ∎

4.21. Proposition. *Let $(X, \widetilde{\tau}, A)$ be a soft topological space and $(G, A) \in SS(X, A)$. Then $(G, A) \in (GO)_S^\theta$ if and only if $(U, A) = \mathbf{1}_A$ whenever $(U, A) \in \widetilde{\tau}$ and $Int_S^\theta(G, A) \sqcup (G, A)^c \sqsubseteq (U, A)$.*

Proof. *Necessity.* Follows from Proposition 4.8.

Sufficiency. Obvious. ∎

4.22. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(G, A) \in SS(X, A)$. *Then* $(G, A) \in (GC)_S^\theta$ *if and only if* $Cl_S^\theta(G, A) \backslash (G, A) \in (GO)_S^\theta$.

Proof. *Necessity.* Follows from Propositions 4.8 and 4.15.

Sufficiency. Suppose that $(U, A) \in \widetilde{\tau}$ such that $(G, A) \sqsubseteq (U, A)$ or $(U, A)^c \sqsubseteq (G, A)^c$. Now, $Cl_S^\theta(G, A) \sqcap (U, A)^c \sqsubseteq Cl_S^\theta(G, A) \sqcap (G, A)^c = Cl_S^\theta(G, A) \backslash (G, A)$ and since $Cl_S^\theta(G, A) \sqcap (U, A)^c \in \widetilde{\tau}^c$ and $Cl_S^\theta(G, A) \backslash (G, A) \in (GO)_S^\theta$, it follows that $Cl_S^\theta(G, A) \sqcap (U, A)^c \sqsubseteq Int_S^\theta(Cl_S^\theta(G, A) \backslash (G, A)) = \mathbf{0}_A$. Therefore $Cl_S^\theta(G, A) \sqcap (U, A)^c = \mathbf{0}_A$ or $Cl_S^\theta(G, A) \sqsubseteq (U, A)$. Hence $(G, A) \in (GC)_S^\theta$. ∎

4.23. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(F, A), (G, A) \in SS(X, A)$. *If* $(G, A) \in \widetilde{\tau}^c$ *and* $(G, A) \in (GO)_S^\theta$, *then* $(G, A) \in \widetilde{\tau}_\theta$.

Proof. Obvious. ∎

4.24. Definition. A soft set (F, A) in a soft topological space $(X, \widetilde{\tau}, A)$ is said to be soft Λ-set if $(F, A) = (F, A)^\Lambda$, where $(F, A)^\Lambda = \sqcap\{(G, A) \in \widetilde{\tau} : (F, A) \sqsubseteq (G, A)\}$.

4.25. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(F, A), (H, A), (F_i, A) \in SS(X, A), i \in I$. *Then the following statements are true.*

(1) $(F, A) \sqsubseteq (F, A)^\Lambda$.

(2) If $(F, A) \sqsubseteq (H, A)$, *then* $(F, A)^\Lambda \sqsubseteq (H, A)^\Lambda$.

(3) $((F, A)^\Lambda)^\Lambda = (F, A)^\Lambda$.

(4) $(\sqcap_{i \in I} (F_i, A))^\Lambda \sqsubseteq \sqcap_{i \in I} (F_i, A)^\Lambda$.

(5) $(\sqcup_{i \in I} (F_i, A))^\Lambda = \sqcup_{i \in I} (F_i, A)^\Lambda$.

Proof. Clear. ∎

The following example shows that the equality does not hold in Proposition 4.25 (4).

4.26. Example. Let us consider the soft topological space $(X, \widetilde{\tau}, A)$ over X in Example 3.4 (2). One can deduce that $(F, A)^\Lambda \sqcap (G, A)^\Lambda \neq ((F, A) \sqcap (G, A))^\Lambda$.

4.27. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space. Then the following statements are true.*

(1) $\mathbf{0}_A$ *and* $\mathbf{1}_A$ *are soft* Λ*-sets.*

(2) Every soft union of soft Λ*-sets is a soft* Λ*-set.*

(3) Every soft intersection of soft Λ*-sets is a soft* Λ*-set.*

Proof. Follows from Proposition 4.25. ∎

4.28. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(F, A) \in SS(X, A)$. *Then* $(F, A) \in (GC)_S^\theta$ *if and only if* $Cl_S^\theta(F, A) \sqsubseteq (F, A)^\Lambda$.

Proof. Clear. ∎

4.29. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(F, A) \in SS(X, A)$. *Let* (F, A) *be a soft* Λ*-set. Then* $(F, A) \in (GC)_S^\theta$ *if and only if* $(F, A) \in \widetilde{\tau}_\theta^c$.

Proof. *Necessity.* Follows from Proposition 4.28.

Sufficiency. Follows from the fact that every soft θ-closed set is soft θ-generalized closed (Proposition 4.2(1)). ∎

4.30. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *be a soft topological space and* $(F, A) \in SS(X, A)$. *If* $(F, A)^{\Lambda} \in (GC)_S^{\theta}$, *then* $(F, A) \in (GC)_S^{\theta}$.

Proof. Clear. ∎

5. Soft strongly pu-θ-continuity

In this section, we introduce the notion of soft strongly pu-θ-continuity of functions induced by two mappings $u : X \to Y$ and $p : A \to B$ on soft topological spaces $(X, \widetilde{\tau}, A)$ and $(Y, \widetilde{\tau^*}, B)$.

5.1. Definition. Let $(X, \widetilde{\tau}, A)$ and $(Y, \widetilde{\tau^*}, B)$ be two soft topological spaces, $u : X \to Y$ and $p : A \to B$ be mappings, and $e_F \in \mathbf{SP}(X)$.

(1) The map $f_{pu} : SS(X, A) \to SS(Y, B)$ is soft strongly pu-θ-continuous at e_F if for each $(G, B) \in N_{\widetilde{\tau^*}}(f_{pu}(e_F))$, there exists $(H, A) \in N_{\widetilde{\tau}}(e_F)$ such that $f_{pu}(Cl_S(H, A)) \sqsubseteq (G, B)$.

(2) The map $f_{pu} : SS(X, A) \to SS(Y, B)$ is soft strongly pu-θ-continuous on X if f_{pu} is soft strongly pu-θ-continuous at each soft point in X.

5.2. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *and* $(Y, \widetilde{\tau^*}, B)$ *be two soft topological spaces. Then the following statements are equivalent.*

(1) The map $f_{pu} : SS(X, A) \to SS(Y, B)$ *is soft strongly pu-*θ*-continuous;*

(2) For each $(G, B) \in \widetilde{\tau^*}$, $f_{pu}^{-1}(G, B) \in \widetilde{\tau}_{\theta}$*;*

(3) For each $(H, B) \in (\widetilde{\tau^*})^c$, $f_{pu}^{-1}(H, B) \in \widetilde{\tau}_{\theta}^{c}$.

Proof. Similar to the proof of [29, Theorem 6.3]. ∎

5.3. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *and* $(Y, \widetilde{\tau^*}, B)$ *be two soft topological spaces,* $u : X \to Y$, $p : A \to B$ *and* $f_{pu} : SS(X, A) \to SS(Y, B)$ *be mappings. Then the following statements are true.*

(1) If f_{pu} *is soft strongly pu-*θ*-continuous, then* f_{pu} *is soft pu-continuous.*

(2) If f_{pu} *is soft strongly pu-*θ*-continuous, then* f_{pu} *is soft pu-*θ*-continuous.*

Proof. (1) Obvious.

(2) Follows from (1) and [10, Proposition 5.26]. ∎

The converses of (1) and (2) in Proposition 5.3 are not true as illustrated by the following example.

5.4. Example. Let $X = \{h_1, h_2, h_3\}, Y = \{m_1, m_2, m_3\}, A = \{e_1, e_2\}$, and $B = \{u_1, u_2\}$. We consider the soft topology $\widetilde{\tau} = \{\mathbf{0}_A, \mathbf{1}_A, \{(e_1, \{h_3\}), (e_2, \{h_1, h_2\})\}, \{(e_1, \phi), (e_2, \{h_3\})\}, \{(e_1, \{h_3\}), (e_2, X)\}\}$ over X and the soft topology $\widetilde{\tau^*} = \{\mathbf{0}_B, \mathbf{1}_B, \{(u_1, \{m_1\}), (u_2, \{m_3\})\}, \{(u_1, \{m_1, m_2\}), (u_2, \{m_3\})\}\}$ over Y. Let $u : X \to Y$ be the map such that $u(h_1) = u(h_2) = m_1$ and $u(h_3) = m_3$ and $p : A \to B$ be the map such that $p(e_1) = u_2$ and $p(e_2) = u_1$. Then, the map $f_{pu} : SS(X, A) \to SS(Y, B)$ is both soft pu-continuous and soft pu-θ-continuous but it is not soft strongly pu-θ-continuous.

5.5. Proposition. *Let* $(X, \widetilde{\tau}, A)$ *and* $(Y, \widetilde{\tau^*}, B)$ *be two soft topological spaces and* S^* *be a soft subbase of* $\widetilde{\tau^*}$. *A map* $f_{pu} : SS(X, A) \to SS(Y, B)$ *is soft strongly pu-*θ*-continuous if and only if for each* $(G, B) \in \mathrm{S}^*, f_{pu}^{-1}(G, B) \in \widetilde{\tau}_{\theta}$.

Proof. *Necessity.* Follows from Proposition 5.2.

Sufficiency. Follows from [10, Proposition 5.7] and Proposition 5.2. ∎

5.6. Proposition. *Let* $(X,\widetilde{\tau},A)$ *and* $(Y,\widetilde{\tau^*},B)$ *be two soft topological spaces. Then the following statements are equivalent.*
(1) The map $f_{pu}: SS(X,A)\to SS(Y,B)$ *is soft strongly pu-*θ*-continuous;*
(2) For each $(F,A)\in SS(X,A), f_{pu}(Cl_S^{\theta}(F,A))\sqsubseteq Cl_S(f_{pu}(F,A))$*;*
(3) For each $(G,B)\in SS(Y,B), Cl_S^{\theta}(f_{pu}^{-1}(G,B))\sqsubseteq f_{pu}^{-1}(Cl_S(G,B))$*.*

Proof. (1)⇒(2) Follows from Proposition 5.2 (3).
(2)⇒(3) This is trivial.
(3)⇒(1) Let $e_F\in \mathbf{SP}(X)$ and $(M,B)\in N_{\widetilde{\tau^*}}(f_{pu}(e_F))$. Since $(M,B)^c\in(\widetilde{\tau^*})^c$, we have $Cl_S^{\theta}(f_{pu}^{-1}(M,B)^c)\sqsubseteq f_{pu}^{-1}(Cl_S(M,B)^c)=f_{pu}^{-1}(M,B)^c$. Therefore $f_{pu}^{-1}(M,B)^c=(f_{pu}^{-1}(M,B))^c\in\widetilde{\tau}_{\theta}^{c}$ and so $f_{pu}^{-1}(M,B)\in\widetilde{\tau}_{\theta}$. Moreover, $e_F\widetilde{\in}f_{pu}^{-1}(M,B)$. There exists $(U,A)\in N_{\widetilde{\tau}}(e_F)$ such that $Cl_S(U,A)\sqsubseteq f_{pu}^{-1}(M,B)$. Therefore $f_{pu}(Cl_S(U,A))\sqsubseteq(M,B)$. Hence f_{pu} is soft strongly pu-θ-continuous. ∎

5.7. Definition. A soft set (F,A) in a soft topological space $(X,\widetilde{\tau},A)$ is called a soft θ-neighborhood of a soft point $e_F\in\mathbf{SP}(X)$ if there exists a soft open set (G,A) such that $e_F\widetilde{\in}(G,A)\sqsubseteq Cl_S(G,A)\sqsubseteq(F,A)$. The soft θ-neighborhood system of a soft point e_F, denoted by $N_{\widetilde{\tau}_{\theta}}(e_F)$, is the family of all its soft θ-neighborhoods.

Note that a soft θ-neighborhood is not necessarily a soft neighborhood in the soft θ-topology.

5.8. Proposition. *The soft* θ*-neighborhood system* $N_{\widetilde{\tau}_{\theta}}(e_F)$ *at* e_F *in a soft topological space* $(X,\widetilde{\tau},A)$ *has the following properties:*
(1) If $(F,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*, then* $e_F\widetilde{\in}(F,A)$*.*
(2) If $(F,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$ *and* $(F,A)\sqsubseteq(G,A)$*, then* $(G,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*.*
(3) If $(F,A),(G,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*, then* $(F,A)\sqcap(G,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*.*
(4) If $(F,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*, then there is a* $(H,A)\in N_{\widetilde{\tau}_{\theta}}(e_F)$ *such that* $(F,A)\in N_{\tau_{\theta}}(e'_M)$ *for each* $e'_M\widetilde{\in}(H,A)$*.*

Proof. Similar to the proof of [29, Theorem 4.10]. ∎

The main results can be paraphrased as follows: soft pu-θ-continuity corresponds to f_{pu}^{-1} (soft θ-neighborhood)= soft θ-neighborhood and strong pu-θ-continuity corresponds to f_{pu}^{-1} (soft neighborhood)= soft θ-neighborhood.

5.9. Proposition. *Let* $(X,\widetilde{\tau},A)$ *and* $(Y,\widetilde{\tau^*},B)$ *be two soft topological spaces,* $u:X\to Y$ *and* $p:A\to B$ *be mappings. Then the following statements are equivalent.*
(1) f_{pu} *is soft pu-*θ*-continuous;*
(2) For each $e_F\in\mathbf{SP}(X)$ *and* $(H,B)\in N_{\widetilde{\tau^*}}(f_{pu}(e_F)), f_{pu}^{-1}(H,B)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*.*

Proof. (1)⇒(2) Follows from Proposition 2.8 (2) and (3).
(2)⇒(1) Follows from Propositions 5.8 (2) and 2.8 (1) and (4). ∎

5.10. Proposition. *Let* $(X,\widetilde{\tau},A)$ *and* $(Y,\widetilde{\tau^*},B)$ *be two soft topological spaces,* $u:X\to Y$ *and* $p:A\to B$ *be mappings. Then the following statements are equivalent.*
(1) f_{pu} *is soft strongly pu-*θ*-continuous;*
(2) For each $e_F\in\mathbf{SP}(X)$ *and* $(H,B)\in N_{\widetilde{\tau^*}}(f_{pu}(e_F)), f_{pu}^{-1}(H,B)\in N_{\widetilde{\tau}_{\theta}}(e_F)$*.*

Proof. Similar to the proof of Proposition 5.9. ∎

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