Hacettepe Journal of Mathematics and Statistics
Volume 43 (3) (2014), 461 – 471

Modified ratio estimators using stratified ranked set sampling

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Abstract

Stratified Ranked Set Sampling (SRSS) combines the advantages of stratification and Ranked set sampling (RSS) to obtain an unbiased estimator for the population mean, with potentially significant gains in efficiency. The present paper deals with modified ratio estimators of finite population mean using information on coefficient of variation and co-efficient of kurtosis of auxiliary variable in Stratified Ranked Set Sampling. It has been shown that these methods are highly beneficial to the estimation based on Stratified Simple Random Sampling (SSRS). The bias and mean squared error of the proposed estimators with large sample approximation are derived. Theoretically, it is shown that these suggested estimators are asymptotically more efficient than the estimators in stratified simple random sampling. The results have been illustrated by numerical example.

Keywords: Stratified ranked set sampling, Ratio-type estimators, Ranked set sampling, Auxiliary variables, Mean squared error, Population mean, Coefficient of variation, Coefficient of kurtosis, Efficiency.

1. Introduction

The literature on ranked set sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Ranked set sampling (RSS) was first suggested by McIntyre (1952) and Stratified Ranked Set Sampling was introduced by Samawi (1996) to increase the efficiency of estimator of population mean. The performance of the combined and the separate ratio estimates using the stratified ranked set sample (SRSS) was given by Samawi and Siam (2003). Kadilar et al. (2009) used ranked set sampling to improve ratio estimator given by Prasad (1989). Here we use the idea of SRSS instead of SSRS to improve the precision of ratio estimators given by Kadilar and Cingi (2003).

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The usual ratio estimator given by Cochran (1977) for the population mean $\overline{Y}$ in stratified random sampling is defined by

$$\overline{y}_{SSRS} = \overline{y}_{st}(\frac{\overline{X}}{\overline{x}_{st}}) \tag{1.1}$$

where $\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$ and $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$ are the unbiased estimators of population mean $\overline{Y}$ and $\overline{X}$ respectively. Here $W_h = \frac{N_h}{N}$ is the weight of h^{th} stratum, where N_h is the h^{th} stratum size and N is the total population size $(h = 1, 2, \ldots, L)$ and L is the total number of strata in the population.

When the population coefficient of variation for h^{th} stratum C_{x_h} is known and motivated by Sisodia and Dwivedi(1981),Kadilar and Cingi(2003) suggested a modified ratio estimator for $\overline{Y}$ in stratified random sampling as

$$\overline{y}_{stSD} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h + C_{x_h})}{\sum_{h=1}^{L} W_h(\overline{x}_h + C_{x_h})} \tag{1.2}$$

When coefficient of kurtosis for h^{th} stratum, $\beta_{2h}(x)$ is known and motivated by Singh and Kakran(1993), Kadilar and Cingi(2003) developed ratio-type estimator for $\overline{Y}$ as

$$\overline{y}_{stSK} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h + \beta_{2h}(x))}{\sum_{h=1}^{L} W_h(\overline{x}_h + \beta_{2h}(x))} \tag{1.3}$$

Kadilar and Cingi (2003) considered ratio type estimators based on Upadhyaya and Singh (1999), using both coefficient of variation and kurtosis in stratified random sampling as

$$\overline{y}_{stUS1} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h \beta_{2h}(x) + C_{x_h})}{\sum_{h=1}^{L} W_h(\overline{x}_h \beta_{2h}(x) + C_{x_h})} \tag{1.4}$$

$$\overline{y}_{stUS2} = \overline{y}_{st} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h C_{x_h} + \beta_{2h}(x))}{\sum_{h=1}^{L} W_h(\overline{x}_h C_{x_h} + \beta_{2h}(x))} \tag{1.5}$$

To the first degree of approximation the mean squared error(MSE) of the estimators $\overline{y}_{SSRS}, \overline{y}_{stSD}, \overline{y}_{stSK}, \overline{y}_{stUS1}$ and $\overline{y}_{stUS2}$ respectively are

$$(1.6)\qquad MSE(\overline{y}_{SSRS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}(S_{y_h}^2 + R^2 S_{x_h}^2 - 2RS_{x_h y_h})$$

$$(1.7)\qquad MSE(\overline{y}_{stSD}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}(S_{y_h}^2 + R^2 \lambda_1^2 S_{x_h}^2 - 2R\lambda_1 S_{x_h y_h})$$

$$(1.8)\qquad MSE(\overline{y}_{stSK}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}(S_{y_h}^2 + R^2 \lambda_2^2 S_{x_h}^2 - 2R\lambda_2 S_{x_h y_h})$$

$$(1.9)\qquad MSE(\overline{y}_{stUS1}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}(S_{y_h}^2 + R^2 \gamma_1^2 S_{x_h}^2 - 2R\gamma_1 S_{x_h y_h})$$

$$(1.10)\qquad MSE(\overline{y}_{stUS2}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}(S_{y_h}^2 + R^2 \gamma_2^2 S_{x_h}^2 - 2R\gamma_2 S_{x_h y_h})$$

where $\lambda_1 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h}{\sum_{h=1}^{L} W_h(\overline{X}_h + C_{x_h})}$, $\lambda_2 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h}{\sum_{h=1}^{L} W_h(\overline{X}_h + \beta_{2h}(x))}$, $\gamma_1 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h \beta_{2h}(x)}{\sum_{h=1}^{L} W_h(\overline{X}_h \beta_{2h}(x) + C_{x_h})}$, $\gamma_2 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h(\overline{X}_h C_{x_h} + \beta_{2h}(x))}$, $S_{y_h}^2 = \frac{\sum_{i=1}^{N_h}(Y_{hi} - \overline{Y}_h)^2}{N_h - 1}$, $S_{x_h}^2 = \frac{\sum_{i=1}^{N_h}(X_{hi} - \overline{Y}_h)^2}{N_h - 1}$ and $S_{y_h x_h} = \frac{\sum_{i=1}^{N_h}(Y_{hi} - \overline{Y}_h)(X_{hi} - \overline{X}_h)}{N_h - 1}$.

2. Stratified ranked set sample

In ranked set sampling, r independent random sets, each of size r and each unit in the set being selected with equal probability and without replacement , are selected from the population. The members of each random set are ranked with respect to the characteristic of the study variable or auxiliary variable. Then, the smallest unit is selected from the first ordered set and the second smallest unit is selected from the second ordered set. By this way, this procedure is continued until the unit with the largest rank is chosen from the r^{th} set. This cycle may be repeated m times, so $mr(= n)$ units have been measured during this process.

In stratified ranked set sampling, for the h^{th} stratum of the population, first choose r_h independent samples each of size r_h , $h = 1, 2, \ldots, L$. Rank each sample, and use RSS scheme to obtain L independent RSS samples of size r_h , one from each stratum. Let $r_1 + r_2 + \ldots + r_L = r$. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ elements have been obtained. A modification of the above procedure is suggested here to be used for the estimation of the ratio using stratified ranked set sample. For the h^{th} stratum, first choose r_h independent samples each of size r_h of independent bivariate elements from the h^{th} subpopulation (Stratum) , $h = 1, 2, \ldots, L$. Rank each sample with respect to one of the variables say Y or X. Then use the RSS sampling scheme to obtain L independent RSS samples of size r_h one from each stratum. This complete one cycle of stratified ranked set sample. The cycle may be repeated m times until $n = mr$ bivariate elements have been obtained. We will use the following notation for the stratified ranked set sample when the ranking is on the variable X. For the k^{th} cycle and the h^{th} stratum, the SRSS is denoted by $\{(Y_{h[1]k}, X_{h(1)k}), (Y_{h[2]k}, X_{h(2)k}), \ldots, (Y_{h[r_h]k}, X_{h(r_h)k}) : k = 1, 2, \ldots, m; h = 1, 2, \ldots, L\}$, where $Y_{h[i]k}$ is the i^{th} Judgment ordering in the i^{th} set for the study variable and $X_{h(i)k}$ is the i^{th} order statistic in the i^{th} set for the auxiliary variable.

The combined ratio estimator of population mean $\overline{Y}$ given by Samawi and Siam (2003), using stratified ranked set sampling is defined as

$$(2.1) \qquad \overline{y}_{SSRS} = \overline{y}_{[SRSS]}\left(\frac{\overline{X}}{\overline{x}_{(SRSS)}}\right)$$

where $\overline{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \overline{y}_{h[r_h]}$ and $\overline{x}_{(SRSS)} = \sum_{h=1}^{L} W_h \overline{x}_{h(r_h)}$.

The Bias and MSE of the estimator $\overline{y}_{SRSS}$ to the first degree of approximation are respectively given by

$$(2.2) \qquad B(\overline{y}_{SRSS}) = \overline{Y}\Big[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{S_{x_h}^2}{\overline{X}^2} - \frac{S_{x_h y_h}}{\overline{X}\overline{Y}}\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{m}{n_h}(\sum_{i=1}^{r_h} D_{x_h(i)}^2 - \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]})\}\Big]$$

and

$$(2.3) \qquad MSE(\overline{y}_{SRSS}) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\big[\{S_{y_h}^2 + R^2 S_{x_h}^2 - 2RS_{x_h y_h}\} - \overline{Y}^2\{\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - D_{x_h(i)})^2\}\big]$$

where $n_h = mr_h, D_{y_h[i]}^2 = \frac{\tau_{y_h[i]}^2}{\overline{Y}^2}, D_{x_h(i)}^2 = \frac{\tau_{x_h(i)}^2}{\overline{X}^2}$ and $D_{x_h(i) y_h[i]} = \frac{\tau_{x_h(i) y_h[i]}}{\overline{Y}\overline{X}}$. Here we would also like to remind that $\tau_{x_h(i)} = \mu_{x_h(i)} - \overline{X}_h, \tau_{y_h[i]} = \mu_{y_h[i]} - \overline{Y}_h$ and $\tau_{x_h(i) y_h[i]} = (\mu_{x_h(i)} - \overline{X}_h)(\mu_{y_h[i]} - \overline{Y}_h)$ where $\mu_{x_h(i)} = E[x_{h(i)}], \mu_{y_h(i)} = E[y_{h(i)}], \overline{X}_h$ and $\overline{Y}_h$ are the means of the h^{th} stratum for the variables X and Y, respectively.

3. Proposed estimators in stratified ranked set sampling

Motivated by estimators given from (1.2) to (1.5) that incorporation of more and more parameters on auxiliary variable help increase the efficiencies of estimators and motivated by Kadilar and Cingi (2003), we propose ratio-type estimator for $\overline{Y}$ using stratified ranked set sampling, when the population coefficient of variation C_{x_h} of auxiliary variable from stratum to stratum $(h = 1, 2, \ldots, L)$, is known, as

$$(3.1) \qquad \overline{y}_{strMM1} = \overline{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h + C_{x_h})}{\sum_{h=1}^{L} W_h(\overline{x}_{h(r_h)} + C_{x_h})}$$

where $\overline{y}_{[SRSS]} = \sum_{h=1}^{L} W_h \overline{y}_{h[r_h]}$ and $\overline{x}_{(SRSS)} = \sum_{h=1}^{L} W_h \overline{x}_{h(r_h)}$ are the stratified ranked set sample means for variables and respectively.

To obtain bias and MSE of $\overline{y}_{strMM1}$, we put $\overline{y}_{[SRSS]} = \overline{Y}(1+\delta_0)$ and $\overline{x}_{(SRSS)} = \overline{X}(1+\delta_1)$ so that $E(\delta_0) = E(\delta_1) = 0$.

Now $V(\delta_0) = E(\delta_0^2) = \frac{V(\overline{y}_{[SRSS]}}{\overline{Y}^2} = \sum_{h=1}^{L} W_h^2 \frac{1}{mr_h} \frac{1}{\overline{Y}^2}[S_{y_h}^2 - \frac{m}{n_h} \sum_{i=1}^{r_h} \tau_{y_h[i]}^2]$
$= \sum_{h=1}^{L} \frac{W_h^2}{n_h}[\frac{S_{y_h}^2}{\overline{Y}^2} - \frac{m}{n_h} \sum_{i=1}^{r_h} D_{y_{h[i]}}^2]$.

Similarly,$E(\delta_1^2) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}[\frac{S_{x_h}^2}{\overline{X}^2} - \frac{m}{n_h} \sum_{i=1}^{r_h} D_{x_{h(i)}}^2]$

and $E(\delta_0, \delta_1) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}[\frac{S_{x_h y_h}}{\overline{X}\overline{Y}} - \frac{m}{n_h} \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]}]$

Further to validate first degree of approximation, we assume that the sample size is large enough to get $|\delta_0|$ and $|\delta_1|$ as small so that the terms involving δ_0 and or δ_1 with degree greater than two will be negligible.

The Bias and MSE of the estimator $\overline{y}_{strMM1}$ to the first degree of approximation are respectively, given by

$B(\overline{y}_{strMM1}) = E(\overline{y}_{strMM1}) - \overline{Y}$

Here $\overline{y}_{strMM1} = \overline{Y}(1+\delta_0)(1+\lambda_1\delta_1)^{-1}$, where $\lambda_1 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h}{\sum_{h=1}^{L} W_h(\overline{X}_h + C_{x_h})}$.

Now $E(\overline{y}_{strMM1}) = \overline{Y}[1 + \lambda_1^2 E(\delta_1^2) - \lambda_1 E(\delta_0\delta_1)]$,because$E(\delta_0) = E(\delta_1) = 0$.

(using taylor series expansion, where $O(\delta_1)$are power terms of δ_1 with powers more than 2 are neglected)

$$(3.2) \qquad B(\overline{y}_{strMM1}) = \overline{Y}[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{\lambda_1^2 S_{x_h}^2}{\overline{X}^2} - \frac{\lambda_1 S_{x_h y_h}}{\overline{X}\overline{Y}}\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{m}{n_h}(\lambda_1^2 \sum_{i=1}^{r_h} D_{x_h(i)}^2 - \lambda_1 \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]})\}]$$

Now $MSE(\overline{y}_{strMM1}) = E(\overline{y}_{strMM1} - \overline{Y})^2 = \overline{Y}^2 E[\delta_0^2 + \lambda_1^2\delta_1^2 - 2\lambda_1\delta_0\delta_1]$

$= \overline{Y}^2[\sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{y_h}^2}{\overline{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_{h[i]}}^2) + \lambda_1^2 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h}^2}{\overline{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}}^2)$
$-2\lambda_1 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h y_h}}{\overline{XY}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}y_{h[i]}})]$

$$(3.3) \qquad \implies MSE(\overline{y}_{strMM1}) = \sum_{h=1}^{L} \frac{{W_h}^2}{n_h}\big[(S_{y_h}{}^2 + R^2\lambda_1^2 S_{x_h}^2 - 2R\lambda_1 S_{x_h y_h}) - \overline{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \lambda_1 D_{x_h(i)})^2\big]$$

Adapting the estimators in (1.3) given by Kadilar and Cingi (2003), we propose another new ratio type estimator in stratified ranked set sampling as follows

$$(3.4) \qquad \overline{y}_{strMM2} = \overline{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h[\overline{X}_h + \beta_{2h}(x)]}{\sum_{h=1}^{L} W_h[\overline{x}_{h(r_h)} + \beta_{2h}(x)]}$$

The Bias and MSE of $\overline{y}_{strMM2}$ can be found as follows-

$B(\overline{y}_{strMM2}) = E(\overline{y}_{strMM2}) - \overline{Y}$

Here $\overline{y}_{strMM2} = \overline{Y}(1+\delta_0)(1+\lambda_2\delta_1)^{-1}$, where $\lambda_2 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h}{\sum_{h=1}^{L} W_h(\overline{X}_h + \beta_{2h}(x))}$.

Now $E(\overline{y}_{strMM2}) = \overline{Y}[1 + \lambda_2^2 E(\delta_1^2) - \lambda_2 E(\delta_0\delta_1)]$,because$E(\delta_0) = E(\delta_1) = 0$.

$$(3.5) \qquad \implies B(\overline{y}_{strMM2}) = \overline{Y}\Big[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{\lambda_2^2 S_{x_h}^2}{\overline{X}^2} - \frac{\lambda_2 S_{x_h y_h}}{\overline{XY}}\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{m}{n_h}(\lambda_2^2 \sum_{i=1}^{r_h} D_{x_h(i)}^2 - \lambda_2 \sum_{i=1}^{r_h} D_{x_h(i)y_h[i]})\}\Big]$$

Now $MSE(\overline{y}_{strMM2}) = E(\overline{y}_{strMM2} - \overline{Y})^2 = \overline{Y}^2 E[\delta_0^2 + \lambda_2^2\delta_1^2 - 2\lambda_2\delta_0\delta_1]$

$= \overline{Y}^2[\sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{y_h}^2}{\overline{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_{h[i]}}^2) + \lambda_2^2 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h}^2}{\overline{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}}^2)$
$-2\lambda_2 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h y_h}}{\overline{XY}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}y_{h[i]}})]$

$$(3.6) \qquad \implies MSE(\overline{y}_{strMM2}) = \sum_{h=1}^{L} \frac{{W_h}^2}{n_h}\big[(S_{y_h}{}^2 + R^2\lambda_2^2 S_{x_h}^2 - 2R\lambda_2 S_{x_h y_h}) - \overline{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \lambda_2 D_{x_h(i)})^2\big]$$

Motivated by estimators (1.4) and (1.5) by Kadilar and Cingi (2003), we now propose two more ratio ÔÇôtype estimators , considering both coefficients of variation and kurtosis in stratified ranked set sampling as follows

$$(3.7)\qquad \overline{y}_{strMM3} = \overline{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h(\overline{X}_h \beta_{2h}(x) + C_{x_h})}{\sum_{h=1}^{L} W_h(\overline{x}_{h(r_h)} \beta_{2h}(x) + C_{x_h})}$$

$$(3.8)\qquad \overline{y}_{strMM4} = \overline{y}_{[SRSS]} \frac{\sum_{h=1}^{L} W_h[\overline{X}_h C_{x_h} + \beta_{2h}(x)]}{\sum_{h=1}^{L} W_h[\overline{x}_{h(r_h)} C_{x_h} + \beta_{2h}(x)]}$$

The Bias and MSE of $\overline{y}_{strMM3}$ can be found as follows-

$B(\overline{y}_{strMM3}) = E(\overline{y}_{strMM3}) - \overline{Y}$

Here $\overline{y}_{strMM2} = \overline{Y}(1+\delta_0)(1+\gamma_1\delta_1)^{-1}$, where$\gamma_1 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h \beta_{2h}(x)}{\sum_{h=1}^{L} W_h(\overline{X}_h \beta_{2h}(x) + C_{x_h})}$

Now $E(\overline{y}_{strMM3}) = \overline{Y}[1 + \gamma_1^2 E(\delta_1^2) - \gamma_1 E(\delta_0\delta_1)]$, because$E(\delta_0) = E(\delta_1) = 0$.

$$(3.9)\qquad \implies B(\overline{y}_{strMM3}) = \overline{Y}[\sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{\gamma_1^2 S_{x_h}^2}{\overline{X}^2} - \frac{\gamma_1 S_{x_h y_h}}{\overline{X}\overline{Y}}\} - \sum_{h=1}^{L} \frac{W_h^2}{n_h}\{\frac{m}{n_h}(\gamma_1^2 \sum_{i=1}^{r_h} D_{x_h(i)}^2 - \gamma_1 \sum_{i=1}^{r_h} D_{x_h(i) y_h[i]})\}]$$

Now $MSE(\overline{y}_{strMM3}) = E(\overline{y}_{strMM3} - \overline{Y})^2 = \overline{Y}^2 E[\delta_0^2 + \gamma_1^2\delta_1^2 - 2\gamma_1\delta_0\delta_1]$

$= \overline{Y}^2[\sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{y_h}^2}{\overline{Y}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{y_{h[i]}}^2) + \gamma_1^2 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h}^2}{\overline{X}^2} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_{h(i)}}^2)$
$-2\gamma_1 \sum_{h=1}^{L} \frac{W_h^2}{n_h}(\frac{S_{x_h y_h}}{\overline{X}\overline{Y}} - \frac{m}{n_h}\sum_{i=1}^{r_h} D_{x_h(i) y_h[i]})]$

$$(3.10)\qquad \implies MSE(\overline{y}_{strMM3}) = \sum_{h=1}^{L} \frac{{W_h}^2}{n_h}\left[(S_{y_h}{}^2 + R^2\gamma_1^2 S_{x_h}^2 - 2R\gamma_1 S_{x_h y_h}) - \overline{Y}^2 \frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \gamma_1 D_{x_h(i)})^2\right]$$

Similarly bias and mean squared error of the estimator $\overline{y}_{strMM4}$ can be obtained respectively by changing the place of coefficient of kurtosis and coefficient of variation, as

$$(3.11)\quad B(\overline{y}_{strMM4}) = \overline{Y}\Big[\sum_{h=1}^{L}\frac{W_h^2}{n_h}\{\frac{\gamma_2^2 S_{x_h}^2}{\overline{X}^2} - \frac{\gamma_2 S_{x_h y_h}}{\overline{XY}}\} - \sum_{h=1}^{L}\frac{W_h^2}{n_h}\{\frac{m}{n_h}(\gamma_2^2\sum_{i=1}^{r_h} D_{x_h(i)}^2 - \gamma_2\sum_{i=1}^{r_h} D_{x_h(i)y_h[i]})\}\Big]$$

and

$$(3.12)\quad MSE(\overline{y}_{strMM4}) = \sum_{h=1}^{L}\frac{{W_h}^2}{n_h}\big[({S_{y_h}}^2 + R^2\gamma_2^2 S_{x_h}^2 - 2R\gamma_2 S_{x_h y_h}) - \overline{Y}^2\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \gamma_2 D_{x_h(i)})^2\big]$$

where $\gamma_2 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h(\overline{X}_h C_{x_h} + \beta_{2h}(x))}$.

4. Efficiency Comparison

On comparing $(1.7), (1.8), (1.9)$ and (1.10) with $(3.3), (3.6), (3.10)$ and (3.12) respectively, we obtain

1) $MSE(\overline{y}_{stSD}) - MSE(\overline{y}_{strMM1}) = A_1 \geq 0$,
where $A_1 = \overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \lambda_1 D_{x_h(i)})^2$
$\implies MSE(\overline{y}_{stSD}) \geq MSE(\overline{y}_{strMM1})$

2) $MSE(\overline{y}_{stSK}) - MSE(\overline{y}_{strMM2}) = A_2 \geq 0$,
where $A_2 = \overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \lambda_2 D_{x_h(i)})^2$
$\implies MSE(\overline{y}_{stSK}) \geq MSE(\overline{y}_{strMM2})$

3) $MSE(\overline{y}_{stUS1}) - MSE(\overline{y}_{strMM3}) = A_3 \geq 0$,
where $A_3 = \overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \gamma_1 D_{x_h(i)})^2$
$\implies MSE(\overline{y}_{stUS1}) \geq MSE(\overline{y}_{strMM3})$

4) $MSE(\overline{y}_{stUS2}) - MSE(\overline{y}_{strMM4}) = A_4 \geq 0$,
where $A_4 = \overline{Y}^2\sum_{h=1}^{L}\frac{W_h^2}{n_h}\frac{m}{n_h}\sum_{i=1}^{r_h}(D_{y_h[i]} - \gamma_2 D_{x_h(i)})^2$
$\implies MSE(\overline{y}_{stUS2}) \geq MSE(\overline{y}_{strMM4})$

It is easily seen that the MSE of the proposed estimators given in (3.1) ,(3.4), (3.7), and (3.8) are always smaller than the estimator given in (1.2) to (1.5) respectively, because A_1, A_2, A_3 and A_4 all are non-negative values and thus it is shown that the proposed ratio types estimators $\overline{y}_{strMM1}, \overline{y}_{strMM2}$, $\overline{y}_{strMM3}$ and $\overline{y}_{strMM4}$ for the population mean using stratified ranked set sampling are asymptotically more efficient than the ratio estimators $\overline{y}_{stSD}, \overline{y}_{stSK}$, $\overline{y}_{stUS1}$ and $\overline{y}_{stUS2}$ given by Kadilar and Cingi (2003).

Table 1. Population Statistics

Stratum 1	Stratum 2	Stratum 3
$N_1 = 12$	$N_2 = 30$	$N_3 = 17$
$n_1 = 9$	$n_2 = 15$	$n_3 = 12$
$W_1 = 0.2034$	$W_2 = 0.5085$	$W_3 = 0.2881$
$\overline{X}_1 = 5987.83$	$\overline{X}_2 = 11682.73$	$\overline{X}_3 = 68662.29$
$\overline{Y}_1 = 11788$	$\overline{Y}_2 = 16862.27$	$\overline{Y}_3 = 227371.53$
$S^2_{x_1} = 27842810.5$	$S^2_{x_2} = 760238523$	$S^2_{x_3} = 12187889050$
$S^2_{y_1} = 153854583$	$S^2_{y_2} = 2049296094$	$S^2_{y_3} = 372428238550$
$S_{y_1x_1} = 62846173.1$	$S_{y_2x_2} = 1190767859$	$S_{y_3x_3} = 27342963562$
$C_{x_1} = 0.8812$	$C_{x_2} = 2.3601$	$C_{x_3} = 1.6079$
$\beta_{21}(x) = 1.8733$	$\beta_{22}(x) = 10.7527$	$\beta_{23}(x) = 8.935$
$R_1 = 1.97$	$R_2 = 1.44$	$R_3 = 3.31$

5. Numerical Illustration

To compare efficiencies of various proposed estimators of our study, here, we take a stratified population with 3 strata with sizes 12,30 & 17, respectively on page 1119 (Appendix) given by Singh(2003). The example considers the data of Tobacco for Area and Production in specified countries during 1998, where y is production (study variable) in metric tons and x is area (auxiliary variable) in hectares.

For the above population, the parameters are summarized in Table 1. Note that total population size $N = 59, \overline{Y} = 76485.42, \overline{X} = 26942.29$

From this population we took 5 ranked set samples of sizes $r_1 = 3, r_2 = 5$ and $r_3 = 4$ from stratum 1st , 2nd and 3rd respectively. Further each ranked set sample from each stratum were repeated with number of cycles $m = 3$.Hence sample sizes of stratified ranked set samples equivalent to stratified simple random samples of sizes $n_h(= mr_h)$ on considering arbitrary allocation.

The relative efficiency of the estimator is given by $RE = \frac{\overline{y}_{SSRS}}{\overline{y}_{SRRS}} * 100$

The estimated relative efficiencies of various stratified ranked set estimators in comparison with different stratified SRS estimators are shown in Table 2.

From Table 2, we see that stratified ranked set estimators are more efficient than corresponding stratified SRS estimators. Thus, if coefficient of variation and coefficient of kurtosis are known for auxiliary variable x,then, these proposed estimators can be used in practice.

Table 2

Variances of various stratified SRS estimators	$\overline{y}_{SSRS}$ 2245922261	$\overline{y}_{stSD}$ 2245878377	$\overline{y}_{stSK}$ 2245739510	$\overline{y}_{stUS1}$ 2245913748	$\overline{y}_{stUS2}$ 2245816148
Variances of corresponding stratified ranked set sampling estimators & Relative Efficiencies in % for	$\overline{y}_{SRSS}$	$\overline{y}_{strMM1}$	$\overline{y}_{strMM2}$	$\overline{y}_{strMM3}$	$\overline{y}_{strMM4}$
Sample 1	1938096069 115.8229	1938091889 115.8809	1938094290 115.8736	1938092227 115.8827	1938095047 115.8775
Sample 2	22135875617 105.1523	2135831913 105.1524	2135693676 105.1527	2135870922 105.1523	2135769977 105.1525
Sample 3	1414957661 158.7272	1414914192 158.729	1414776801 158.7345	1414952994 158.727361	1414852647 158.7315
Sample 4	2017429690 111.3259	2017405990 111.3251	2017338937 111.3219	2017427349 111.32582	2017377001 111.3236
Sample 5	1523423535 147.426	1523375065 147.4278	1523219867 147.4337	1523418280 147.426191	1523305279 147.4305

6. Conclusion

We have proposed new ratio-type estimators for strati´$\frac{1}{4}$üed ranked set sampling on the lines of the estimators of Kadilar and Cingi (2003) and obtained their MSE equations. By these equations, the MSEÔÇÖs of proposed estimators have been compared with corresponding stratified simple random sampling estimators given by Kadilar and Cingi (2003) and found that the proposed estimators have smaller MSEÔÇÖs than the corresponding estimators. These theoretical results have been supported by the above numerical example and thus it is concluded that the proposed ratio type estimators $\overline{y}_{SRSS}$, $\overline{y}_{strMM1}$, $\overline{y}_{strMM2}$, $\overline{y}_{strMM3}$ and $\overline{y}_{strMM4}$, for the population mean using stratified ranked set sampling are more efficient asymptotically than the corresponding ratio estimators $\overline{y}_{SSRS}$, $\overline{y}_{stSD}$, $\overline{y}_{stSK}$, $\overline{y}_{stUS1}$ and $\overline{y}_{stUS2}$ using SRS. With this conclusion, we hope to develop new estimators in other sampling methods in the forthcoming studies.

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