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Warped product pseudo-slant submanifolds of $(LCS)_n$-manifolds

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Abstract: The object of the present paper is to study warped product pseudo-slant submanifolds of $(LCS)_n$-manifolds. We study the existence or non-existence of such submanifolds. The existence is also ensured by an example.

Keywords: Warped product, pseudo-slant submanifold, $(LCS)_n$-manifold.

1 Introduction

The notion of warped product manifolds were introduced by Bishop and O'Neill [4] and later it was studied by many mathematicians and physicists. These manifolds are generalization of Riemannian product manifolds. The existence or non-existence of warped product manifolds plays some important role in differential geometry as well as physics.

The notion of slant submanifolds in a complex manifold was introduced and studied by B.-Y. Chen [8], which is a natural generalization of both invariant and anti-invariant submanifolds. B.-Y. Chen [8] also found examples of slant submanifolds of complex Euclidean space C^2 and C^4. Then Lotta [14] has defined and studied slant immersions of a Riemannian manifold into an almost contact metric manifold and proved some properties of such immersions. Thereafter, many authors studied slant submanifolds of almost contact metric manifolds.

In [18], N. Papaghiuc introduced the notion of semi-slant submanifolds of almost Hermitian manifolds. Then Cabrerizo et. al [5] defined and investigated semi-slant submanifolds of Sasakian manifolds. In this connection, it may be mentioned that Sahin [19] studied warped product semi-slant submanifolds of Kaehler manifolds. Also in [1], Atceken studied warped product semi-slant submanifolds in locally Riemannian product manifolds. Again Atceken [2] studied warped product semi-slant submanifolds in Kenmotsu manifolds. Beside these, Uddin and his co-authors studied warped product submanifolds in different context such as ([13], [28]) etc. Recently, Hui and Atceken [10] studied warped product semi-slant submanifolds of $(LCS)_n$-manifolds.

Next, A. Carriazo [7] defined and studied bi-slant submanifolds in almost Hermitian manifolds and simultaneously gave the notion of pseudo-slant submanifolds in almost Hermitian manifolds. The contact version of pseudo-slant submanifolds has been defined and studied by Khan and Khan in [12]. In this connection it may be mentioned that Atceken and Hui [3] studied slant and pseudo-slant submanifolds of $(LCS)_n$-manifolds. Recently, Khan and Chahal [11] have been studied warped product pseudo-slant submanifold of trans-Sasakian manifolds.

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In 2003, Shaikh [20] introduced the notion of Lorentzian concircular structure manifolds (briefly, $(LCS)_n$-manifolds), with an example, which generalizes the notion of LP-Sasakian manifolds introduced by Matsumoto [15] and also by Mihai and Rosca [16]. Then Shaikh and Baishya ([22], [23]) investigated the applications of $(LCS)_n$-manifolds to the general theory of relativity and cosmology. The $(LCS)_n$-manifolds is also studied by Hui [9], Hui and Atceken ([3], [10]), Shaikh and his co-authors ([21], [24], [25], [26], [27]) and many others.

Motivated by the studies the object of the present paper is to study warped product pseudo-slant submanifolds of $(LCS)_n$-manifolds. The paper is organized as follows. Section 2 is concerned with preliminaries. In section 3, we study warped product pseudo-slant submanifolds of $(LCS)_n$-manifolds. It is shown that there do not exist warped product pseudo-slant submanifolds of an $(LCS)_n$-manifold $\overline{M}$ of the type $M = N_\perp \times_f N_\theta$ such that $N_\perp$ and N_θ are anti-invariant and proper slant submanifolds of $\overline{M}$, respectively such that ξ is tangent to N_θ, where as the warped products of the form $M = N_\perp \times_f N_\theta$ exist, whenever ξ is tangent to N_θ. Finally, the existence of such submanifolds is ensured by an interesting example.

2 Preliminaries

An n-dimensional Lorentzian manifold $\overline{M}$ is a smooth connected paracompact Hausdorff manifold with a Lorentzian metric g, that is, $\overline{M}$ admits a smooth symmetric tensor field g of type (0,2) such that for each point $p \in \overline{M}$, the tensor $g_p : T_p\overline{M} \times T_p\overline{M} \to \mathbb{R}$ is a non-degenerate inner product of signature $(-,+,\cdots,+)$, where $T_p\overline{M}$ denotes the tangent vector space of $\overline{M}$ at p and $\mathbb{R}$ is the real number space. A non-zero vector $v \in T_p\overline{M}$ is said to be timelike (resp., non-spacelike, null, spacelike) if it satisfies $g_p(v,v) < 0$ (resp, ≤ 0, = 0, > 0) [17].

Definition 1. *In a Lorentzian manifold* $(\overline{M}, g)$ *a vector field* P *defined by*

$$g(X,P) = A(X),$$

for any $X \in \Gamma(T\overline{M})$, *is said to be a concircular vector field* [30] *if*

$$(\overline{\nabla}_X A)(Y) = \alpha\{g(X,Y) + \omega(X)A(Y)\}$$

where α *is a non-zero scalar and* ω *is a closed 1-form and* $\overline{\nabla}$ *denotes the operator of covariant differentiation with respect to the Lorentzian metric* g.

Let $\overline{M}$ be an n-dimensional Lorentzian manifold admitting a unit timelike concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

$$g(\xi,\xi) = -1. \tag{1}$$

Since ξ is a unit concircular vector field, it follows that there exists a non-zero 1-form η such that for

$$g(X,\xi) = \eta(X), \tag{2}$$

the equation of the following form holds

$$(\overline{\nabla}_X \eta)(Y) = \alpha\{g(X,Y) + \eta(X)\eta(Y)\} \quad (\alpha \neq 0) \tag{3}$$

$$\overline{\nabla}_X \xi = \alpha\{X + \eta(X)\xi\}, \quad \alpha \neq 0 \tag{4}$$

for all vector fields X, Y, where $\overline{\nabla}$ denotes the operator of covariant differentiation with respect to the Lorentzian metric g and α is a non-zero scalar function satisfies

$$\overline{\nabla}_X \alpha = (X\alpha) = d\alpha(X) = \rho\eta(X), \tag{5}$$

ρ being a certain scalar function given by $\rho = -(\xi\alpha)$. If we put

$$\phi X = \frac{1}{\alpha}\overline{\nabla}_X \xi, \tag{6}$$

then from (4) and (6) we have

$$\phi X = X + \eta(X)\xi, \tag{7}$$

$$g(\phi X, Y) = g(X, \phi Y) \tag{8}$$

from which it follows that ϕ is a symmetric (1,1) tensor and called the structure tensor of the manifold. Thus the Lorentzian manifold $\overline{M}$ together with the unit timelike concircular vector field ξ, its associated 1-form η and an (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly, $(LCS)_n$-manifold), [20]. Especially, if we take $\alpha = 1$, then we can obtain the LP-Sasakian structure of Matsumoto [15]. In a $(LCS)_n$-manifold $(n > 2)$, the following relations hold [20].

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y), \tag{9}$$

$$\phi^2 X = X + \eta(X)\xi, \tag{10}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X), \tag{11}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y], \tag{12}$$

$$R(\xi,Y)Z = (\alpha^2 - \rho)[g(Y,Z)\xi - \eta(Z)Y], \tag{13}$$

$$(\overline{\nabla}_X \phi)Y = \alpha\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\}, \tag{14}$$

$$(X\rho) = d\rho(X) = \beta\eta(X), \tag{15}$$

$$R(X,Y)Z = \phi R(X,Y)Z + (\alpha^2 - \rho)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\xi, \tag{16}$$

for all $X, Y, Z \in \Gamma(T\overline{M})$ and $\beta = -(\xi\rho)$ is a scalar function, where R is the curvature tensor and S is the Ricci tensor of the manifold.

Let M be a submanifold of a $(LCS)_n$-manifold $\overline{M}$ with induced metric g. Also let ∇ and $\nabla^{\perp}$ are the induced connections on the tangent bundle TM and the normal bundle $T^{\perp}M$ of M respectively. Then the Gauss and Weingarten formulae are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X,Y) \tag{17}$$

and

$$\overline{\nabla}_X V = -A_V X + \nabla^{\perp}_X V \tag{18}$$

for all $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$, where h and A_V are second fundamental form and the shape operator (corresponding to the normal vector field V) respectively for the immersion of M into $\overline{M}$. The second fundamental form h and the shape operator A_V are related by

$$g(h(X,Y),V) = g(A_V X, Y), \tag{19}$$

for any $X, Y \in \Gamma(TM)$ and $V \in \Gamma(T^{\perp}M)$.

For any $X \in \Gamma(TM)$, we can write

$$\phi X = EX + FX, \tag{20}$$

where EX is the tangential component and FX is the normal component of ϕX.

Also, for any $V \in \Gamma(T^{\perp}M)$, we have

$$\phi V = BV + CV, \tag{21}$$

where BV and CV are also the tangential and normal components of ϕV respectively. From (20) and (21), we can derive the tensor fields E, F, B and C are also symmetric, because ϕ is symmetric. Also from (8) and (20) we have

$$g(EX,Y) = g(X,EY) \tag{22}$$

for any $X, Y \in \Gamma(TM)$.

Throughout the paper, we consider ξ to be tangent to M. The submanifold M is said to be invariant if F is identically zero, i.e., $\phi X \in \Gamma(TM)$ for any $X \in \Gamma(TM)$. Also M is said to be anti-invariant if E is identically zero, that is $\phi X \in \Gamma(T^{\perp}M)$ for any $X \in \Gamma(TM)$.

For any $X, Y \in \Gamma(TM)$, we have from (7), (17) and (20) that

$$\nabla_X \xi = \alpha EX, \tag{23}$$

$$h(X,\xi) = \alpha FX. \tag{24}$$

Definition 2. *Let M be a submanifold of $(LCS)_n$-manifold $\overline{M}$. For each non-zero vector X tangent to M at x, the angle $\theta(x)$, $0 \leq \theta(x) \leq \frac{\pi}{2}$ between ϕX and EX is called the slant angle or the Wirtinger angle. If the slant angle is constant then the submanifold is called the slant submanifold. Invariant and anti-invariant submanifolds are particular slant submanifolds with slant angle $\theta = 0$ and $\theta = \frac{\pi}{2}$ respectively. A slant submanifold is said to be proper slant if the slant angle θ lies strictly between 0 and $\frac{\pi}{2}$, i.e., $0 < \theta < \frac{\pi}{2}$* [6].

Theorem 1.[2] *Let M be a submanifold of a $(LCS)_n$-manifold $\overline{M}$ such that ξ is tangent to M. Then M is slant submanifold if and only if there exists a constant $\lambda \in [0,1]$ such that*

$$E^2 = \lambda(I + \eta \otimes \xi). \tag{25}$$

Moreover, if θ is the slant angle of M, then $\lambda = \cos^2\theta$.
Also from (25) we have

$$g(EX,EY) = \cos^2\theta[g(X,Y) + \eta(X)\eta(Y)], \tag{26}$$

$$g(FX,FY) = \sin^2\theta[g(X,Y) + \eta(X)\eta(Y)] \tag{27}$$

for any X, Y tangent to M.

Definition 3. *Let $\overline{M}$ be a $(LCS)_n$-manifold and M be an immersed submanifold in $\overline{M}$. Then M is said to be pseudo-slant submanifold of $\overline{M}$ if there exist two orthogonal complementary distributions D_θ and $D^\perp$ such that*

(i) $TM = D^\perp \oplus D_\theta \oplus < \xi >$,

(ii) *the distribution $D^\perp$ is anti-invariant, that is, $\phi(D^\perp) \subseteq (T^\perp M)$,*

(iii) *the distribution D_θ is slant with slant angle $\theta \neq \frac{\pi}{2}$.*

From the above definition, it is obvious that if $\theta = 0$ and $\theta = \frac{\pi}{2}$, then the pseudo slant submanifold becomes semi-invariant submanifold and anti-invariant submanifold, respectively. On the other hand, if we denote the dimensions of D_θ and $D^\perp$ by d_1 and d_2, respectively, then we have the following cases.

(i) if $d_1 = 0$, then M is an anti-invariant submanifold,
(ii) if d_2 and $\theta = 0$, then M is an invariant submanifold,
(iii) if $d_2 = 0$ and $\theta \neq 0$, then M is a proper slant submanifold.

A pseudo submanifold is called proper if $d_1.d_2 \neq 0$, $\theta \neq 0$ and $\theta \neq \frac{\pi}{2}$.

In this connection it may be mentioned that Atceken and Hui [3] studied pseudo-slant submanifolds of $(LCS)_n$-manifolds.

The notion of warped product manifolds were introduced by Bishop and O'Neill [4].

Definition 4. *Let (N_1, g_1) and (N_2, g_2) be two Riemannian manifolds with Riemannian metric g_1 and g_2 respectively and f be a positive definite smooth function on N_1. The warped product of N_1 and N_2 is the Riemannian manifold $N_1 \times_f N_2 = (N_1 \times N_2, g)$, where*

$$g = g_1 + f^2 g_2. \tag{28}$$

A warped product manifold $N_1 \times_f N_2$ is said to be trivial if the warping function f is constant.
More explicitely, if the vector fields X and Y are tangent to $N_1 \times_f N_2$ at (p,q) then

$$g(X,Y) = g_1(\pi_1 * X, \pi_1 * Y) + f^2(p) g_2(\pi_2 * X, \pi_2 * Y),$$

where π_i $(i = 1,2)$ are the canonical projections of $N_1 \times N_2$ onto N_1 and N_2 respectively and * stands for the derivative map.

Let $M = N_1 \times_f N_2$ be warped product manifold, which means that N_1 and N_2 are totally geodesic and totally umbilical submanifolds of M respectively.
For warped product manifolds, we have [17].

Proposition 1. *Let $M = N_1 \times_f N_2$ be a warped product manifold. Then*

(i) $\nabla_X Y \in TN_1$ *is the lift of* $\nabla_X Y$ *on* N_1,
(ii) $\nabla_U X = \nabla_X U = (X \ln f)U$,
(iii) $\nabla_U V = \nabla'_U V - g(U,V)\nabla \ln f$, *for any* $X, Y \in \Gamma(TN_1)$ *and* $U, V \in \Gamma(TN_2)$, *where* ∇ *and* ∇' *denote the Levi-Civita connections on* N_1 *and* N_2, *respectively.*

3 Warped product pseudo-slant submanifolds of $(LCS)_n$-manifolds

Let us suppose that $M = N_1 \times_f N_2$ be a warped product pseudo-slant submanifold of a $(LCS)_n$-manifold $\overline{M}$. Such submanifolds are always tangent to the structure vector field ξ. If N_θ and $N_\perp$ are proper slant submanifolds and anti-invariant submanifolds of a $(LCS)_n$-manifold $\overline{M}$ then their warped product pseudo-slant submanifolds may be given by one of the following:
(i) $N_\perp \times_f N_\theta$, (ii) $N_\theta \times_f N_\perp$.
We now prove the following.

Theorem 2. *Let* $\overline{M}$ *be a* $(LCS)_n$*-manifold. Then there does not exist warped product pseudo-slant submanifold of* $\overline{M}$ *of the type* $M = N_\perp \times_f N_\theta$ *in* $\overline{M}$ *where* $N_\perp$ *is an anti-invariant submanifold and* N_θ *is a proper slant submanifold of* $\overline{M}$ *such that* ξ *is tangent to* N_θ.

Proof. From Proposition 1, we have
$$\nabla_X Z = \nabla_Z X = (Z \ln f)X \tag{29}$$
for any vector fields $X \in \Gamma(TN_\theta)$ and $Z \in \Gamma(TN_\perp)$. If $\xi \in \Gamma(TN_\theta)$, then we have
$$\nabla_Z \xi = (Z \ln f)\xi. \tag{30}$$
On the other hand, from (4), (17) and Proposition 1, we have
$$Z(\ln f)\xi = \alpha Z \tag{31}$$
Taking the inner product with ξ in (31), we get $Z(\ln f) = 0$, which means that f is constant on M and hence the proof is complete.

Theorem 3. *Let* $\overline{M}$ *be a* $(LCS)_n$*-manifold. Then there exist warped product pseudo-slant submanifolds of* $\overline{M}$ *of the type* $M = N_\theta \times_f N_\perp$ *in* $\overline{M}$ *such that* N_θ *is a proper slant submanifold tangent to* ξ *and* $N_\perp$ *is an anti-invariant submanifold of* $\overline{M}$.

Proof. For any vector fields $X \in \Gamma(TN_\theta)$ and $Z \in \Gamma(TN_\perp)$, from Proposition 1, we get the relation (29). Then for $\xi \in \Gamma(TN_\theta)$ we have from (29) that
$$\nabla_Z \xi = (\xi \ln f)Z. \tag{32}$$
Again, from (4) and (17), we get
$$\nabla_Z \xi = \alpha Z, \tag{33}$$
$$h(Z,\xi) = 0. \tag{34}$$
From (32) and (33), we get $\xi \ln f = \alpha(\neq 0)$ for all $Z \in \Gamma(TN_\perp)$. That means we get a non-zero and non-constant warping function f. Hence such a structure exist and consequently the theorem is proved.

Example 1. Consider the semi-Euclidean space $\mathbb{R}^{11}$ with the cartesian coordinates $(x_1, y_1 \cdots, x_5, y_5, t)$ and paracontact structure

$$\phi\left(\frac{\partial}{\partial x_i}\right) = \frac{\partial}{\partial y_i},\ \phi\left(\frac{\partial}{\partial y_j}\right) = \frac{\partial}{\partial x_j},\ \phi\left(\frac{\partial}{\partial t}\right) = 0,\ \ 1 \leq i, j \leq 5.$$

It is clear that $\mathbb{R}^{11}$ is a Lorentzian metric manifold manifold with usual semi-Euclidean metric tensor. Let M be a submanifold of $\mathbb{R}^{11}$ defined by

$$\chi(u, v, w, t) = (v\cos u,\ v\sin u,\ w\cos u,\ w\sin u,\ v+2w,\ -2v+w,\ -w\cos u,\ w\sin u,\ -v\cos u,\ v\sin u,\ t)$$

with non-zero u, v, w and $u \in \left(0, \frac{\pi}{2}\right)$. Then the tangent space of M is spanned by the following vectors

$$\begin{aligned}
Z_1 &= -v\sin u\frac{\partial}{\partial x_1} + v\cos u\frac{\partial}{\partial y_1} - w\sin u\frac{\partial}{\partial x_2} + w\cos u\frac{\partial}{\partial y_2} + w\sin u\frac{\partial}{\partial x_4} + w\cos u\frac{\partial}{\partial y_4} + v\sin u\frac{\partial}{\partial x_5} + v\cos u\frac{\partial}{\partial y_5},\\
Z_2 &= \cos u\frac{\partial}{\partial x_1} + \sin u\frac{\partial}{\partial y_1} + \frac{\partial}{\partial x_3} - 2\frac{\partial}{\partial y_3} - \cos u\frac{\partial}{\partial x_5} + \sin u\frac{\partial}{\partial y_5},\\
Z_3 &= \cos u\frac{\partial}{\partial x_2} + \sin u\frac{\partial}{\partial y_2} + 2\frac{\partial}{\partial x_3} + \frac{\partial}{\partial y_3} - \cos u\frac{\partial}{\partial x_4} + \sin u\frac{\partial}{\partial y_4},\\
Z_4 &= \frac{\partial}{\partial t}.
\end{aligned}$$

Then with respect to paracontact structure on $\mathbb{R}^{11}$, we get

$$\begin{aligned}
\phi Z_1 &= -v\sin u\frac{\partial}{\partial y_1} + v\cos u\frac{\partial}{\partial x_1} - w\sin u\frac{\partial}{\partial y_2} + w\cos u\frac{\partial}{\partial x_2} + w\sin u\frac{\partial}{\partial y_4} + w\cos u\frac{\partial}{\partial x_4} + v\sin u\frac{\partial}{\partial y_5} + v\cos u\frac{\partial}{\partial x_5},\\
\phi Z_2 &= \cos u\frac{\partial}{\partial y_1} + \sin u\frac{\partial}{\partial x_1} + \frac{\partial}{\partial y_3} - 2\frac{\partial}{\partial x_3} - \cos u\frac{\partial}{\partial y_5} + \sin u\frac{\partial}{\partial x_5},\\
\phi Z_3 &= \cos u\frac{\partial}{\partial y_2} + \sin u\frac{\partial}{\partial x_2} + 2\frac{\partial}{\partial y_3} + \frac{\partial}{\partial x_3} - \cos u\frac{\partial}{\partial y_4} + \sin u\frac{\partial}{\partial x_4},\\
\phi Z_4 &= 0.
\end{aligned}$$

It is easy to see that $\mathscr{D}^\theta = \mathrm{Span}\{Z_2, Z_3\}$ is a slant distribution with slant angle $\theta = \cos^{-1}\left(\frac{3}{7}\right)$ and $\mathscr{D}^\perp = \mathrm{Span}\{Z_1\}$ is an anti-invariant distribution. Thus M is a pseudo-slant submanifold of $\mathbb{R}^{11}$. It is easy to see that both the distributions are integrable. We denote the integral manifolds of $\mathscr{D}^\theta$ and $\mathscr{D}^\perp$ by M_θ and $M_\perp$, respectively. Then the product metric g of M is given by

$$g = -dt^2 + 7\left(dv^2 + dw^2\right) + 2\left(v^2 + w^2\right)du^2.$$

Hence M is a warped product pseudo-slant submanifold of $\mathbb{R}^{11}$ of the type $M_\theta \times_f M_\perp$ with warping function $f = \sqrt{2\left(v^2 + w^2\right)}$.

4 Conclusion

Let N_θ and $N_\perp$ be proper slant and anti-invariant submanifolds of a $(LCS)_n$-manifold $\overline{M}$ then their warped product pseudo-slant submanifolds may be given by one of the following.

(i) $N_\perp \times_f N_\theta$,

(ii) $N_\theta \times_f N_\perp$. Here we prove two theorems. Theorem 2 states that there does not exist warped product pseudo-slant submanifolds of $(LCS)_n$-manifold $\overline{M}$ of the type $M = N_\perp \times_f N_\theta$ such that $N_\perp$ and N_θ are anti-invariant and proper slant submanifolds of $\overline{M}$ so that ξ is tangent to N_θ. And theorem 3 states that there exist warped product pseudo-slant submanifolds of a $(LCS)_n$-manifold $\overline{M}$ of the type $M = N_\theta \times_f N_\perp$ such that N_θ is a proper slant submanifold tangent to ξ and $N_\perp$ is an anti-invariant submanifold of $\overline{M}$. The example 1 also support the Theorem 3. So there is a natural question arises. Does there exist warped product pseudo-slant submanifolds of a $(LCS)_n$-manifold $\overline{M}$ of the type,

(iii) $M = N_\perp \times_f N_\theta$ and (iv) $M = N_\theta \times_f N_\perp$ such that N_θ is a proper slant submanifold and $N_\perp$ is an anti-invariant submanifold tangent to ξ of $\overline{M}$? These problems are still open.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have contributed to all parts of the article. All authors read and approved the final manuscript.

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