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BIBLIOMETRICS OR INFORMETRICS: DISPLAYING REGULARITY IN SCIENTIFIC PATTERNS BY USING STATISTICAL DISTRIBUTIONS

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Abstract

The application of Statistical and Mathematical methods in Library and Information Sciences is called Bibliometrics or Informetrics. The fundamental topic in this area is the study of distributions which display regularity in scientific patterns, such as in the productivity of authors, or in articles, words or citations. In this study these distributions will be summarized and discussed after describing briefly the concept of Informetrics. Also an original applications will be given where the fit of the data to these distributions is tested.

Key Words: Informetrics, Bibliometrics, statistical distributions, Zipf's law, Lotka's law, Bradford's law, GIGP.

1. Introduction

Areas of applications of Statistics to various disciplines have from time to time been given special names, such as Econometry, Biostatistics, etc. which reflect their importance in relation to specific methods and interpretations. Informetrics or Bibliometrics was formed "to develop statistical and mathematical methods in order to study and analyze the characteristics of documentation or information in Library and Information Sciences" [10]. The term Informetrics is preferred in this study.

The historical background of Informetrics started with the tables of cases cited in the 1700's and 1800's [26]. The first study in terms of statistical analysis was by Cole and Eales in 1917. Hulme in 1922 used the term "Statistical Bibliography" to denote studies in Library science. Rangethan used "Librametry" in 1948 as a term embracing the quantitative methods applied to library management and services [21]. Pritchard [20] suggested "Bibliometrics" as a better name. He defined it

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as “the application of mathematical and statistical methods to books and other media communication”. In the same year, Nalimov and Mulchenko used the term “Scientometrics” in the same sense as Pritchard’s definition [15]. Meanwhile, E. Garfield played an important role at this point with his citation analysis [12]. In 1984, Brookes used “Informetrics” while investigating the relations between laws or distributions [4]. Almind and Ingwerson [1] gave the name “Webometrics” to the applications of Informetrics to Web pages. Diadato [7] pointed out that Informetrics can be used synonymously with Bibliometrics. Rao and Neelameghan [21] also put forward the view that Informetrics should cover Bibliometrics and Scientometrics.

Brookes [5] explained the term “Informetrics” in terms of laws or distributions. In general, these distributions are empirical laws. They correspond to discrete variables and form a family of highly skewed and long-tailed distributions. Their moments are infinite. They are also called Zipfian or Hyperbolic distributions, and include the Lotka, Zipf and Bradford Laws. Informetric distributions are generally based on ranks. Pao [17] has summarized the general aims of Informetric studies as measuring activities and processes with respect to information, as completing the deficiencies in information systems, as planing and managing information services, so as to improve the designs of documentation and information retrieval systems and predict future trends and uses.

According to Egghe and Rousseau [8], Informetrics is generally related to information production and/or information production processes (IPPs). According to these authors, the basic properties of information are called “source” and “item”, such as, authors as sources and their publications as items, or journals as sources and articles as their items. Hence, Informetrics involves the proof of source-item relations and the definition of regularities in information production patterns. Meanwhile Diadato [7] has described the topics involved in Informetrics under three headings: Informetric/Bibliometric distributions, citation analysis and Informetric indicators. On the other hand, Erar [9] reported on the frequencies of use of advanced statistical methods and some mathematical and operations research methods in Library and Information Sciences. Moreover some definitions, history, and fields of application of Informetrics have been summarized briefly in a preceding study [10]. In this study, Informetric distributions will be examined and some original applications will be given.

2. Informetric Distributions: From Social Principles to Laws

Informetric studies in Library and Information Sciences are based on the mathematical expression of three principles relating to social life: the **Principle of Least Effort**, the **80/20 rule** and the **Principle of Success Breeds Success**.

The Principle of Least Effort means that a person will strive to solve his problems in such a way as to minimize the total work that he must expend in solving both his immediate problems and his probable future problems [31]. Zipf had used the term least effort to describe the least average rate of probable work. This principle emphasizes the importance of summarizing an article using “little words with substance," authors feeling free to repeat certain words instead of using new ones. To express with many words what can be expressed with a few is meaningless [31].

A general observation in social and economic life is called the 80/20 rule, also well known as Pareto's rule. In Informetric studies, for example, it can be expected that 80% of the citations refer to a core of 20% of the titles in journals. Likewise, it can be stated that approximately 80% of the circulation are accounted for by about 20% of the collection or 80% of the articles in journals belong to about 20% of the authors. Thus they produce graphs which represent J-curves, or reverse J-curves, which give an important view of the distributional scheme [6].

The social phenomena "success breeds success" (SBS) is also considered in the form "failure results in failure" from time to time. In the context of Informetrics, the rule means that a paper which has been cited many times is more likely to be cited again than one which has been little cited. An author of many papers is more likely to publish again than one who has been less prolific. A journal that has been frequently consulted for some purpose is more likely to be turned to again than one of previously infrequent use [19]. So according to this rule, success in the past increases the chances of success in the future. The SBS principle was used to define a general theory of Informetrics by Solla-Price [19]. He named this theory the *Cumulative Advantage Process*. Many authors such as Simon [27], Egghe and Rousseau [8] have concerned themselves with the structure of the stochastic process of the SBS principle.

These principles have been embedded in the laws of Zipf, Lotka and Bradford. These laws have been studied and expressed in mathematical form, so that they can be used in practice. Also some scientists such as Fairthorne [11], Leimkuhler [16] and Simon [27] have investigated the similarities of these laws to standard statistical distributions.

Zipf's law investigates the frequency of the occurrences of words in a text. Lotka's law studies the productivity of authors in terms of scientific papers. Bradford's law examines the scatter of articles over different journals. These distributions and some evaluations corresponding to these laws can be given as follows.

Zipf's Law: Also known as the productivity law of words. The words in a book are ranked in decreasing order according to their number of occurrences. The number of occurrences of a word is inversely proportional to its rank. That is, the r^{th} ranking word appears approximately c/r times, where c is a constant. So Zipf stated in 1935 the relationship approximately as

$$r \times f \cong \text{ constant}, \quad \text{or} \quad rf = c, \tag{1}$$

where r is the rank of the word that occurs f times [31]. According to Zipf, there is a correlation between the number of different words in a book and the frequency of their usage. And this relation is a simple hyperbolic equation as given by (1). The graphical representation of Zipf's law is given in Figures 1a and 1b.

Figure 1a. The graph of Rank - Frequency.

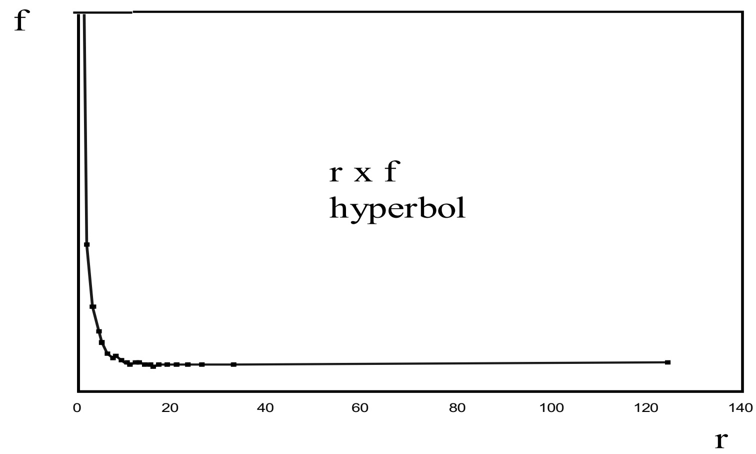
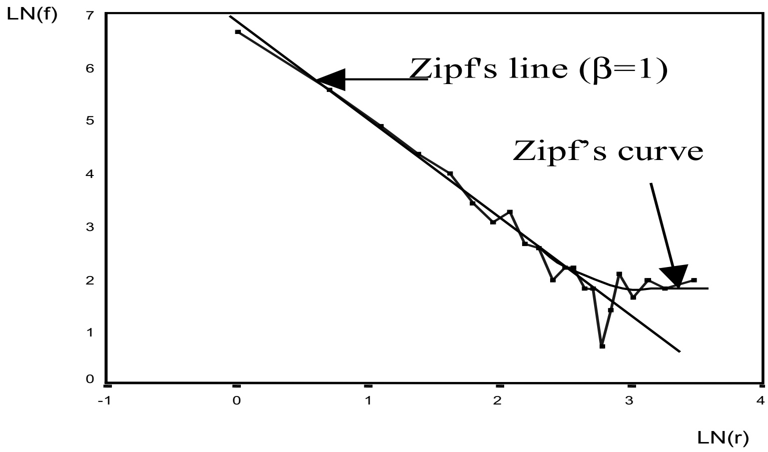


Figure 1b. The graph of $\ln(\mathbf{r}) - \ln(\mathbf{f})$



Bookstein (1976) expressed this law in a cumulative form where the number of occurrences of the most frequent r words in the text is proportional to $\ln(r)$. A generalization of Zipf's law is given as $r^{\beta} f = c$ by Wyllys [30]. Then the mathematical form can be written

$$f(r) = cr^{-beta}, \ r = 1, 2, 3, \ldots, \tag{2}$$

where c and β are parameters. The parameters c and β can be found by the method of least squares. But Pao [17] suggested finding the parameters after discarding observations on the tails of the distributions because the extreme points form a slightly curved line, such as in Figure 1b. Zipf's law is also called a rank-frequency law. But Equation 2 gives the classical size-frequency form of Zipf's law. Here, $f(r)$ is the number of words used r times. Since rank-frequency tabulation reverses the order of size-frequency tabulation, most authors such as Simon [27], Brookes

[3], Fairthorne [11] preferred the size-frequency form. In addition, Fairthore (1969) explained the relation of Zipf's law to the principle of least effort.

In general, a diagram of the $\ln(f) - \ln(r)$ relationship shows a downward concavity for the low rank or high frequency words. In this case, Wyllys [30] defined the parameter as $\beta < 1$. B. Mandelbrot in 1953 developed the law with three parameters as [30]

$$f(r) = c(r+a)^{-\beta},\ c > 0,\ \beta > 0,\ a \geq 0. \tag{3}$$

Constant a has its greatest effect when r is small.

On the other hand, many authors have claimed that Zipf's law is probabilistic. That is, this approach is concerned with the probability that a word occurs with a certain frequency in a book or a long text. The frequencies can be ranked. They imply the probabilities that a certain rank r is associated with a certain frequency f as explained above. When a random variable r, relating to the number of times a specific word is used, is discrete, the probability distribution will be given as

$$P(x = r) = [\zeta(\gamma+1)]^{-1} r^{-(\gamma+1)},\ r = 1, 2, \ldots, \tag{4}$$

where $\zeta(\,.\,)$ is the Riemann zeta function. This distribution is called Zeta by Johnson and Kotz ([13], p.240). They suggested its use for Linguistic studies.

Kendall [14] pointed out that the primary law in social phenomena is Pareto's law and that zipf's law can be regarded as a special form of Pareto's law, which is well known in Economics today. Brookes [3] has given the interrelations of Zipf with the Lotka and Bradford distributions. Brooks [3], Bookstein [2] and Price [19] have also shown that Zipf's law is a special case of the general distributions they suggested.

On the other hand, the fit of the statistical distribution to the Zipfian data has also been studied. Kendall [14] examined the fit of the Good distribution to these data. He suggested a more general form, which he called the Yule distribution, defined as follows

$$P(x = r) = [\Sigma B(r, \rho+1)]^{-1} B(r, \rho+1),\ r = 1, 2, \ldots, \tag{5}$$

where $B(r, \rho+1)$ is the Beta function of r and ρ. Simon [27] defined

$$P(x = r) = [r(r+1)]^{-1},\ r = 1, 2, \ldots, \tag{6}$$

by using the relation $f(r) = A \cdot B(r, r+1)$, which is a class of functions yielding the Negative Binomial (NB), or Fisher's log-series distributions (FLS), where A and r are constants. Simon [27] obtained more appropriate expected frequencies then the others on some samples. Simon's suggestion is a special case of Equation (5) for $\rho = 1$. According to this distribution, $f(2)/f(1) \cong 1/3$ and $f(1)/n \cong 0.5$. Besides, Simon [27] determined that these proportions are appropriate in the case of word frequencies, publications and even biological genera.

Some authors such as Price [19] used a Negative Binomial distribution for Zipfian data. But the most pretentious distribution is given as the zero-truncated

Generalized Inverse Gaussian-Poisson (GIGP) by Sichel [24, 25] as follows

$$P(x=r) = \left[((1-\theta)^{1/2})^{-\gamma} K_\gamma(\alpha(1-\theta)^{1/2}) - K_\gamma(\alpha)\right]^{-1} \cdot \frac{(\frac{\alpha\theta}{2})^r K_{r+\gamma}(\alpha)}{r!}, \quad r = 1, 2, \ldots, \tag{7}$$

where $K_\gamma(z)$ is the Bessel function of the second kind of order γ and argument z. The estimation of parameters α and θ under the special case $\gamma = -\frac{1}{2}$ and some other assumptions are given in [24]. Sichel [24] proposed this distribution as a new family of compound Poisson distribution of the reverse J-shaped type with an extraordinarily long-tail. Sichel [25] showed that the distributions NB and FLS are limiting distributions of GIGP for special values of the parameters and determined that the GIGP becomes a Zipfian distribution, like those of Zipf, Yule and Lotka, for some properties of the parameter by using a new parameterization. An advantage of this distribution is that all the moments exist as long as $\theta < 1$ [25].

Zipf described his law as explaining Linguistics in terms of the Least Effort principle. Zipf's law uses a definition of the descriptive words of literature obtained by discarding groups of up and down words, and is used in Linguistic researches, in automatic indexing, in estimating the algorithmic performance of textual retrieval from databases and even in studies of some relationships in biological systems.

The goodness of fit of some of the distributions given above are compared and tested on data in Table 1. This data belongs to a frequency distribution of 1476 different words, which was drawn with random sampling from the book *Matemetiğin Aydınlık Dünyası* by Sinan Sertöz [23]. In this sampling twenty nine pages were first drawn at random out of the 116 pages, and then a total of 5197 words were taken from these pages. The frequency graph of these words is given in Figures 1a and 1b. The observed and expected frequencies are also given in Table 1. In this table, r shows how many times a word has occurred and $f(r)$ is the number of words occurring r times.

Equation (1) can be written as $rf =$ constant $+$ error. Because the errors are additive, in this study, Zipf's expected frequencies f'(Zipf) are obtained from a nonlinear regression estimation using equation (2) in the form

$$f'(r) = 751,56 r^{-1,668}.$$

Zeta's frequencies are found from the equation

$$P(x=r) = 0,580 r^{-1,907}$$

using (3). For this distribution, the maximum likelihood estimator $\hat{\gamma}$ of γ is found from

$$n^{-1}\sum_{i=1}^{n} \log x_i = -\frac{\zeta'(\hat{\gamma}+1)}{\zeta(\hat{\gamma}+1)}, \tag{8}$$

which is given by Johnson and Kotz [13]. But the table of $\zeta'(\hat{\gamma}+1)/\zeta(\hat{\gamma}+1)$ given by Rousseau [22] is used instead of Johnson and Kotz's table for the term on the right hand side of (8).

Simon's expected frequencies are obtained from Equation (6). The GIGP frequencies are calculated from (7) using $\gamma = \frac{1}{2}$ and the method of moments, which is given by Sichel. For these calculations, the recurrence formula for Bessel's function given by Sichel [24] is used. So the probability function from Equation (7) is as follows:

$$P(x = r) = 0,96337(1 - 1,5r^{-1})P(x = r - 1) + \left[\frac{0,090274}{4r(r-1)}\right] P(x = r - 2).$$

Table 1. Observed and expected frequencies from a book written by Sertöz.

r	f(r)	f′(Zipf)	f′(Zeta)	f′(Simon)	f′(Sichel)
1	773	751,5	856,1	738,0	773,0
2	258	236,5	236,2	246,0	244,2
3	128	120,2	106,3	123,0	125,0
4	75	74,4	63,5	73,8	75,7
5	52	51,0	39,8	49,2	51,2
6	30	37,0	29,5	35,1	36,0
7	21	29,2	22,1	26,3	27,0
8	25	23,4	17,7	20,5	22,0
9	14	19,2	13,3	16,4	17,0
10	13	16,1	11,8	13,4	15,0
11	7	12,7	8,9	11,2	12,0
12	9	10,7	7,5	9,5	10,0
13	9	10,4	6,5	8,1	10,0
14	6	9,0	5,9	7,0	7,2
15	6	8,2	5,2	6,1	6,2
16	2	7,0	4,4	5,4	5,4
17	4	6,2	4,4	4,8	4,8
18-19	8	9,1	7,5	8,2	7,9
20-21	5	8,6	5,6	6,7	6,3
22-24	7	11,2	6,6	8,1	7,0
25-27	6	9,0	5,9	6,3	7,3
28-32	7	9,3	6,2	7,1	5,5
33 +	11	5,8	4,7	38,0	4,5
χ^2		26,53	35,14	29,74	19,47
Result ($a = 0.05$)		accept	reject	accept	accept

For the parameters in Equation (7), the values $\hat{\alpha} = 0,31188$ and $\hat{\theta} = 0,97337$ were obtained. If the proportions of the frequencies are taken into account, it can be seen that $\frac{f(1)}{f(2)} \cong 3,0$ and $\frac{f(1)}{n} \cong 0,50$. Moreover, there is a concavity in the observed distribution as shown in Figure 1b. So, with the exception of Zeta, the other three distribution's fit the observations quiet well. In particular, GIGP gives the best fit.

Lotka's Law: Known as the productivity law for authors in the realm of scientific papers, it can be seen as a fundamental law when researching the regularity of the scatter of items to their sources. Lotka formulated this law, which was inspired by the principle of SBS, in 1926. The first form of this law was expressed as

$$x^{\beta} y = c, \tag{9}$$

where y is the number of authors who have published x papers, and c is a constant. Lotka used $\beta = 2$. If in addition $c = 1$ is accepted, the number of authors who have published x papers becomes proportional to $\frac{1}{x^2}$. For this reason, some authors also called this law the Inverse Square Law [18]. The general expression of Lotka's law may also be expressed in the form

$$y = cx^{-\beta},\ c, \beta > 0, \tag{10}$$

where β is the Lotka parameter and c is the number of authors who have published one paper. This equation is similar in general form to (1) and (2), but the meanings of the parameters or variables are different. Also the limits on β can be different. While $\beta = 1$ for Zipf's law, $\beta > 1$ for Lotka [21]. Here, the greater values of β are associated with the case where a large number of authors have published one paper, because then only a few authors will have written a large number of papers. Small values of β show that many authors have published a great many papers. Pao [17] applied the method given for Zipf's law to calculate β.

Fairthrone [11] expressed this law as a probability distribution in the form

$$P(x) = cx^{-(\beta+1)}, \tag{11}$$

where c and β are parameters. Simon's suggestion for Lotka is the same as in equation (6) [27]. Moreover, Simon [27] also gave the more general distribution form

$$P(x) = cx^{-\alpha} b^{x},\ c, b > 0, \alpha > 0, x \geq 1. \tag{12}$$

Brookes [5] defined the equation as

$$P(x) = k\Sigma(\frac{1}{x^2}),\ x \geq 1, \tag{13}$$

where $x \to \infty$ and $k \to \frac{6}{\pi^2}$. Sichel [24] has also used Equation (7) for Lotka.

As an application, the data used is related to the number of papers using advanced statistical methods published in journals in the category of Library and Information Sciences, Science Citation Index (SCI), between 1990-1999 [9]. Table 2 shows the frequency distribution of the number of authors $f(x)$ who published x papers. The expected values of $f(x)$ are also given in this table using the following equations.

Lotka model (equation 10): $f'(x) = 709,042x^{-2,96374}$,
Brookes model (equation 13): $P(x) = \frac{0,60793}{x^2}$,
Simon Model (equation 12): $f'(x) = 722,6137x^{-2,93389}(0,9812)^x$,
GIGP (equation 8): $P(x) = 0,6064(1-1,5x^{-1})P(x-1) + \left[\frac{0,000018863}{(x(x-1))}\right] P(x-2)$.

Table 2. Observed and expected frequencies of authors who have published x papers.

x	f(x)	f′(Lotka)	f′(Brookes)	f′(Simon)	f′(GIGP)	f′(Yule)
1	709,00	709,04	519,17	710,00	693,02	468,7
2	92,00	90,89	129,79	91,04	106,56	156,2
3	24,00	27,33	57,69	27,19	32,31	78,12
4	13,00	11,65	32,45	11,47	12,24	46,87
5	6,00	6,01	20,77	5,85	5,94	31,25
6+	10,00	8,04	36,00	8,36	4,50	72
χ^2		1,053	141,020	0,815	11,264	285,36
K-S		0,0027	0,222	0,0038	0,019	0,336
Result ($\alpha = 0.05$)		accept	reject	accept	reject	reject

Simon's expected values are calculated from (6). Only the equations of Lotka and Simon yield a good fit, because of the linear relation between $\ln x$ and $\ln f(x)$. The lack of fit of GIGP could be due to the lack of concavity in the double logarithmic plot, and to the small number of groups. Besides, the proportions in Table 1 do not validate in this sample.

Bradford's Law: Bradford used this law to express the article productivity of journals in 1934. The law is also called the "Scattering Law". According to Bradford, there is a regularity, which is observed in the retrieval or use of published information. This regularity is determined by the dispersion of specific items of information over different sources of information. Then for a given topic, a large number of the relevant articles will be concentrated in a small number of journals. The remaining articles will be dispersed over a large number of journals. The Law can be expressed in the form: "if scientific journals are arranged in order of decreasing productivity of articles on a given subject, they may be divided into a nucleus of journals more particularly devoted to the subject and several groups or zones containing the same numbers of articles as the nucleus, when the number of journals in the nucleus and succeeding zones will satisfy $1 : n : n^2 : \ldots$" [3]. Bradford has illustrated this verbal definition with graphics. He also illustrated the 80 / 20 rule with a plot of the cumulative number of article to the cumulative number of journals [18].

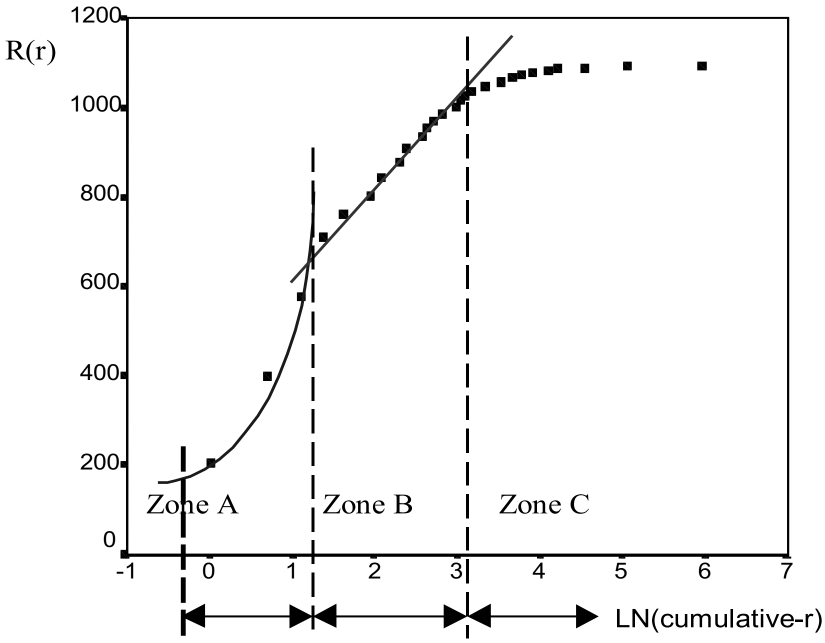
Bradford's law is obtained as follows from its graphical expression:

$$R(r) = k \ln(r). \tag{14}$$

Here $R(r)$ is the cumulative number of articles corresponding to the cumulative number of journals r or its rank. The constant k is called the Bradford factor [17].

According to Bradford, zone A in Figure 2 shows the core sources in a special topic, that is the nuclear journals. Zone B gives the journals with secondary productivity, or the specified sources in relevant topics. Finally, zone C shows the journals not classified as good, that is the sources for irrelevant topics. However, while Bradford divided the sources into three regions, some authors have divided them into 4, 5, 8 and even 14 regions [17, 29].

Figure 2. The Bradford zones on references in Statistical journals published in 1999.



The exact mathematical form of this law was established by Vickery [28]. Vickery gave the equation (14) as

$$R(r) = k\ln(\frac{r}{s}), \quad \text{or} \quad R(r) = a + k\ln(r), \tag{15}$$

where a, k, and s are constants. Leimkuhler (1967) answered the question "what is the probabilistic model of this law?" He gave the probability density function of x with parameter β as

$$f(x) = \frac{\beta}{[(1+\beta x)\ln(1+\beta)]}, \ 0 \leq x \leq 1, \tag{16}$$

where x is the proportion of the most productive journals.

The majority of studies in this area are concerned with obtaining the number of zones and the ratios or frequencies of the articles in these zones. The existence of a well defined Bradford law helps in the planning of Library databases.

The data used in Figure 2 involves journals cited in articles drawn randomly from some statistical journals published in 1999. The numbers of sources and references are given in Table 3. The scattering of the statistical journals gave a bad fit with the given equations, and a constant Bradford factor was not found. This result may have been caused by too small a number of references or/and an inappropriate view of the content.

Table 3. The numbers of sources (journals) and items (references).

Journal (Source)	Number of Sources	Number of References	Journal (Source)	Number of Sources	Number of References
JASA	1	205	···	1	13
Biometrika	1	195	···	1	12
Ann Statistics	1	179	···	2	11
JRSS B	1	132	···	4	10
Technometrics	1	48	···	6	9
Biometrics	2	44	···	5	8
Econometrica			···	5	7
Stat Med	1	39	···	6	6
Ann Math Stat	2	35	···	10	5
App Statistics			···	7	4
Stat Science	1	32	···	28	3
JRSS A	2	26	···	62	2
············	3	14	···	229	1

3. Conclusions

Principles inspired by econometry, which were initially expressed verbally, were expressed mathematically in the 1930's–1940's, and modelled statistically in the 1950's. Zipf's law is the mathematical expression of principles relating to word production, Lotka's law expresses those principles relating to the productivity of authors, while the law relating to journals is called Bradford's law. As a result of the proliferation of information relating to scientific studies, and the development of the Internet, studies have been performed to use these laws in order to explain statistically the general patterns of information production and communication in terms of the relations between sources and items.

On the other hand, the fit of Informetric distributions to data shows high variability from data to data and from topic to topic. Generally, these distributions give the best fit for large data sets. There are a great many distributions in any specific area, but because the empirical distributions are very long-tailed and the regularities in the tail can not be sufficiently determined, a larger data set is required. The GIGP distribution has some advantages for goodness of fit when there is concavity in the data. But the difficulties in computing the GIGP parameters leads us to prefer the basic forms, such as Lotka. Meanwhile in researches involving statistical journals, the sampling data does not produce a good fit to the suggested distributions. It could be that the sample sizes are insufficient and/or that there are differences in citation patterns between classical sciences and new areas such as statistical science.

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