Hacettepe Journal of Mathematics and Statistics
Volume 31 (2002), 127–137

COMPARISON OF PERFORMANCE AMONG INFORMATION CRITERIA IN VAR AND SEASONAL VAR MODELS

Cem Kadılar* and Cenap Erdemir*

Received 14. 08. 2002

Abstract

The aim of this paper is to show the effect of seasonality on the performance of traditional information criteria (such as Akaike, Schwarz, Shibata, and Quinn), used for determining the order of vector autoregressive (VAR) models. Using a simulation study, we find that the performances of these criteria decrease in seasonal VAR (SVAR) models.

Key Words: VAR models, Information criteria, Seasonality

1. Introduction

Autoregressive modelling has been used successfully in many fields of application like economics, agriculture, biomedics, geophysics etc. The VAR(p) model can be written as follows:

$$\mathbf{Z}_t = \boldsymbol{\phi}_p\,(B)\,\mathbf{Z}_t + \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t$, the $d \times 1$ vector of disturbances to the system, is assumed to be a white noise error (ie. with a mean of zero, a constant variance and zero covariances) and is a identically and independently distributed random variable, $\mathbf{Z}_t$ is a d-dimensional stationary vector time series, and

$$\boldsymbol{\phi}_p\,(B) = \boldsymbol{\phi}_1 B + \boldsymbol{\phi}_2 B^2 + \cdots + \boldsymbol{\phi}_p B^p$$

is the autoregressive $d \times d$ matrix polynomial of order p, with all its roots outside the unit circle, and B is the backshift operator: $B^j Z_t = Z_{t-j}$ [13]. It should be noted that before the VAR model is estimated it is necessary to identify the properties of the variables used in the model. In other words, it should be determined whether the underlying data processes are stationary, or not. If the time series are found to be non-stationary, the order of integration will need to be determined, and the stationary form of the variable should be added to the VAR model.

*Hacettepe University, Faculty of Science, Department of Statistics, Beytepe, Ankara

One of the major problems in VAR modelling is the optimal selection of the model order. In the last two decades several order determination criteria have been proposed to estimate the order of the VAR(p) model, and occupy an important place in the literature (see [2–4,8,9,11]). In the multivariate case, the general form of most criteria is

$$\Delta_k = \log\left|\widehat{\mathbf{\Sigma}}_k\right| + \frac{k}{T}\lambda\left(T\right), \qquad (1)$$

where $\widehat{\mathbf{\Sigma}}_k$ is the residual covariance matrix associated with the fitted VAR(k) structure and is obtained from least squares residuals, T is the sample size, and the function $\lambda(T)$ varies from criterion to criterion. Various order determination criteria can be expressed in the form of equation (1). Lütkepohl [10] showed that in the AIC [1], $\lambda(T) = 2d^2$, where d is the dimension (i.e., the number of series), and that $\lambda(T) = d^2 \log T$ in the criterion for VAR models of Schwarz [14]. Hannan and Quinn [5] suggested a criterion that used $\lambda(T) = 2c \log\log T$, where c is a constant greater than 1. Subsequently, Quinn [12] generalized the Hannan-Quinn criterion for VAR(p) models by setting $c = d^2$. Shibata [15] investigated the asymptotic properties of Akaike's estimate and proved that the AIC does not produce a consistent estimate of the order of an autoregressive model. In a subsequent paper Shibata [16] developed the following criterion:

$$S_k = \left(1 + 2\frac{dk+1}{T}\right)^d \left|\widehat{\mathbf{\Sigma}}_k\right|.$$

The use of order selection criteria, based on fitting vector autoregressive models of various orders $k = 0, 1, \ldots$ to a series, was also proposed and studied by Pukkila and Krishnaiah [11] as a testing procedure for assessing whether a series is a white noise process. In their procedure, the white noise null hypothesis for the series is accepted if the AR order selected by the use of the given order determination criterion (eg. AIC, SBIC, HQ) is equal to zero. Besides, Pukkila and Krishnaiah [11] also developed a new criterion that is less dependent on the number of series by modifying the Quinn criterion as

$$QPK_k = log\left|\widehat{\mathbf{\Sigma}}_k\right| + \frac{kd^2 + kd\left[\log\left(T\right) - 1\right]}{T - 0.5d\left(k+1\right)}.$$

The procedure of Pukkila and Krishnaiah [11] was extended by Koreisha and Pukkila [8,9] for selection of the order of a vector autoregressive model in the following way: First, VAR models of given orders $k = 0, 1, \ldots$ are fitted by least squares and the residuals from the fitted VAR model of order k are obtained. Then the procedure of Pukkila and Krishnaiah [11] is applied to this residual series, and if the order selection criterion leads to selection of an autoregressive model of order greater than zero for the residual series then the residuals are viewed as not having satisfied the white noise test and hence the order k is rejected. The smallest VAR order k for which the corresponding the residual series is accepted to be a white noise process, is selected as the appropriate VAR order model.

In another study on VAR model orders, Hurvich and Tsai [6] found a criterion derived from the Akaike information criterion as follows:

$$HT_k = \log\left|\widehat{\mathbf{\Sigma}}_k\right| + \frac{1 + k/T}{1 - (k+2)\ /T}.$$

Univariate seasonal models have received considerably more attention than multivariate seasonal models for vector time series [13]. However, many of the time series encountered in signal processing, economics, meteorology, and environmental studies exhibit strong seasonal behaviour. These situations require the use of seasonal VAR (SVAR) models of the form:

$$\mathbf{Z}_t = \mathbf{\Phi}_p\left(B^s\right)\mathbf{Z}_t + \varepsilon_t,$$

where s is the period of the seasonal behavior, and

$$\mathbf{\Phi}_p\left(B^s\right) = \mathbf{\Phi}_1 B^s + \mathbf{\Phi}_2 B^{2s} + \cdots + \mathbf{\Phi}_p B^{ps}$$

is the seasonal autoregressive operator [13].

2 Simulation Study

In this simulation study, we investigated the performance of the traditional information criteria such as Akaike, Schwarz, Shibata and Quinn in VAR and seasonal VAR models. We performed a simulation with 10000 trials for each of a variety of model structures, involving many series with varying parameter values. The sample size (T) was taken to be 100, the period (s) for the seasonal models was 4, the number of series (d) was 2 to 5 and the lag number (k) was taken to be 1 to 7. Data was generated using the VAR(1), VAR(2), SVAR(1) and SVAR(2) stationary models. Following the method of Koreisha and Pukkila [8], the parameters and the covariance matrices of the residuals of the models were defined in the same way:

For the VAR(1) model:

$$\begin{aligned}
\phi_{1ii} &= 0,5 \\
\phi_{1ij} &= \begin{cases} (0,5)(-1)^{j-i} & ;\ j > i \\ 0 & ;\ j < i \end{cases}
\end{aligned}$$

For the VAR(2) model:

$$\begin{aligned}
\phi_{1ii} &= \begin{cases} 1,42 & ;\ i \text{ odd} \\ 0,50 & ;\ i \text{ even} \end{cases} \\
\phi_{2ii} &= \begin{cases} -0,73 & ;\ i \text{ odd} \\ 0 & ;\ i \text{ even} \end{cases} \\
\phi_{1ij} &= \begin{cases} (0,5)(-1)^{j-i} & ;\ j > i \\ 0 & ;\ j < i \end{cases} \\
\phi_{2ij} &= \begin{cases} (-0,5)(-1)^{j-i} & ;\ j > i \\ 0 & ;\ j < i \end{cases}
\end{aligned}$$

where i indicates the i^{th} row and j indicates the j^{th} column of the matrix of parameters. For the SVAR models, the parameters were selected to be the same as in the VAR models to allow comparison of the performance of the criteria in the VAR and SVAR models under the same conditions. The residuals were obtained from a multivariate normal distribution with mean $\mathbf{0}$ and covariance matrix $\boldsymbol{\Sigma}$. The elements of the covariance matrix was defined as $\sigma_{ij} = 0,5^{|i-j|}$.

As is shown in Table 1, in this simulation study we have 4 models, and in each model there are $2+3+4+5=14$ regression equations because the number of series was taken as 2, 3, 4 and 5. Also it will be seen that we tried 7 fitted VAR model orders for each regression equation since $k = 1, 2, ..., 7$. Thus, we applied the least squares method $4 \times 14 \times 7 = 392$ times in each trial in order to find the least square residuals, and in this way we obtained the covariance matrices of residuals corresponding to related sub-models, and computed the determinant values of these matrices.

Table 1. The Models in the Simulation Study

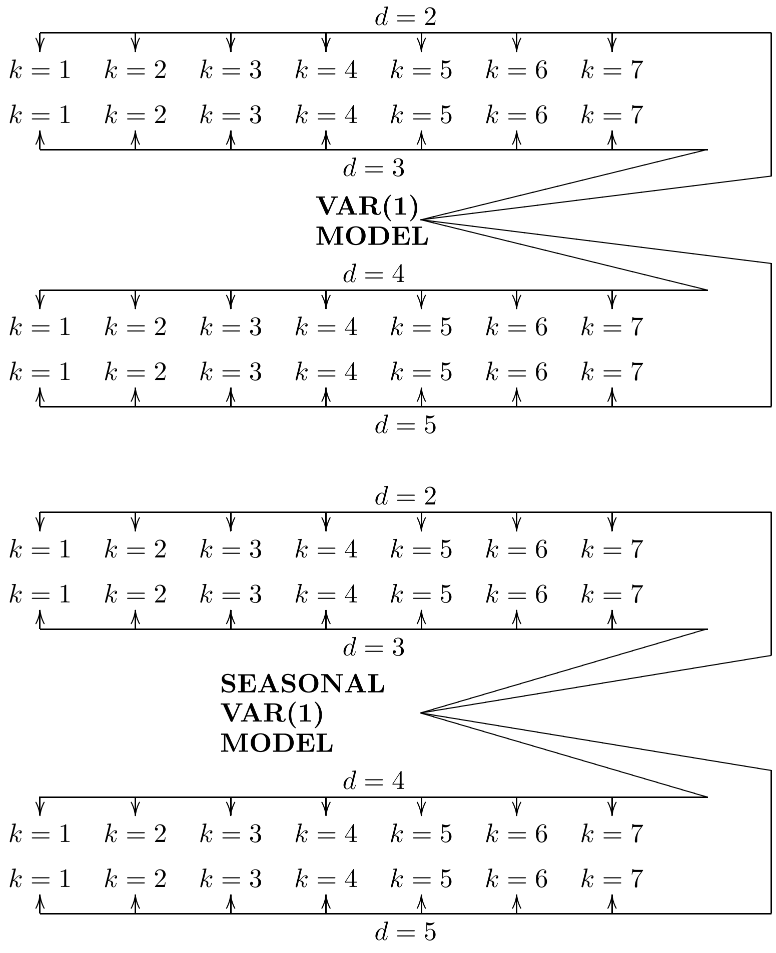
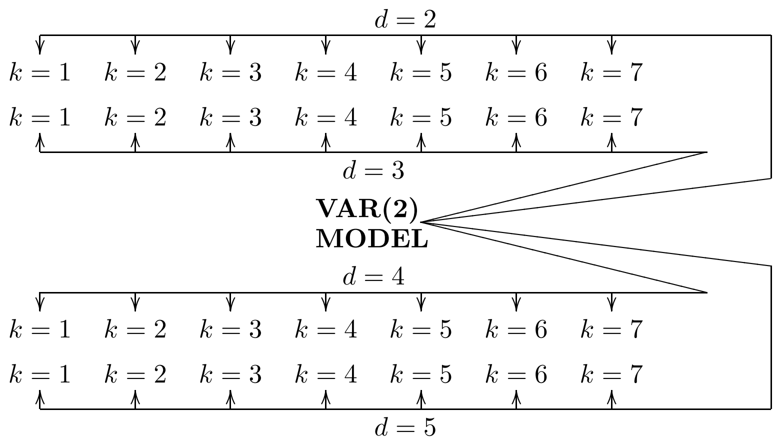
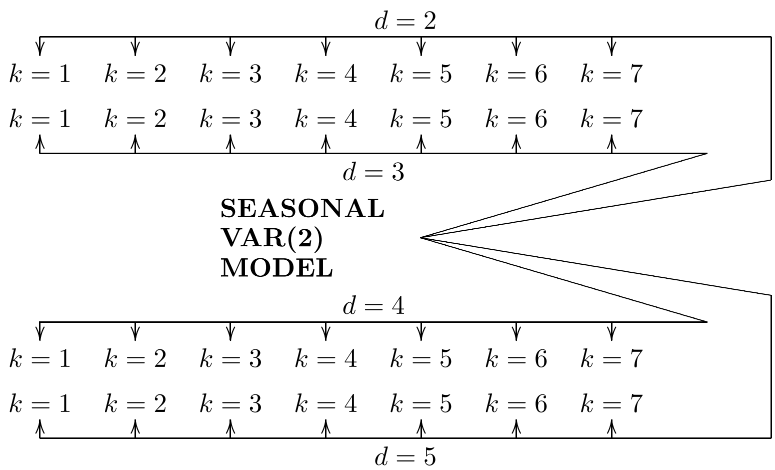


Table 1. (Continued)





As there are 4 models, 4 different numbers of series and 7 fitted VAR orders, we have $4 \times 4 \times 7 = 112$ submodels in total, so we tried to find 112 different determinant values of the 112 covariance matrices of residuals for the 4 information criteria formulae in each trial. Therefore, this simulation program calculated $112 \times 4 \times 10000 = 4480000$ criteria results, these results as shown tabulated in Table 2. Using the output of the program we can analyze 4480000 results and thereby contrast the performances of the criteria for each model. Kadilar presented graphs and a detail analysis of these results in [7] .

Table 2 lists the frequency distribution of choosing the lags of AIC, Schwarz, Shibata, and Quinn in VAR(1), SVAR(1), VAR(2), and SVAR(2) models. For example, in Table 2, when 10000 trials were performed from VAR(1) for $d = 2$, the AIC chose VAR(1) as the best model in 7711 trials, VAR(2) in 1122 trials, VAR(3) in 490 trials, VAR(4) in 249 trials, VAR(5) in 150 trials, VAR(6) in 132 trials, and VAR(7) in 146 trials. Given that the correct model is VAR(1), the performance of AIC is 77% (7711/10000 = 0.7711) when $d = 2$ in VAR(1) models. In another example, when the trials were performed from SVAR(2) for $d = 5$, in 10000 trials, the AIC chose SVAR(1) as the best model in 0 trials, SVAR(2) in 509 trials, SVAR(3) in 48 trials, SVAR(4) in 18 trials, SVAR(5) in 31 trials, SVAR(6) in 144 trials, and SVAR(7) in 9250 trials. Given that in this example the correct model is SVAR(2), the performance of AIC is 5% (509/10000 = 0.0509) when $d = 5$ in SVAR(2) models.

From Table 2, we observe that there is a decrease in the performances of each criterion for SVAR(1) with respect to VAR(1) models. For example, when the number of series is 2, the performance of AIC decreases from 77.11% to 46.36%, the performance of the Schwarz criterion decreases from 99.57% to 97.54%, the performance of the Shibata criterion decreases from 67.11% to 34.82%, and the performance of the Quinn criterion decreases from 95.76% to 83.58%.

Table 2. Frequency Distribution of the Identified Model Structures for the Model Selection Criteria

VAR(1) MODEL

Akaike	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$
$d = 2$	0,7711	0,1122	0,0490	0,0249	0,0150	0,0132	0,0146
$d = 3$	0,8852	0,0717	0,0199	0,0088	0,0051	0,0037	0,0056
$d = 4$	0,9448	0,0380	0,0071	0,0028	0,0029	0,0022	0,0022
$d = 5$	0,9666	0,0212	0,0037	0,0018	0,0015	0,0017	0,0035
Schwarz							
$d = 2$	0,9957	0,0041	0,0002	0,0000	0,0000	0,0000	0,0000
$d = 3$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
$d = 4$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
$d = 5$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Shibata							
$d = 2$	0,6711	0,1132	0,0588	0,0359	0,0301	0,0358	0,0551
$d = 3$	0,7244	0,0798	0,0357	0,0228	0,0216	0,0318	0,0839
$d = 4$	0,6525	0,0447	0,0191	0,0110	0,0158	0,0381	0,2188
$d = 5$	0,3486	0,0168	0,0037	0,0026	0,0062	0,0302	0,5919
Quinn							
$d = 2$	0,9576	0,0349	0,0057	0,0013	0,0003	0,0001	0,0001
$d = 3$	0,9955	0,0043	0,0002	0,0000	0,0000	0,0000	0,0000
$d = 4$	0,9995	0,0005	0,0000	0,0000	0,0000	0,0000	0,0000
$d = 5$	0,9999	0,0001	0,0000	0,0000	0,0000	0,0000	0,0000

Table 2. (Continued)

SVAR(1) MODEL

Akaike	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$d=2$	0,4636	0,0994	0,0605	0,0473	0,0523	0,0767	0,2002
$d=3$	0,5038	0,0694	0,0339	0,0250	0,0313	0,0536	0,2830
$d=4$	0,4061	0,0297	0,0116	0,0090	0,0113	0,0379	0,4944
$d=5$	0,1660	0,0066	0,0017	0,0011	0,0017	0,0123	0,8106
Schwarz							
$d=2$	0,9754	0,0211	0,0024	0,0006	0,0003	0,0000	0,0002
$d=3$	0,9986	0,0013	0,0001	0,0000	0,0000	0,0000	0,0000
$d=4$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
$d=5$	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Shibata							
$d=2$	0,3482	0,0794	0,0516	0,0459	0,0565	0,0980	0,3204
$d=3$	0,2223	0,0320	0,0225	0,0164	0,0245	0,0756	0,6067
$d=4$	0,0441	0,0036	0,0016	0,0020	0,0032	0,0225	0,9230
$d=5$	0,0002	0,0000	0,0001	0,0000	0,0002	0,0012	0,9983
Quinn							
$d=2$	0,8358	0,0790	0,0256	0,0134	0,0121	0,0112	0,0229
$d=3$	0,9577	0,0267	0,0042	0,0023	0,0012	0,0012	0,0067
$d=4$	0,9905	0,0067	0,0002	0,0001	0,0001	0,0002	0,0022
$d=5$	0,9939	0,0009	0,0001	0,0000	0,0001	0,0000	0,0050

VAR(2) MODEL

Akaike	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
d=2	0,0000	0,7478	0,1192	0,0534	0,0324	0,0237	0,0235
d=3	0,0000	0,8443	0,0874	0,0302	0,0159	0,0106	0,0116
d=4	0,0000	0,9053	0,0560	0,0149	0,0067	0,0073	0,0098
d=5	0,0000	0,9141	0,0400	0,0101	0,0058	0,0090	0,0210
Schwarz							
d=2	0,0001	0,9966	0,0032	0,0001	0,0000	0,0000	0,0000
d=3	0,0000	0,9998	0,0002	0,0000	0,0000	0,0000	0,0000
d=4	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000
d=5	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Table 2. (Continued)

VAR(2) MODEL

Shibata							
d=2	0,0000	0,6221	0,1253	0,0706	0,0526	0,0533	0,0761
d=3	0,0000	0,6063	0,0980	0,0535	0,0462	0,0611	0,1349
d=4	0,0000	0,4577	0,0589	0,0335	0,0344	0,0686	0,3469
d=5	0,0000	0,1386	0,0140	0,0099	0,0138	0,0401	0,7836
Quinn							
d=2	0,0000	0,9513	0,0405	0,0060	0,0016	0,0005	0,0001
d=3	0,0000	0,9914	0,0081	0,0005	0,0000	0,0000	0,0000
d=4	0,0000	0,9989	0,0011	0,0000	0,0000	0,0000	0,0000
d=5	0,0000	0,9995	0,0005	0,0000	0,0000	0,0000	0,0000

SVAR(2) MODEL

Akaike	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$
d=2	0,0000	0,4047	0,1134	0,0758	0,0682	0,1012	0,2367
d=3	0,0000	0,3841	0,0776	0,0470	0,0469	0,0839	0,3605
d=4	0,0000	0,2502	0,0318	0,0179	0,0199	0,0568	0,6234
d=5	0,0000	0,0509	0,0048	0,0018	0,0031	0,0144	0,9250
Schwarz							
d=2	0,0004	0,9661	0,0255	0,0049	0,0020	0,0004	0,0007
d=3	0,0000	0,9968	0,0031	0,0001	0,0000	0,0000	0,0000
d=4	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000
d=5	0,0000	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Shibata							
d=2	0,0000	0,2886	0,0925	0,0667	0,0680	0,1226	0,3616
d=3	0,0000	0,1382	0,0326	0,0255	0,0321	0,0891	0,6825
d=4	0,0000	0,0138	0,0029	0,0023	0,0038	0,0265	0,9507
d=5	0,0000	0,0000	0,0000	0,0000	0,0001	0,0005	0,9994
Quinn							
d=2	0,0000	0,7800	0,0946	0,0384	0,0237	0,0241	0,0392
d=3	0,0000	0,9105	0,0457	0,0111	0,0072	0,0059	0,0196
d=4	0,0000	0,9557	0,0168	0,0031	0,0016	0,0024	0,0204
d=5	0,0000	0,9285	0,0055	0,0004	0,0007	0,0018	0,0631

A similar trend is observed in the performance of the criteria for the second order models VAR(2) and SVAR(2). For example, when the number of series is 3, the performance of AIC declines from 84.43% to 38.41%, the performance of the Schwarz criterion declines from 99.98% to 99.68%, the performance of the Shibata criterion declines from 60.63% to 13.82%, and the performance of the Quinn criterion declines from 99.14% to 91.05%.

The consistently poor performance of the AIC and Shibata for the SVAR models in comparison to the VAR models indicates that there is a serious problem in the determination of the lag number in multivariate seasonal models for AIC and Shibata. However, the Quinn and especially the Schwarz criteria demonstrate high performance also in seasonal vector autoregressive models.

3 Conclusion

The results from the simulation can be given briefly as follows:

- For all the models, the order of the performances of the criteria from higher to lower: Schwarz, Quinn, Akaike and Shibata.
- For all criteria, the performances are high in VAR models with respect to SVAR models.
- For all criteria (except the Schwarz criterion when $d = 2$), the performances are higher in VAR(1) than in VAR(2) models.
- For all criteria, the performances are higher in SVAR(1) than in SVAR(2) models.
- Except for the VAR(1) model when $d = 3$, the performance of the Shibata criterion decreases when the number of series increases.
- The performance of AIC increases when the number of series increases in the VAR models.
- Except for the SVAR(1) model when $d = 3$, the performances of AIC decreases when the number of series increases in the SVAR models.
- Our results indicate that AIC and Shibata should not be used in SVAR models because of their very poor performance.

In planning future studies, one could change the values of the parameters in the models and define the covariance matrix of residuals in a different way. Also, while generating data, the multiplicative seasonal models for a vector time series could be used. Besides, the period in the seasonal models could be taken as 6 and 12 instead of 4, the sample size can be selected as 150 and 200, the fitted VAR model orders extended to 10, and the number of series increased to 7. Probably, when these are done it will be confirmed that there is a serious problem in determining the correct order of SVAR models. Consequently, it is clear that new criteria for determining the order of multivariate seasonal models should be developed.

References

[1] Akaike, H. A new look at the statistical model identification, IEEE Transactions on Automatic Control. AC-19, 716–723, 1974.

[2] Broersen, P. M. T. and Wensink, H. E. On finite sample theory for autoregressive model order selection, IEEE Transactions on Signal Processing **41 (1)**, 194–204, 1993.

[3] Broersen, P. M. T. and Wensink, H. E. On the penalty factor for autoregressive order selection in finite samples, IEEE Transactions on Signal Processing **44 (3)**, 748–752, 1996.

[4] Çiftçioglu, Ö., Hoogenboom, J. E. and Dam, H V. A consistent estimator for the model order of an autoregressive process, IEEE Transactions on Signal Processing **42 (6)**, 1471–1477, 1994.

[5] Hannan, E. J. and Quinn, B. G. The determination of the order of an autoregression, Journal of Royal Statistical Society **B41**, 190–195, 1979.

[6] Hurvich, C. M. and Tsai, C. L. Regression and time series model selection in small samples, Biometrika **76 (2)**, 297–307, 1989.

[7] Kadilar, C. The identification problem in vector autoregressive models, Ph.D. Dissertation. Department of Statistics, Hacettepe University, 2000.

[8] Koreisha, S. and Pukkila, T. Determining the order of a vector autoregression when the number of component series is large, Journal of Time Series Analysis **14**, 47–69, 1993.

[9] Koreisha, S. and Pukkila, T. The selection of the order and identification of non-zero elements in the polynomial matrices of vector autoregressive processes, Journal of Statistical Computation and Simulation **62 (3)**, 207–235, 1999.

[10] Lütkepohl, H. Comparison of criteria for estimating the order of a vector autoregressive process, Journal of Time Series Analysis **6 (1)**, 35–52, 1985, Correction: 1987, 373.

[11] Pukkila, T. M. and Krishnaiah, P. R. On the use of autoregressive order determination criteria in multivariate white noise tests, IEEE Transactions on Acoustics, Speech and Signal Processing **36 (9)**, 1396–1403, 1988.

[12] Quinn, B. Order determination for a multivariate autoregression, Journal of Royal Statistical Society **B42**, 182–185, 1980.

[13] Reinsel, G. C. Elements of multivariate time series analysis, Springer-Verlag, 1995.

[14] Schwarz, G. Estimating the dimension of a model, Annals of Statistics **6**, 461-464, 1978.

[15] Shibata, R. Selection of the order of an autoregressive model by Akaike's information criterion, Biometrika **63 (1)**, 117–126, 1976.

[16] Shibata, R. Asymptotically efficient selection of the order of the model for estimating parameters of a linear process, Annals of Statistics **8**, 147–164, 1980.