Hacettepe Journal of Mathematics and Statistics
Volume 44 (3) (2015), 707 – 713

Composite quantile regression for linear errors-in-variables models

Rong Jiang *

Abstract

Composite quantile regression can be more efficient and sometimes arbitrarily more efficient than least squares for non-normal random errors, and almost as efficient for normal random errors. Therefore, we extend composite quantile regression method to linear errors-in-variables models, and prove the asymptotic normality of the proposed estimators. Simulation results and a real dataset are also given to illustrate our the proposed methods.

2000 AMS Classification: primary 60G08, 62G10; secondary 62G20.

Keywords: Errors-in-variables; composite quantile regression.

Received 25/05/2014 : Accepted 17/07/2014 Doi : 10.15672/HJMS.2014237474

1. Introduction

Consider a linear errors-in-variables model as follows:

$$\begin{cases} Y = x^T\beta_0 + \varepsilon, \\ X = x + u, \end{cases} \tag{1.1}$$

where x is a p-dimensional vector of unobserved latent covariates which is measured in an error-prone way, X is the observed surrogate of x, β_0 is a p-dimensional unknown parameter vector, Y are responses vector, $(\varepsilon, u^T)^T$ is a p+1-dimensional spherical error vector, and they are independent with a common error distribution that is spherically symmetric. Spherically symmetric implies that ε and each component u have the same distribution, which ensures model identifiability. We restrict ourselves to structural models where x are independently and identically distributed random variables. If x stem from non-stochastic designs, the model is said to have a functional relationship, see Fuller (1987) for details. Model (1.1) belongs to a kind of model called the errors-in-variables model or measurement error model which was proposed by Deaton (1985) to correct for the effects of sampling error and is somewhat more practical than the ordinary regression model. Fuller (1987) gave a systematic survey on this research topic and present many

*Department of Applied Mathematics, Donghua University, Shanghai, 201620, China
Email: jrtrying@dhu.edu.cn

applications of measurement error data. Other references can see Cui (1997a), He and Liang (2000), Huang and Wang (2001), Ma and Tsiatis (2006), Schennach (2007), Liang and Li (2009), Wei and Carroll (2009), Hu and Cui (2009), Jiang et al. (2012a) and so on.

The composite quantile regression (CQR) was first proposed by Zou and Yuan (2008) for estimating the regression coefficients in the classical linear regression model. Zou and Yuan (2008) showed that the relative efficiency of the CQR estimator compared with the least squares estimator is greater than 70% regardless of the error distribution. Furthermore, the CQR estimator could be more efficient and sometimes arbitrarily more efficient than the least squares estimator. Other references about CQR method can see Kai, Li and Zou (2010), Kai, Li and Zou (2011), Tang et al. (2012a), Tang et al. (2012b), Guo et al. (2012) and Jiang et al. (2012b, 2012c, 2013, 2014a, 2014b). These nice theoretical properties of CQR in linear regression motivate us to consider linear errors-in-variables models based on CQR method so as to make the method of CQR more effective and convenient.

This paper is organized as follows. The main results are given in Section 2. Some simulations and a real data application are conducted in Section 3 to illustrate our methodology. Final remarks are given in Section 4. All the conditions and technical proofs are collected in the Appendix.

2. Methodology and main results

If the true covariates x are observed, the parameters β in model (1.1) can be estimated through (Zou and Yuan, 2008)

$$(\tilde{b}_1, ..., \tilde{b}_K, \tilde{\beta}) = argmin_{b_1,...,b_K,\beta} \sum_{k=1}^{K} \sum_{i=1}^{n} \rho_{\tau_k} \left(Y_i - b_k - x_i^T \beta \right),$$

where $\rho_{\tau_k}(r) = \tau_k r - rI(r < 0), k = 1, 2, \ldots, K$, be K check loss functions with $0 < \tau_1 < \tau_2 < \cdots \tau_K < 1$. Typically, we use the equally spaced quantiles: $\tau_k = \frac{k}{K+1}$ for $k = 1, 2, \ldots, K$. $\tilde{b}_k$ is estimator of b_{τ_k}, where $P(\varepsilon \leq b_{\tau_k} | x_i) = \tau_k$, b_{τ_k} is the τ_k quantile of ε.

Taking into account the measurement error in X, we consider estimating β as follows

$$(\hat{a}_1, ..., \hat{a}_K, \hat{\beta}) = argmin_{a_1,...,a_K,\beta} \sum_{k=1}^{K} \sum_{i=1}^{n} \rho_{\tau_k} \left(\frac{Y_i - a_k - X_i^T \beta}{\sqrt{1 + \|\beta\|^2}} \right), \tag{2.1}$$

where $\hat{a}_k$ is the estimator of $b_{\tau_k}\sqrt{1 + \|\beta_0\|^2}$, $k = 1, \ldots, K$. The measurement error correction factor $\frac{1}{\sqrt{1+\|\beta\|^2}}$ is widely used in linear models with additive errors (see Ma and Yin, 2011) . The main intuition is the following. In the usual regression, one minimizes the vertical standardized distance $d\{(Y - a_k - X^T\beta)/s.d.(Y - a_k - X^T\beta)\}$ where d stands for a suitable distance measure and s.d. is the standard deviation, because only the vertical Y direction has errors. However, in the measurement error situation, errors also occur along the horizontal X direction, hence a distance containing both vertical and horizontal components should be favored. In fact, the minimization of the same standardized distance with X replaced by x automatically corrects for this. If we denote the variance of ε as Σ_ε and the variance-covariance matrix of u as Σ_u, we have

$$\frac{(Y - a_k - X^T\beta)}{s.d.(Y - a_k - X^T\beta)} = \frac{(Y - a_k - X^T\beta)}{\sqrt{\Sigma_\varepsilon + \beta^T \Sigma_u \beta}}$$

which is proportional to $(Y - a_k - X^T\beta)/\sqrt{1 + \|\beta\|^2}$ under the spherical symmetry assumption. The following theorem gives the the asymptotic normality for the composite quantile regression estimator $\hat{\beta}$.

Theorem 1 Assuming Conditions A1-A2 in the Appendix are satisfied, then

$$\sqrt{n}(\hat{\beta} - \beta_0) \xrightarrow{L} N\left(0, \left(\sum_{k=1}^{K} f(b_{\tau_k})\right)^{-2} (1 + \|\beta_0\|^2)\Sigma_x^{-1} S \Sigma_x^{-1}\right),$$

where $\xrightarrow{L}$ stands for convergence in distribution, $\Sigma_x = E(xx^T)$ and $S = \sum_{k,k'=1}^{K} \min(\tau_k, \tau_{k'})(1 - \max(\tau_k, \tau_{k'}))\Sigma_x + Cov\left[\sum_{k=1}^{K} \psi_{\tau_k}\left(\frac{\varepsilon - u^T\beta_0}{\sqrt{1+\|\beta_0\|^2}} - b_{\tau_k}\right)\left(u + \frac{(\varepsilon - u^T\beta_0)\beta_0}{1+\|\beta_0\|^2}\right)\right]$.

Remark 1: In practice, there is constant term in model (1.1), then model (1.1) can be write as

$$\begin{cases} Y = \alpha_0 + x^T\beta_0 + \varepsilon, \\ X = x + u. \end{cases}$$

The parameter α_0 and β_0 can be estimated as follows (Cui, 1997b)

$$(\hat{a}_1^*, ..., \hat{a}_K^*, \hat{\beta}^*) = argmin_{a_1,...,a_K,\beta} \sum_{k=1}^{K}\sum_{i=1}^{n} \rho_{\tau_k}\left(\frac{Y_i - \bar{Y} - a_k - (X_i - \bar{X})^T\beta}{\sqrt{1 + \|\beta\|^2}}\right),$$

$$\hat{\alpha} = \bar{Y} - \bar{X}^T\hat{\beta}^*,$$

where $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

3. Numerical studies

In this section, we conduct simulation studies to assess the finite sample performance of the proposed procedures and illustrate the proposed methodology on AIDS clinical trials. Furthermore, we compare CQR method with least square (LS) method proposed by Fuller (1987), t-type (TT) method proposed by Hu and Cui (2009) and quantile regression method with $\tau = 0.5$ ($\mathrm{QR}_{0.5}$) proposed by He and Liang (2000).

3.1. Simulation example. We conduct a small simulation study with n = 100 and the data are generated from model (1.1), where the random error variables are taken to be 0.5*N(0,1), 0.2*t(3) and 0.05*C(0,1) distribution. The covariate vector $x = (x_1, x_2, \ldots, x_p)$ are generated from standard normal distribution N(0,1) and p=1,2,5 are considered. We focus on $K = 5$, $K = 9$ and $K = 19$ for composite quantile regression, respectively. The mean squared errors (MSE) and their standard deviations (STD) over 1000 simulations are summarized in Table 1, where MSE=$\|\hat{\beta} - \beta_0\|^2$. It can be seen from Table 1 that the CQR estimators have better performance than LS for heavy-tailed error distributions (t(3) and C(0,1)), but is less efficient when the error is normal distribution N(0,1). Moreover, the results show that our method is more efficient than TT and $\mathrm{QR}_{0.5}$ in most cases. The three CQR estimators perform very similarly. Further, one can see that the three CQR estimators are close to the true value.

3.2. Real data example. In this section, we present an analysis of an AIDS clinical trial group (ACTG 315) study. One of the purposes of this study is to investigate the relationship between virologic and immunologic responses in AIDS clinical trials. In general, it is believed that the virologic response RNA (measured by viral load) and immunologic response (measured by CD4+ cell counts) are negatively correlated during treatment. Our preliminary investigations suggested that viral load depends linearly on CD4+ cell count. We therefore model the relationship between viral load and CD4+ cell counts by model (1.1). Let Y_i be the viral load and let x_i be the CD4+ cell count

Table 1 Simulation results for simulation example.

p	Method	N(0,1) MSE	STD	t(3) MSE	STD	C(0,1) MSE	STD
p=1	LS	0.0056	0.0077	0.0037	0.0063	1.8657	8.2551
	TT	0.0072	0.0082	0.0025	0.0035	0.0214	0.1407
	$QR_{0.5}$	0.0086	0.0095	0.0030	0.0039	0.0209	0.1422
	CQR_5	0.0064	0.0081	0.0022	0.0033	0.0219	0.1399
	CQR_9	0.0064	0.0084	0.0023	0.0034	0.0217	0.1407
	CQR_{19}	0.0063	0.0083	0.0022	0.0033	0.0315	0.1686
p=2	LS	0.0192	0.0204	0.0198	0.1749	3.2342	10.1476
	TT	0.0257	0.0284	0.0065	0.0074	0.6477	5.7348
	$QR_{0.5}$	0.0295	0.0330	0.0072	0.0083	0.6295	5.6614
	CQR_5	0.0216	0.0225	0.0061	0.0064	0.1525	0.5327
	CQR_9	0.0209	0.0216	0.0062	0.0066	0.1512	0.5065
	CQR_{19}	0.0206	0.0216	0.0061	0.0066	0.1586	0.5313
p=5	LS	0.1081	0.0724	0.0796	0.1203	4.7832	8.6034
	TT	0.1498	0.1010	0.0426	0.0309	0.8189	1.9204
	$QR_{0.5}$	0.1613	0.1100	0.0446	0.0326	0.7865	1.9108
	CQR_5	0.1203	0.0775	0.0397	0.0286	0.6306	0.9306
	CQR_9	0.1170	0.0765	0.0390	0.0283	0.6560	0.9254
	CQR_{19}	0.1156	0.0772	0.0396	0.0286	0.7147	0.9592

for subject i. To reduce the marked skewness of CD4+ cell counts, and make treatment times equal space, we take log-transformations of both variables. The x_i are measured with error (Liang et al., 2003). The model we used is

$$Y = x^T\beta_0 + \varepsilon, \quad X = x + u,$$

where X is the observed CD4+ cell counts. The performances of CQR method with different K are very similar (see Table 1), and considering computing time, K=5 is a good choice in practice. Therefore, K=5 for CQR method is considered in this example. The parameter estimator by using our proposed method is -0.0717. Moreover, the standard deviation of the parameter is 0.0078 and the 90% confidence interval is [-0.0824,-0.0576] by using the random weighting method (see Jiang et al., 2012a).

4. Conclusion

In this work, we have focused on the CQR method for linear errors-in-variables models and proven its nice theoretical properties. Moreover, the proposed approaches are demonstrated by simulation examples and a real data application.

Appendix

To prove main results in this paper, the following technical conditions are imposed.

A1. Assume (ε, u^T) is spherically symmetric with finite first moment, and the distribution functions F of ε are absolutely continuous, with continuous densities f uniformly bounded away from 0 and ∞ at the points $b_{\tau_k}, k = 1, \ldots, K$ and $E\varepsilon^2 < \infty$.

A2. $E(x) = 0$ and $\Sigma_x = E(xx^T)$ is positive definite.

Remark 2: Conditions A1-A2 are standard conditions, see He and Liang (2000).

Now we proceed to prove the theorems.

Proof of Theorem 1. Denote

$$
\begin{aligned}
f_{1ik}(b_k,\beta) =&\rho_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta}{\sqrt{1+\|\beta\|^2}}-b_{\tau_k}-\frac{x_i^T(\beta-\beta_0)}{\sqrt{1+\|\beta\|^2}}-(b_k-b_{\tau_k})\right)-\rho_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\\
&-\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\left(\frac{\varepsilon_i-u_i^T\beta}{\sqrt{1+\|\beta\|^2}}-\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-\frac{x_i^T(\beta-\beta_0)}{\sqrt{1+\|\beta\|^2}}-(b_k-b_{\tau_k})\right),\\
f_{2ik}(b_k,\beta) =&\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\left(\frac{\varepsilon_i-u_i^T\beta}{\sqrt{1+\|\beta\|^2}}-\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-\frac{x_i^T(\beta-\beta_0)}{\sqrt{1+\|\beta\|^2}}-(b_k-b_{\tau_k})\right).
\end{aligned}
$$

$(\hat a_1,...,\hat a_K,\hat\beta)$ is the minimizer of the following criterion:

$$
\begin{aligned}
&\sum_{k=1}^K\sum_{i=1}^n\left[\rho_{\tau_k}\left(\frac{Y_i-a_k-X_i^T\beta}{\sqrt{1+\|\beta\|^2}}\right)-\rho_{\tau_k}\left(\frac{Y_i-a_{\tau_k}-X_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}\right)\right]\\
=&\sum_{k=1}^K\sum_{i=1}^n f_{1ik}(b_k,\beta)+\sum_{k=1}^K\sum_{i=1}^n f_{2ik}(b_k,\beta)\\
\equiv&Q_n(b_1,...,b_K,\beta)
\end{aligned}
$$

Therefore, by applying the identity in Knight (1998)

$$
\rho_\tau(x-y)-\rho_\tau(x)=-y\psi_\tau(x)+\int_0^y\{I(x\le z)-I(x\le 0)\}dz.
$$

We have

$$
\begin{aligned}
EQ_n(b_1,...,b_K,\beta)&=\sum_{k=1}^K\sum_{i=1}^n E\left[\rho_{\tau_k}\left(\frac{Y_i-a_k-X_i^T\beta}{\sqrt{1+\|\beta\|^2}}\right)-\rho_{\tau_k}\left(\frac{Y_i-a_{\tau_k}-X_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}\right)\right]\\
&=\sum_{k=1}^K\sum_{i=1}^n E\left[\rho_{\tau_k}\left(\varepsilon_i-b_{\tau_k}-\frac{x_i^T(\beta-\beta_0)}{\sqrt{1+\|\beta\|^2}}-(b_k-b_{\tau_k})\right)-\rho_{\tau_k}\left(\varepsilon_i-b_{\tau_k}\right)\right]\\
&=\sum_{k=1}^K\sum_{i=1}^n E\left[\int_0^{\frac{x_i^T(\beta-\beta_0)}{\sqrt{1+\|\beta\|^2}}+(b_k-b_{\tau_k})}\{F(\varepsilon_i\le b_{\tau_k}+z|x_i)-F(\varepsilon_i\le z|x_i)\}\right]dz\\
&\to\frac{1}{2}\sum_{k=1}^K f(b_{\tau_k})(\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0))\left[\begin{array}{cc}1 & 0\\ 0 & \frac{\Sigma_x}{1+\|\beta_0\|^2}\end{array}\right](\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0)^T)^T.
\end{aligned}
$$

Next, we study $f_{2ik}(b_k,\beta)$,

$$
\begin{aligned}
f_{2ik}(b_k,\beta)=&-\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\frac{1}{\sqrt{1+\|\beta_0\|^2}}\left(x_i+u_i+\frac{(\varepsilon_i-u_i^T\beta_0)\beta_0}{1+\|\beta_0\|^2}\right)^T(\beta-\beta_0)\\
&-\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)(b_k-b_{\tau_k})+R.
\end{aligned}
$$

Similar to the proof of Theorem 3 in Cui (1997a), we can obtain

$$
\begin{aligned}
&\sum_{i=1}^n[f_{1ik}(b_k,\beta)-Ef_{1ik}(b_k,\beta)]=o_p\left(\left\|\left(\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0)\right)\right\|\right)\\
&\sum_{i=1}^n[R-ER]=o_p\left(\left\|\left(\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0)\right)\right\|\right).
\end{aligned}
$$

Thus it follows that

$$
\begin{aligned}
&Q_n(b_1,...,b_K,\beta) \to Q_0(b_1,...,b_K,\beta)\\
=&-\frac{1}{\sqrt{n}}\sum_{k=1}^{K}\sum_{i=1}^{n}\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\frac{1}{\sqrt{1+\|\beta_0\|^2}}\left(x_i+u_i+\frac{(\varepsilon_i-u_i^T\beta_0)\beta_0}{1+\|\beta_0\|^2}\right)^T\sqrt{n}(\beta-\beta_0)\\
&-\frac{1}{\sqrt{n}}\sum_{k=1}^{K}\sum_{i=1}^{n}\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\sqrt{n}(b_k-b_{\tau_k})\\
&+\frac{1}{2}\sum_{k=1}^{K}f(b_{\tau_k})(\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0))\begin{bmatrix}1 & 0\\ 0 & \frac{\Sigma_x}{1+\|\beta_0\|^2}\end{bmatrix}(\sqrt{n}(b_k-b_{\tau_k}),\sqrt{n}(\beta-\beta_0)^T)^T.
\end{aligned}
$$

The convexity of the limiting objective function, $Q_0(b_1,...,b_K,\beta)$, assures the uniqueness of the minimizer and, consequently, that

$$
\sqrt{n}(\hat{\beta}-\beta_0)=\frac{\Sigma_x^{-1}\sqrt{1+\|\beta_0\|^2}}{\sqrt{n}\sum_{k=1}^{K}f(b_{\tau_k})}\sum_{k=1}^{K}\sum_{i=1}^{n}\psi_{\tau_k}\left(\frac{\varepsilon_i-u_i^T\beta_0}{\sqrt{1+\|\beta_0\|^2}}-b_{\tau_k}\right)\left(x_i+u_i+\frac{(\varepsilon_i-u_i^T\beta_0)\beta_0}{1+\|\beta_0\|^2}\right)+o_p(1).
$$

The proof is completed.

References

[1] Cui, H. J. (1997a). Asymptotic normality of M-estimates in the EV model. Systems Science and Mathematical Sciences, 10, 225-236.

[2] Cui, H. J. (1997b). Asymptotic properties of generalized minimum L1-norm estimates in EV model. Science in China (Series A), in Chinese, 2, 119-131.

[3] Deaton, A. (1985). Panel data from a time series of cross-sections. Journal of Econometrics, 30, 109-126.

[4] Fuller, W. A. (1987). Measurement Error Models. Wiley, New York.

[5] Guo, J., Tian, M. and Zhu, K. (2012). New Efficient and robust estimation in varying-coefficient models with heteroscedasticity. Statistica Sinica, 22, 1075-1101.

[6] He, X. and Liang, H. (2000). Quantile regression estimates for a class of linear and partially linear errors-in-variables models. Statistica Sinica, 10, 129-140.

[7] Huang, Y. J. and Wang, C. Y. (2001). Consistent functional methods for logistic regression with errors in covariates. Journal of the American Statistical Association, 96, 1469-1482.

[8] Hu, T. and Cui, H. J. (2009). t-type estimators for a class of linear errors-in-variables models. Statistica Sinica, 19, 1013-1036.

[9] Jiang, R., Yang, X. H. and Qian, W. M. (2012a). Random weighting M-estimation for linear errors-in-variables models. Journal of the Korean Statistical Society, 41, 505-514.

[10] Jiang, R., Zhou, Z. G., Qian, W. M. and Shao, W. Q. (2012b). Single-index composite quantile regression. Journal of the Korean Statistical Society, 3, 323-332.

[11] Jiang, R., Qian, W. M. and Zhou, Z. G. (2012c). Variable selection and coefficient estimation via composite quantile regression with randomly censored data. Statistics & Probability Letters, 2, 308-317.

[12] Jiang, R., Zhou, Z. G., Qian, W. M. and Chen, Y. (2013). Two step composite quantile regression for single-index models. Computational Statistics and Data Analysis, 64, 180-191.

[13] Jiang, R., Qian, W. M. and Li, J. R. (2014a). Testing in linear composite quantile regression models, Computational Statistics, In Print.

[14] Jiang, R. Qian, W. M. and Zhou, Z. G. (2014b). Test for single-index composite quantile regression. Hacettepe Journal of Mathematics and Statistics, Accepted.

[15] Kai, B., Li, R. and Zou, H. (2011). New efficient estimation and variable selection methods for semiparametric varying-coefficient partially linear models. Annals of Statistics, 39, 305-332.

[16] Kai, B., Li, R. and Zou, H. (2010). Local composite quantile regression smoothing: an efficient and safe alternative to local polynomial regression. Journal of the Royal Statistical Society, Series B, 72, 49-69.

[17] Knight, K. (1998). Limiting distributions for L_1 regression estimators under general conditions. Annals of Statistics, 26, 755-770.

[18] Liang H., Wu H. L. and Carroll R. J. (2003). The relationship between virologic and immunologic responses in AIDS clinical research using mixed-effect varying-coefficient semiparametric models with measurement error. Biostatistics, 4, 297-312.

[19] Liang, H. and Li, R. (2009). Variable selection for partially linear models with measurement errors. Journal of the American Statistical Association, 104, 234-248.

[20] Ma, Y. and Yin, G. (2011). Censored quantile regression with covariate measurement errors. Statistica Sinica, 21, 949-971.

[21] Ma, Y. and Tsiatis, A. A. (2006). Closed form semiparametric estimators for measurement error models. Statistica Sinica, 16, 183-193.

[22] Schennach, M. (2007). Instrumental variable estimation of nonlinear errors-in-variables models. Econometrica, 75, 201-239.

[23] Tang, L., Zhou, Z. and Wu, C. (2012a). Weighted composite quantile estimation and variable selection method for censored regression model. Statistics & Probability Letters, 3, 653-663.

[24] Tang, L., Zhou, Z. and Wu, C. (2012b). Efficient estimation and variable selection for infinite variance autoregressive models. Journal of Applied Mathematics and Computing, 40, 399-413.

[25] Wei, Y. and Carroll, R. J. (2009). Quantile regression with measurement error. Journal of the American Statistical Association, 104, 1129-1143.

[26] Zou, H. and Yuan, M. (2008). Composite quantile regression and the oracle model selection theory. Annals of Statistics, 36, 1108-1126.