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An Approach for the Time-Dependent Thermoeconomic Modeling and Optimization of Energy System Synthesis, Design and Operation
Part II: Reliability and Availability

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Abstract

Details of the reliability and availability aspects of the thermoeconomic methodology presented in Part I (Olsommer et al., 1999) are given here in Part II of our series of two articles. These system details cannot be forgotten, particularly if the superconfiguration envisioned offers the choice of several technologies for the same function (redundancy) or if there are guarantees of availability which must be met. An original method, which is an extension of the event space method, is presented here for automating the reliability and/or availability calculations of a repairable system chosen from a flexible structure of equipment in active and/or passive redundancy, connected in series and/or parallel. This methodology permits a more realistic evaluation of performance, flows and costs at both the structural (synthesis-design) and operational levels and leads, thus, to more rational decisions on system synthesis, design and operation. Results for the application of this methodology to a waste incineration cogeneration facility with a gas turbine topping cycle are given in Part I (Olsommer et al., 1999).

Keywords: reliability, availability, Markov chains, energy systems optimization

1. Introduction

When the reliability and/or the availability of a system must be enhanced or must meet a particular goal, it is common practice to place several key pieces of equipment in parallel (redundancy). Sometimes it is a matter of meeting a given heat demand to be delivered to a district heating network within a given period and with a minimal probability of not meeting the demand. Or it can be a matter of improving the productivity of a system. In both cases, these considerations have an immediate impact on the costs of the system and on its multiple flows (materials, energy). Furthermore, as soon as one admits that a piece of equipment is not perfectly reliable such considerations should be taken into account in a thermoeconomic analysis and/or optimization. In fact, when a system is composed of more than one piece of equipment (in series and/or in parallel), a *comprehensive* reliability and/or availability analysis must be completed if one wishes to calculate with accuracy the costs, the performance and the flows of the system.

In the context of simultaneously modeling and optimizing from a thermoeconomic standpoint the structure and operation of a system, there is a need for an automated procedure, which can deal with systems for which the structure changes (flexible) with time. Several methods which could be applicable to such an automated procedure have been developed (Villemeur, 1988). Of these the event space method (Roberts, 1964) is well suited for the reliability and availability analysis of repairable systems, taking into account all possible events of the system. The procedure can be easily implemented on a computer when the pieces of equipment are in active redundancy (i.e. no equipment is in stand-by[1]). The method involves constructing a so-called "Markov graph" or event space graph from which the reliability and/or availability can easily be deduced. Unfortunately, this automated procedure cannot be applied in a straightforward manner when in addition to active redundancy passive redundancy (i.e. a system including stand-by equipment) is considered. In this case, there are more than two possible events for each piece of equipment (actually six), leading, a priori, to a more complicated analysis with many more possible events[2]. The methodology developed here to overcome this difficulty (Olsommer, 1998) is presented below and permits use of a standard *binary* events-based Markov graph for repairable systems with active and/or passive redundancy. The algorithm, based on a set of rules, can easily be implemented on a computer.

We begin with a definition of important concepts of the reliability and availability theory.

2. Brief Background on Fundamentals

A system consists of several pieces of equipment placed in series and/or in parallel. Redundancy occurs when the equipment is in parallel. Depending on whether or not a piece of equipment is supposed to be in operation or in stand-by, redundancy is either active or passive, respectively.

2.1 Reliability, availability, maintainability

The reliability R(t) (Eq. (1)) is measured by the probability that an entity E is able to accomplish its required function during a given period t. The availability A(t) (Eq. (2)) is measured by the probability that an entity E is able to accomplish its required function at a given time t. The maintainability M(t) (Eq. (3)) is measured by the probability that the maintenance of an entity E is completed at a given time t, knowing that it had failed at time t=0. In equation form, these are given by

$$R(t)=\mathbb{P}[\text{E does not fail over } [0,t]] \tag{1}$$

$$A(t)=\mathbb{P}[\text{E does not fail at time t}] \tag{2}$$

$$M(t)=\mathbb{P}[\text{E is repaired over } [0,t]] \tag{3}$$

2.2 Stochastic process and Markov chains

A stochastic process is a mathematical model which allows one to associate a random variable Q (representing here a system event) to a random variable T (representing here the time t). Thus, the realization of event Q at time t is a function of the history (the trajectory of Q(t)). A stochastic process is called Markov when the event $Q(t_n)$ is fully determined by the preceding event $Q(t_{n-1})$. It is said that all the history is contained in the previous event. When all events of a Markov process are discrete, this process is called a Markov chain. This chain can be represented graphically as a Markov graph (or event space graph). The event space method (Roberts, 1964) relies on these two main assumptions: a Markov process in a discrete space.

2.3 Transition rates (failure and repair rates)

Transition rates express the probability of passing from a given event to another during the very next time period, knowing that the given event existed as such in the previous time period.

The failure rate λ (Eq. (4)) is defined by the limit, provided it exists, of the ratio between the conditional probability that the failure instance of an entity is included in a given time interval [t, t+Δt] and the time interval when Δt→0. This assumes that the entity has not had a failure over [0, t]. In equation form, this is expressed as

$$\lambda(t)=\lim_{\Delta t\to 0}\frac{1}{\Delta t}\ \mathbb{P}\left[\text{E fails between } [t, t+\Delta t], \text{ knowing that it has not had any failure over } [0,t]\right] \tag{4}$$

With the help of the conditional probabilities theorem, it can be easily shown that the failure rate can be expressed as a function of the reliability as follows:

$$\lambda(t)=\lim_{\Delta t\to 0}\frac{R(t)-R(t+\Delta t)}{\Delta t R(t)}=\frac{-\frac{dR(t)}{dt}}{R(t)}\geq 0 \tag{5}$$

The repair rate μ (Eq. (6)) is defined by the limit (if existing) as Δt→0 of the ratio between the conditional probability that the moment of completion of an entity's repair is included in a given time interval [t, t+Δt] and the time interval itself. This assumes that the entity has failed over [0, t]. Thus,

[1] Equipment engaged only in case of need.

[2] 6^K events, with K the number of pieces of equipment.

$$\mu(t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \; P\big[E \text{ repaired between } [t, t+\Delta t], \text{ knowing that it has failed over } [0,t]\big] \quad (6)$$

In a way similar to that for the failure rate, it can be shown that the repair rate takes the form

$$\mu(t) = \frac{\frac{dM(t)}{dt}}{1 - M(t)} \geq 0 \qquad (7)$$

In graphic form, the failure rate of a piece of equipment is traditionally represented by a curve with a tub shape (see *Fig. 1*), depicting three main periods of its lifetime. They are (i) the early period marked by an initial abrupt decrease in failure rate (early failures); (ii) the useful lifetime period denoted by a constant failure rate; and (iii) the end of life period marked by an abrupt increase in the failure rate (wear). When a quasi-stationary approximation is used (see Part I; Olsommer et al., 1999), the curve of *Fig. 1* can easily be decomposed into several mean constant values.

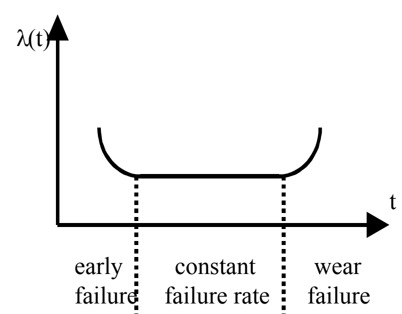


Figure 1. Time dependence of failure rate.

With the preceding definitions, the reliability and availability of a piece of equipment can be treated provided the following parameters are known:

- operating failure rate (λ);
- start-up failure rate (γ);
- stopped failure rate (λ_s);
- repair rate (μ).

TABLE I. TRANSITION RATE ESTIMATORS.

Estimator	Note
$\hat{\lambda} = N_f/T_f$	N_f = number of failures observed while operating T_f = cumulative operating time
$\hat{\gamma} = N_d/N_s$	N_d = number of failures observed at start-up N_s = number of start-ups
$\hat{\mu} = N_r/T_r$	N_r = number of repairs T_r = cumulative repair time

Note that the stopped failure rate can usually be neglected (i.e. λ_s=0). Estimators for the parameters listed above are given in TABLE I. Their values can be found in the literature (e.g., Procaccia and Aufort, 1995).

3. Event Space Method for Systems with Active Redundancy

Let a system be composed of a set of K components, placed in series and/or active redundancy. Each component has two possible events (operation (0) and breakdown (1)). Thus, the number of possible events is 2^K. The transition rates between each event are functions of the transition rates (λ, μ). This information can be depicted in a Markov graph (see *Fig. 2*). Each cell represents an event and is named a summit (V), while each arrow represents a transition rate and is named an edge ($\rightarrow$). Events are grouped in columns defined by the number of broken-down pieces of equipment (e.g., the middle column has one broken-down piece of equipment while those to the left and right have zero and two, respectively). Thus the Markov graph of a system of K components is made of K +1 columns (see *Fig. 2*). It is important to note that there is no edge between event "1" (00) and "4" (11) since the probability of having two transitions at a time is of magnitude dt^2 and can be neglected (Pagès and Gondran, 1980).

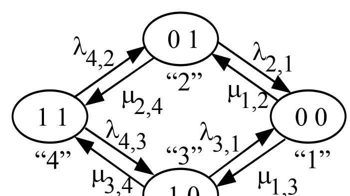


Figure 2. Markov graph for availability analysis of a system with two repairable pieces of equipment (1=operation, 0=breakdown).[3]

3.1 System's state equations

Let a_{ij} be the transition rate between events i and j. In order to calculate the probability $P_i(t+dt)$ of an event i at time (t+dt), there are two possibilities:

(i) If the system was in event i at time t, it must have stayed in that event. The associated probability is given by

[3] Note that this graph can also be used for reliability analysis by noting (based on the definition of reliability) that the system cannot return from a breakdown event. If the breakdown event occur when both pieces of equipment are broken-down (event "1"), it follows that one has to suppress all edges emanating from event "1" (00).

$$1-\sum_{j\neq i}^{I} a_{ij}\ dt \tag{8}$$

(ii) If the system was in event j at time t, then the transition inevitably occurred. The probability is

$$a_{ji}\ dt \tag{9}$$

Thus, the probability $P_i(t+dt)$ can be expressed as the sum of the possible probabilities given above, i.e.

$$\begin{aligned} P_i(t+dt) = \ & P\big[\text{the system is in i at (t)} \\ & \text{and stays in i at (t+dt)}\big] + \\ & \sum_{j\neq i}^{I} P\ \big[\text{the system is in j at (t)} \\ & \text{and in i at (t+dt)}\ \big] \end{aligned} \tag{10}$$

By setting $a_{ii} = -\sum_{j\neq i}^{I} a_{ij}$, the system's state equations become

$$\frac{dP_i(t)}{dt} = \sum_{j=1}^{I} P_j(t)\ a_{ji} \qquad \forall i = 1,2,..,I \tag{11}$$

When initial conditions are known and a_{ji} are fixed values, the system of equations (Eq. (11)) can be integrated. Each function is formed by an unsteady component and by a stationary component. In practice, the unsteady component tends rapidly towards zero and the system thus tends towards the stationary solution denoted by $P_i(\infty)$. In this case, Eq. (11) can be reduced to a homogenous system of equations, i.e.

$$\sum_{j=1}^{I} P_j(\infty)\ a_{ji} = 0 \qquad \forall i = 1,..,I \tag{12}$$

This system can easily be solved by any linear algebra algorithm, taking into account the additional condition that

$$\sum_{j=1}^{I} P_j(\infty) = 1 \tag{13}$$

Laplace's method (Pagès and Gondran, 1980) allows one to express the solution of Eqs. (12) and (13) as the ratio of the determinant of the transition rate matrix[4] **A** (where except for line i which is replaced by a one, the last column is replaced by a column of zeros) to the determinant of **A** (where the last column is replaced by a column of ones). In equation form this is expressed as

$$P_i(\infty) = \frac{\det\begin{bmatrix} a_{1,1} & \cdots & a_{1,I-1} & 0 \\ \vdots & & \vdots & 0 \\ a_{i,1} & & a_{i,I-1} & 1 \\ \vdots & & \vdots & 0 \\ a_{I,1} & \cdots & a_{I,I-1} & 0 \end{bmatrix}}{\det\begin{bmatrix} a_{1,1} & \cdots & a_{1,I-1} & 1 \\ \vdots & & \vdots & 1 \\ a_{i,1} & & a_{i,I-1} & 1 \\ \vdots & & \vdots & 1 \\ a_{I,1} & \cdots & a_{I,I-1} & 1 \end{bmatrix}} \tag{14}$$

When the transition rates are constant and the Markov graph has been established, the above system of homogenous equations (Eq. (14)) determines the asymptotic ($t\rightarrow\infty$) availability and/or reliability of systems composed of equipment in series and/or parallel and in active redundancy.

4. Event Space Method Extended to Passive Redundancy

When considering active redundancy combined with passive, six possible events can occur for each component (see TABLE II). The construction of the Markov graph is, thus, more complicated, since it can contain 6^K events. For this reason, the usual way to do it is to do it "manually" for each system configuration (Alvi, 1993). In an optimization procedure, this is impractical. An alternative consists in automatically constructing the graph as it was done for active redundancy only, resulting in a 6^K events graph. For practical reasons, this is not acceptable for systems with a large number of components. What is needed is an algorithm able to reduce the size of matrix **A** and to automatically construct the Markov graph.

In complex systems, when a piece of equipment breaks down, the measures that have to be taken are usually dictated by predefined procedures (decision tree, automated procedures) called emergency procedures. Depending on the preceding event, these procedures must indicate, for example, if a particular stand-by piece of equipment must be engaged or not.

TABLE II. ACTIVE AND PASSIVE REDUNDANCIES: POSSIBLE EVENTS.

Redundancy	Description	V_E
active	breakdown	0
active	operating	1
passive	waiting, broken-down	2
passive	waiting, ready to operate	3
passive	engaged, breaks down	4
passive	engaged, starts	5

4 Matrix of size IxI composed of the transition rates a_{ij}.

4.1 Problem definition

Let N be the maximum number of pieces of equipment possible in the superconfiguration at time t. When all pieces of equipment are defined by an independent structural variable[5], this number can be identified in the optimization procedure by inspecting the binary and integer components of vector **w**(1,..,$\boldsymbol{N}$) (see Part I; Olsommer et al., 1999). If w_n= 0, the corres-ponding piece of equipment does not exist. A similar inspection of the binary components of the operational independent vector **x**(1,.., $\boldsymbol{N}$) permits one to determine if the corresponding piece of equipment is active or passive in the corresponding sequence. It is active if x_n= 1 and passive if x_n= 0. Thus, vector **w** determines the number K of pieces of equipment effectively present and vector **x** their type of redundancy (active or passive).

In order to construct the Markov graph and its associated transition rate matrix **A**, two separate parts of **A** will be considered: one for the failure analysis and the second for the repair analysis. Before proceeding, however, the following notation is adopted:

- the prefix *"binary"* or the subscript "$_B$" will be added for active redundancy (event, table);
- the prefix *"integer"* or the subscript "$_E$" will be added for passive redundancy (event, table).

4.2 Markov graph and transition rate matrix A (failures):

The proposed algorithm consists of four main steps: (i) to construct the *binary* events table (F_B) and to sort it; (ii) to construct the *integer* events table (F_E) and the associated Markov graph; (iii) to construct the transition rate matrix (**A**); and (iv) to resolve possible conflicts which can occur in the construction of (F_B). These steps are outlined below.

(i) <u>*Binary* events table:</u>

Let K be the number of pieces of equipment. Thus, there are $I=2^K$ *binary* events. The *binary* events table F_B (Eq. (15)) is obtained by:

- associating with each number $i \in \{1,.., I\}$ (denoted #D) the binary number (ÿÿnoteÿÿ#B) equivalentn o i-1 as well as H, the numprr eduiÿÿlent to the sum (bit by bit) of the corresponding binary number (#B);
- sorting the table according to both criteria:
  1. H decreasing;
  2. #D decreasing.

$$F_B = \{\#D,\ \ \#B,\ \ H\} \tag{15}$$

where H will be named *"number of ones"*. Note that event i=I, always has H= K and that F_B is fictive in the sense that it is only helpful to the calculus of the *integer* events table (F_E). Note also that in Markov graphs, columns of events are composed of identical *"number of ones"* (H).

(ii) <u>*Integer* events table and the Markov graph:</u>

The *integer* events table (Eq. (16)) is defined as a binary table in which *binary* events (#B) are replaced by *integer* events (#E), i.e.

$$F_E = \{\#D,\ \ \#E,\ \ H\} \tag{16}$$

where #E is found by replacing each component (i.e. corresponding to each piece of equipment) of #B by the corresponding integer event $\in \{0,..,5\}$ using the procedure described below.

For the failure graph, construction proceeds from left to right and by decreasing #D. Based on the assumption that only one piece of equipment can breakdown at any one time (see above), a transition can only take place between two events for which the H of the event of origin is greater than the one for the summit event ($H_{origin} > H_{summit}$). The method consists of skimming through F_B from top to bottom and determining for each event of origin if one can connect it to each summit event meeting the above criterion ($H_{origin} > H_{summit}$). If the answer is yes, the *integer* event of the summit has to be determined with the help of the set of rules described below (R.1 – R.9). The *binary* and *integer* events can be denoted as a table V_B and V_E, respectively, equal to the second column #B and #E, respectively, in tables F_B and F_E, respectively. For example, $V_E(i,k)$ represents the *integer* event of the k^{th} piece of equipment of the i^{th} event. The rules needed to determine the *integer* event are as follow:

1. <u>Event $i = I = 2^k$</u> : Is the corresponding piece of equipment v in active or in passive redundancy? Apply rule R.1.

For k =1,.., K , if x_v=1, then $V_E(I,\ k)$=1 (i.e. active) if x_v=0, then $V_E(I,\ k)$=3 (i.e. passive)	R.1

2. <u>For each event $i \le I = 2^k$ ($V_E(i)$)</u>: <u>For each event</u> j of $V_B(j)$ (with k =1,..,K) meeting criterion $H_{origin} > H_{summit}$, <u>test if</u> the couple[6] <$V_E(i) \rightarrow V_B(j)$> meets following criteria: a piece of equipment cannot return from a break-

[5] Note that each piece of equipment is not necessarily defined by a structural or operational independent variable.

[6] Here, <$V_E(i,k) \rightarrow V_B(j,k)$> has integer event V_E for its origin and the binary event V_B for its summit.

down event to a operating event[7]; note that the number of transitions from operating events (1 or 5 in TABLE II) to breakdown events can not exceed 1. Apply rules (R.2a and R.2b).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, $\kappa = 1,..., K$, there is no transition $<0\rightarrow 1>$, there is no transition $<2\rightarrow 1>$, there is no transition $<4\rightarrow 1>$.	R.2a
For $<V_E(i, k)\rightarrow V_B(j, k)>$, $k = 1,...,K$, the number of transitions $<1\rightarrow 0>$ + the number of transitions $<5\rightarrow 0> \leq 1$.	R.2b

If rule R.2a or rule R.2b is violated, then there is no possibility of an edge between both events $<V_E(i)\rightarrow V_B(j)>$. Thus, one must go to next event j and start again. If both R.2a and R.2b are satisfied, then an edge can exist. In this case, begin with the calculation of the *integer* event of summit $V_B(j)$ of the couple $<V_E(i)\rightarrow V_B(j)>$ as follows:

<u>Calculation of</u> $V_E(j, k)$: <u>For each</u> $k = 1,...,K$ (i.e. for each piece of equipment) of the *integer* event of origin $V_E(i, k)$, <u>do</u> the following:

a) If the equipment is active ($V_E(i, k) = 0$ or 1), then the *binary* event of the summit $V_B(j, k)$ is not modified (rule R.3).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k) = 0$ or 1, then $V_E(j, k) = V_B(j, k)$.	R.3

b) If the equipment is passive ($V_E(i, k) =$ 2,3,4 or 5), then the *integer* event of the summit $V_B(j, k)$ depends on both the *integer* event of origin $V_E(i, k)$ and the emergency procedure. The latter is described as follows:

I. If equipment k is not engaged and is broken-down ($V_E(i, k)=2$), then it stays in the same event (rule R.4). Note that the transition $<2\rightarrow 1>$ has already been eliminated by rule (R.2a).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k) = 2$, then $V_E(j, k) = 2$.	R.4

II. If equipment k has been engaged and has broken-down ($V_E(i, k)=4$), then it stays in the same event (rule R.5). Note that the transition $<4\rightarrow 1>$ has already been eliminated by rule (R.2a).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k) = 4$, then $V_E(j, k) = 4$.	R.5

III. If equipment k has been engaged and has started ($V_E(i, k)=5$), then it can break down or stay in operation (rule R.6).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k) = 5$ and $V_B(j, k) = 0$, then $V_E(j, k) = 4$, if $V_E(i, k) = 5$ and $V_B(j, k) = 1$, then $V_E(j, k) = 5$.	R.6

IV. If equipment k is waiting and ready to operate ($V_E(i, k)=3$), then look at the emergency procedure. If k has been engaged, it can break down or start (rule R.7a). If it has not been engaged, it can break down or stay in readiness to operate (rule R.7b).

For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k)= 3$ and equipment j must be engaged, if $V_B(j, k) = 0$, then $V_E(j, k) =4$, if $V_B(j, k) = 1$, then $V_E(j, k) =5$.	R.7a
For $<V_E(i, k)\rightarrow V_B(j, k)>$, if $V_E(i, k)=3$ and equipment j has not been engaged, if $V_B(j, k) = 0$, then $V_E(j, k) =2$, if $V_B(j, k) = 1$, then $V_E(j, k) =3$.	R.7b

At this point, F_E and the failure Markov graph can be constructed. It still remains to calculate the weight (or transition rate) a_{ij} of each edge and resolve possible conflicts.

(iii) <u>Transition rate matrix (**A**):</u>

Assuming that the transition rates are independent of the event and based on the aforementioned assumptions, the contribution τ_k for each possible transition is summarized in TABLE III. The weight of each edge is found by multiplying the K transition rate contributions as follows:

$$a_{ij} = \prod_{k=1}^{K} \tau_k \qquad (17)$$

(iv) <u>Possible conflicts:</u>

In some cases, it is possible that the calculation of an *integer* event $V_E(j, k)$ can lead to different values through different edges (ways). This happens sometimes with events $V_E(j, k)=2$ and $V_E(j, k)=4$. In this case, event $V_E(j, k)=4$ supersedes the former (rule R.8). Note that it can easily be shown that this conflict does not affect the numerical results of the availability or reliability analysis at all, but only the Markov graph representation.

For $<V_E(i, k)\rightarrow V_E(j, k)>$, $k = 1,..,K$ if $V_E(j, k)= 2$ or 4 differs from another edge,	R.8

[7] Note that at this point, only the failure graph has been considered. The repair graph will be treated below.

then $V_E(j,\ k) = 4$	

At this point, F_E, and both the failure Markov graph and **A** are fully defined. What remains is to complete the Markov graph and the transition rate matrix **A** for repairs.

TABLE III. TRANSITION RATE τ_k FOR EACH TRANSITION.

Transition $<V_E(i,\ k) \rightarrow V_E(j,\ k)>$	Transition rate τ_k
$<0\rightarrow0>$	1
$<1\rightarrow0>$	λ_k
$<1\rightarrow1>$	1
$<2\rightarrow2>$	1
$<2\rightarrow4>$	1
$<3\rightarrow2>$	0
$<3\rightarrow3>$	1
$<3\rightarrow4>$	γ_k
$<3\rightarrow5>$	$1-\gamma_k$
$<4\rightarrow4>$	1
$<5\rightarrow4>$	λ_k
$<5\rightarrow5>$	1

4.3 Markov graph and transition rate matrix A (repairs):

Assuming that only one piece of equipment can be repaired at a time, the repair Markov graph can easily be built from the *binary* events table (F_B) as follows:

For each event ($V_B(i)$): For each event $V_B(j)$, the couple $<V_B(i) \rightarrow V_B(j)>$ has an edge if the criterion $H_{origin}<H_{summit}$ is met and if one and only one piece of equipment of event $V_B(i)$ is repaired (rule R.9).

For $<V_B(i,\ k)\rightarrow V_B(j,\ k)>$, $k = 1,..,K$ if $H(i) < H(j)$ and if the transition number $<0\rightarrow1> = 1$, then there is an edge.	R.9

The weight of edge a_{ij} is given by the repair rate (μ_k) of the corresponding repaired piece of equipment, i.e.

$$a_{ij} = \mu_k \tag{18}$$

At this point, the Markov graph and **A** are fully defined. The asymptotic probabilities ($P_i(\infty)$, $i=1,..,I$ with $I=2^K$) can easily be deduced through Eq. (14).

5. Exact Costs and Flows

Consider a system of order K and assume that a quasi-stationary approximation can be made and that the solution of Eq. (11) rapidly tends towards a stationary solution (asymptotic probabilities). Then, the exact costs and flows (materials, energy) are easily calculated by considering each event and weighting the cost or flow with the asymptotic probability $P_i(\infty)$ (Eqs (19) and (20)).

$$\dot{\Phi}_{exact} = \sum_{i=1}^{I=2^K} P_i(\infty)\dot{\Phi}_i \tag{19}$$

$$\Phi_{exact} = \left(\sum_{i=1}^{I=2^K} P_i(\infty)\dot{\Phi}_i \right)\Delta t \tag{20}$$

6. Application: Influence of Availability Analysis on Thermoeconomics

Consider the availability analysis of the superconfiguration depicted in *Fig. 1* appearing in Part I (Olsommer et al., 1999).

6.1 Problem definition

In addition to the aforementioned assumptions, the following assumptions are made:

- the RH, T1, T2, C1, C2 and DH are much more reliable than the F1, F2, F3, GT and AB (Procaccia and Aufort, 1995);
- the pumps (P) are already doubled or tripled and are, thus, perfectly reliable;
- the furnaces (F1, F2, F3) are always, when present, active pieces of equipment (i.e. they are much too expensive to be passive).

Thus, the availability depends only on the furnaces (F1, F2, F3), the gas turbine (GT) and the auxiliary boiler (AB).

The minimum availability limit on heat delivery to the district heating network (A_{DH}) has been fixed at 99.9 (%). Below this value, the objective function has been penalized (see TABLE V in Part I (Olsommer et al., 1999). The maximum operational load of the furnaces has been set at 110 % and at 100 % for the GT and AB. For sanitary and security reasons, the maximum storage period in the waste pit has been set to four days for each sequence, assuming that each furnace does not break down more than once per season (4 months; SAIOD, 1997).

For each sequence ($k = 1,\ldots,K$), the problem is defined by the structural vector $\mathbf{w}_k = (w_{F1}, w_{F2}, w_{F3}, w_{GT}, w_{AB})_k$ and the operational vector $\mathbf{x}_k = (x_{F1}, x_{F2}, x_{F3}, x_{GT}, x_{AB})_k$. Note that based on the acquisition date of each piece of equipment, the structure of the system can vary with the sequence k (flexible structure in time). Transition rate estimators (see TABLE IV) have been assumed identical over each sequence.

TABLE IV. TRANSITION RATE ESTIMATORS (Procaccia and Aufort, 1995; SAIOD, 1997).

Equip-ment	Operating failure $\hat{\lambda}$	Start-up failure $\hat{\gamma}$	Repair $\hat{\mu}$
F1,F2,F3	497e-6	100e-3	1/180
GT	1400e-6	15e-3	1/21
AB	220e-6	5e-3	1/82

The emergency procedures can easily be implemented and are as follow:

- **F1:** If F2 or F3 <u>or</u> F2 and F3 are broken-down, then run F1 up to its maximum load.
- **F2:** If F1 or F3 <u>or</u> F1 and F3 are broken-down, then run F2 up to its maximum load.
- **F3:** If F1 or F2 <u>or</u> F1 and F2 are broken-down, then run F3 up to its maximum load.
- **GT:** If F1, F2 and F3 are broken-down, then start the GT and run it up to its maximum load if it is waiting or let it operate at its chosen operating load if it is in operation.
- **AB:** If F1, F2 and F3 are broken-down and the GT is broken-down or has broken down when started, then start the AB and run it up to its maximum load if it is waiting or let it operate at its chosen operating load if it is in operation.

6.2 Numerical results

Figure 3 shows the results of the influence of availability on the overall costs (C_{tpn}) with respect to the number of furnaces in redundance. Although this particular example does not result from an optimization but a parametric sensitivity study, it illustrates quite well that without an availability analysis:

- the overall costs are far from their exact value;
- the optimal structure would not be the right one.

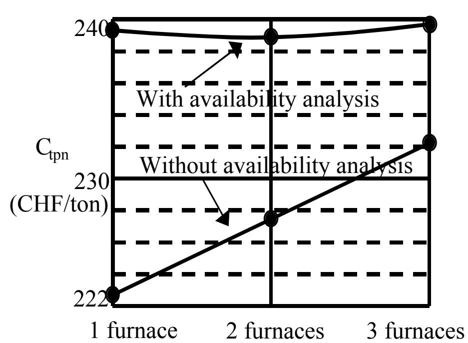


Figure 3. Influence of availability analysis and redundancy on the overall costs of a waste incineration power plant with cogeneration and a topping-cycle.

Figures 4 and *5* show detailed results of the availability analysis for the global optimum solution of the application presented in Part I (Olsommer et al., 1999). This solution is composed of only one furnace (F1), the gas turbine (GT) and the auxiliary boiler (AB). *Figures 4a,b* show the Markov graphs and their associated asymptotic probabilities for the optimal system of each sequence. Looking at Tables VIII, IX in Part I (Olsommer et al., 1999), it can easily be shown that the only event for which the heat demand cannot be fulfilled is the one when all the equipment is broken-down (event "1"). In both cases (see *Fig. 4a,b*), the DH availability constraint is never violated (*Fig. 4a*: $A_{DH} = 1 - 1.8e\text{-}5 \geq 0.999$; *Fig. 4b*: $A_{DH} = 1 - 4.1e\text{-}5 \geq 0.999$). Note that events "1" and "2" of *Fig. 4b* represent the unavailability of a system composed only of the F1 and the GT in active redundancy. In this case, the constraints would have been violated (*Fig. 4b*: $A_{DH} = 1 - (4.1e\text{-}5+2.3e\text{-}3) < 0.999$). This explains why the optimization chose the configuration with at least three steam sources.

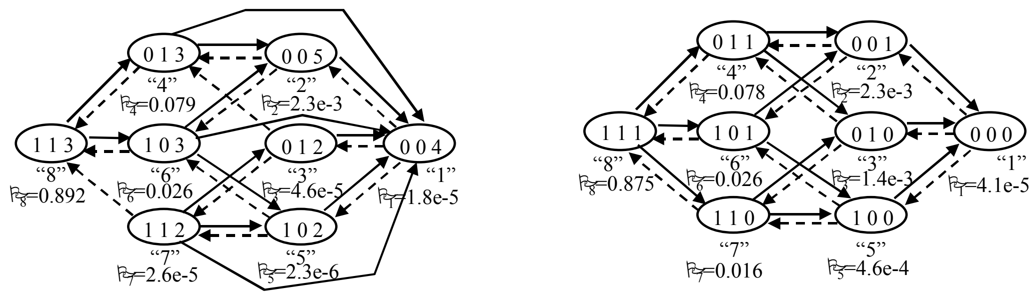
Figures 5a,b show a comparison of the optimal costs at the structural and operational levels with and without availability analysis[8]. At the structural level, the costs (C_{tpn}) are underestimated by more than 18 CHF/ton. At the operational level, the error is between 5 % and 10 %.

7. Conclusion

In the context of system integration, a comprehensive reliability and/or availability analysis leads to a more realistic evaluation of the performance, flows and costs of a given system by accounting for all possible intermediate events. The method developed and presented in this paper can be easily implemented on a computer and used in an optimization procedure. It permits one to treat systems with multiple events (6 in this case) with the number of mathematical operations identical to a procedure for a system with only two events. With realistic assumptions, this method can deal with complex systems with a flexible structure in time and composed of equipment placed in series and/or in parallel and in active and/or passive redundance.

Results show that the availability analysis has a significant influence on both structural and operational decisions and that neglecting such a comprehensive analysis can lead to important errors.

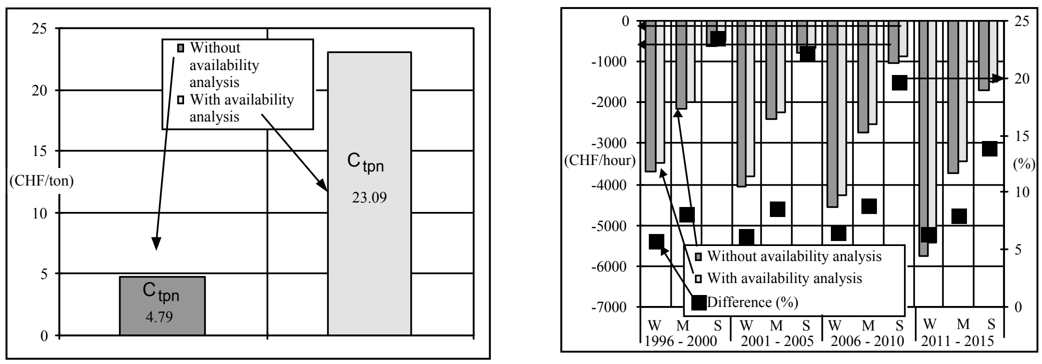
[8] Note that the results without availability analysis are based on the same optimal independent structural and operational variable sets (**w**, **x**) as those with availability analysis.



(a) F1 and GT active, AB in passive redundance (sequences 2,3,4,5,6,8,9,11,12).

(b) F1, GT and AB in active redundance (sequences 1,7,10).

Figure 4. Optimal solution: Markov graphs and asymptotic probabilities ($\Pi_i(\infty)$) for each of the K=12 sequences of the application (see Part I (Olsommer et al., 1999)).



(a) Structural level (L1): objective function (C_{tpn}).

(b) Operational level (L2): objective function ($\dot{C}_k$).

Figure 5. Optimal solution: Influence of availability analysis on the costs.

Nomenclature

A	availability	(-)
A	transition rate matrix	
a_{ij}	transition rate of matrix **A**	
AB	auxiliary boiler	
C	aero-condenser	
C	cost	
CHF	Swiss Franc	
DH	heat exchanger for district heating	
E	entity	
F	furnace plus steam generator	
F_B	*binary* events table	
F_E	*integer* events table	
GT	gas turbine	
H	*"number of ones"* in #B	
I,i	Number, index of event(s)	
i	index for origin	
j	index for summit	
K, k	number, index of sequence(s)	
K, k	number, index of equipment	
M	mid-season	
M	maintainability	(-)
N	maximum number of pieces of equipment in the superconfiguration	
P	probability(-)	
Q	random variable	
R	reliability	(-)
S	summer	
T	turbine	
t	time	(s)
V	summit, event	
W	winter	
w	structural (synthesis-design) independent variable set	
x	operational independent variable set	
#B	binary representation of a *binary* event	
#D	decimal number	
#E	integer representation of an *integer* event	
→	edge	

Greek Symbols

$\dot{\Phi}$, Φ	flux, quantity	
λ	operating failure rate	(1/s)
λ_s	stopped failure rate	(1/s)
γ	start-up failure rate	(-)
μ	repair rate	(1/s)
τ	transition rate	

Subscripts and Superscripts

B	binary
E	integer
tpn	total present net
^	estimator

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