The Non-Isothermal and Non-Stationary Flow of Dry and Condensate Gas in the Vicinity of Well

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Abstract

The paper presents a mathematical model, and its solution, of non-isothermal radial gas flow in porous media. The system of partial differential equations has been converted into two ordinary differential equations and solved numerically. In the case of the gas condensate system the properties of the liquid and gas phases were evaluated using the Peng-Robinson equation of state. The condensation process depends on the simultaneous decrease of pressure and temperature. These two parameters are intrinsically conjugated, and it is not possible to separate each from other. The equilibrium model was solved by the Quasi-Newton Successive Substitution (QNSS) and Dominant Eigenvalue Method (DEM). A new function describing the saturation of drop out condensate for the non-equilibrium process has been proposed.

Key words: non-isothermal flow, porous media, condensate gas

1. Introduction

In the classical engineering analysis of natural gas reservoirs' exploitation the changes of gas temperature have no significant influence on the production schedule.

However in some important practical applications the changes in gas temperature have to be taken into consideration (Siemek et al. 1987, Siemek and Nagy 1993, Siemek and Nagy 2001). Two main cases are important: evaluation of the skin effect of a well during transient testing and the injection/exploitation cycle of Underground Gas Storage (UGS) with high rate in the transient state.

The temperature effect in the near-well zone may be considered as an additional skin effect in the early-time analysis of transient gas well drawdown (or gas injection) tests with high constant rates at a wellhead. In the case of gas-condensate reservoirs the high temperature drop may cause the additional retrograde condensation of heavier components of hydrocarbon mixture.

The detailed discussions and models of non-isothermal flow of gas with axial symmetry were presented by Siemek (1972).

2. Non-isothermal flow model in porous media

The gas flow in porous media is uniquely described by the equations of continuity, motion and energy conservation together with the equation of state. They are given below.

In classical problems the Darcy Law describes the velocity vector of gas flow:

$$\vec{u} = -\frac{k}{\mu} \cdot \text{grad}(p) \qquad (1)$$

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Considering the turbulence effect inside the near-well zone, the velocity vector results from the equation (Firoozabadi and Katz 1979):

$$-\,\text{grad}\ p = \frac{\mu \cdot \bar{u}}{k_o} + F_b \cdot \bar{u} \cdot |\bar{u}| \qquad (2)$$

The equation of continuity for one phase fluid flow in porous media has the form:

$$\text{div}(\rho \cdot \vec{u}) + \phi \cdot \left(\frac{\partial \rho}{\partial t}\right) = 0 \qquad (3)$$

The energy conservation law (Burger et al. 1984):

$$\frac{\partial}{\partial t}(\rho H) + \text{div}(\rho H \bar{u}) = \text{div}(\lambda \cdot \text{grad}\ T) + \\ + \frac{\partial p}{\partial t} + \bar{u} \cdot \text{grad}\ p + \Psi \qquad (4)$$

The Peng-Robinson equation of state (EOS) (Peng, Robinson 1976):

$$p = \frac{RT}{v - b} - \frac{a}{v(v + b) + b(v - b)} \qquad (5)$$

Equations (1-5) represent the mathematical model for the solution of non-isothermal single-phase gas flow in the reservoir and its semianalytical solution for non-isothermal flow without vertical heat exchange has been proposed in the form of function series (Siemek 1972):

$$p^2 = p_k^2 + \eta \cdot p_1(x,t) + \eta^2 \cdot p_2(x,t) + \eta^3 \cdot p_3(x,t) + \dots \qquad (6)$$

$$T^2 = T_k^2 + \eta \cdot T_1(x,t) + \eta^2 \cdot T_2(x,t) + \eta^3 \cdot T_3(x,t) + \dots \qquad (7)$$

where coefficients for radial flow are:

$$p_1(r,t) = A_1 + \frac{2A_1}{\pi}\int_0^\infty \frac{\exp(-r\,u^2 t)}{u} \times \\ \frac{J_o(u)[u \cdot Y_1(uR_o) + B \cdot Y_o(uR_o)] - Y_o(u)[J_1(uR_o) + B \cdot J_o(uR_o)]}{[u \cdot J_1(uR_o) + B \cdot J_o(uR_o)]^2 + [u \cdot Y_1(uR_o) + B \cdot Y_o(uR_o)]^2} du \qquad (8)$$

and

$$T_1(r,t) = \frac{\phi c_p \eta_s}{(\rho c)_o} p_1(r,t) \qquad (9)$$

where

$$\eta_s = \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p \qquad (10)$$

The Bessel function $Y_0(u), Y_1(u), J_0(u), J_1(u)$ is evaluated using tables (Korn G. A and Korn T. M. 1968). The η is the so-called "small parameter".

The method proposed in this paper is different. Combining equations (4) and (2) (after neglecting a dissipation term in (4)) we obtain:

$$\text{div}(k\frac{\rho H}{\mu}\text{grad}p + \lambda\ \text{grad}\ T) \\ + \frac{\partial p}{\partial t} - \frac{k}{\mu}(\text{grad}\ p)^2 = \frac{\partial(\rho H)_o}{\partial t} \qquad (11)$$

where unit volume porous medium and porous rock enthalpy changes with time are expressed as:

$$\frac{\partial(\rho H)_o}{\partial t} = \phi\rho \cdot c_p\left(\frac{\partial T}{\partial t} - \mu_h\frac{\partial p}{\partial t}\right) \\ + (1-\phi)(\rho c)_r\frac{\partial T}{\partial t} \qquad (12)$$

The method of solution is based on the conversion of model equations into a dimensionless set of an ordinary coupled system using the Boltzmann transformation. Assuming that the reservoir is homogeneous with uniform thickness, infinite extends, using the Buckingham theorem and introducing dimensionless variables:

$$\bar{p} = \frac{p}{p_i}, \quad \bar{T} = \frac{T}{T_i}, \quad \xi = r\sqrt{\frac{\phi \cdot \mu}{k \cdot p_i \cdot t}} \qquad (13)$$

the system of model equations comes in following the form:

$$\frac{k \cdot \rho \cdot H \cdot p_i}{\lambda \cdot \mu \cdot T_i}\left\{ \begin{array}{l} \left(\frac{\partial \ln H}{\partial p} - \frac{p_i}{\rho H}\right)\left(\frac{d\bar{p}}{d\xi}\right)^2 + \frac{\partial \ln H}{\partial \bar{T}}\left(\frac{d\bar{p}}{d\zeta}\right)\left(\frac{d\bar{T}}{d\xi}\right) \\ + \frac{\xi}{2}\left[\begin{array}{l}\left(\frac{(\rho c)_r}{\rho H}T_i - \frac{\partial \ln p}{\partial \bar{T}}\right)\left(\frac{d\bar{T}}{d\xi}\right) \\ -\left(\frac{\phi\rho c_p\mu_h + 1}{\phi} \frac{p_i}{\rho H} + \frac{\partial \ln \rho}{\partial \bar{T}}\right)\left(\frac{d\bar{p}}{d\xi}\right)\end{array}\right] \end{array} \right\} \\ + \frac{\left(\frac{d\bar{T}}{d\xi}\right)}{\xi} + \left(\frac{d^2\bar{T}}{d\xi^2}\right) = 0 \qquad (14)$$

$$\frac{\left(\frac{d\bar{T}}{d\xi}\right)}{\xi} + \left(\frac{d^2\bar{T}}{d\xi^2}\right) + \frac{\partial \ln \rho/\mu}{\partial \bar{p}}\left(\frac{d\bar{p}}{d\xi}\right)^2 + \\ + \frac{\partial \ln \rho/\mu}{\partial \bar{T}}\left(\frac{d\bar{T}}{d\xi}\right)\left(\frac{d\bar{p}}{d\xi}\right) + \\ + \frac{\xi}{2}\left[\left(\frac{\partial \ln \rho}{\partial \bar{p}}\right)\left(\frac{d\bar{p}}{d\xi}\right) + \left(\frac{\partial \ln \rho}{\partial \bar{T}}\right)\left(\frac{d\bar{T}}{d\xi}\right)\right] = 0 \qquad (15)$$

$$\rho = \rho(p,T) \qquad (16)$$

To obtain a unique solution, the following initial and boundary conditions have been assumed:

$$\tau=0 \quad r>r_w, \quad p=p_i, \quad T=T_i, \quad \xi\ \infty, \qquad (17)$$

$$\tau>0 \quad r \to \infty, \quad p=p_i, \quad T=T_i\ \xi \to \infty, \qquad (18)$$

3. Single Phase Flow Near Well without Hydrocarbon Condensation

The thermodynamic parameters like heat capacity, the Joule-Thomson coefficient, viscosity, the compressibility coefficient and the heat conduction coefficient were assumed to be constant.

The term representing viscous dissipation in the energy balance equation was regarded as negligible.

The following data have been chosen for calculation:

Case 1: 100% of methane, $(\rho c)_r$=2.82 10^6 J/m^3 K, λ=1.73 W/m K, ϕ=0.2, μ_h= 0.12 K/bar, c_p = 3.1 kJ/kg K, p=23MPa, T=344.8 K, μ= 2 10^{-5} Pa s, k = 24.1 10^{-15} m^2, $m°(r_w)$ = 8165 kg/h.

Figures 1 and *2* show the pressure and temperature changes in time function for various reservoir radii in the vicinity of a well.

4. Single Phase Flow Near a Well with Hydrocarbon Condensation in Porous Media – Non-Equilibrium Condensation Model

To solve the mathematical model the properties of two phases were evaluated using the Peng-Robinson equation of state. For turbulent flow near a well a new function describing non-equilibrium condensation has been proposed. To estimate the percentage of liquid molar fractions really outcropped from gas during turbulent flow, the following model has been proposed (Siemek, Nagy 2001):

Let us assume that $\Delta\tau = \Delta r / u$ is the characteristic time needed for full condensation of $L(r,\tau)$ mole fractions of gas flowing through the element dv with width $\Delta r'$ in time $\Delta\tau' = \Delta r'/u$. For one mole of gas flowing through the dv' element the volume which liquefied is equal to (Siemek et al. 1978):

$$\frac{\Delta r'}{u'} \frac{u}{\Delta r} L'(r',\tau)$$

The correction term representing the non-equilibrium condensation is:

$$\beta_u = \begin{cases} \dfrac{\Delta r'}{u'} \dfrac{u}{\Delta r} & \text{for} \quad \dfrac{\Delta r'}{u'} < \dfrac{\Delta r}{u}, \\ 1 & \text{for} \quad \dfrac{\Delta r'}{u'} \geq \dfrac{\Delta r}{u} \end{cases} \qquad (19)$$

As a criterion of full equilibrium condensation, the reference velocity u is assumed to be at radius r'=10 m for τ=10 days since production starts. The choice of reference velocity depends on pressure and temperature gradients.

The assumed value u was chosen on the results of sensitivity analysis.

The equation which describes saturation of condensate (defined as a percent of liquid volume condensed in the free porous volume) has the form (Siemek, Nagy 2001):

$$S(r,\tau) = S(r,\tau_o) \qquad (20)$$
$$+ \int_{\tau_o}^{\tau} \beta_u \frac{\partial}{\partial p}\left[\frac{M_g}{M_L}\frac{Z_L}{Z_g}L'(r',\tau)\right]\frac{Dp}{D\tau}d\tau$$

where the first derivative is evaluated numerically and $\dfrac{Dp}{D\tau}$ represents substantial derivative of pressure and term $\dfrac{\partial}{\partial p}\left[\dfrac{M_g}{M_L}\dfrac{Z_L}{Z_g}L'(r',\tau)\right]$ describes the change of condensate volume in the element dv during equilibrium condensation as a pressure function. The integral in equation (20) is calculated numerically.

The calculation was carried out for reference filtration velocity value at radius r'=10 m for time τ=10 days. The following data have been chosen: Case 2 (methane 95%, ethane 2%, propane 1%, n-hexane 2%), μ_h=0.4 K/bar, p=10MPa, T=300 K.

The compositions of equilibrium phases were computed using equations:

$$L\cdot x_i + V\cdot y_i - z_i = 0 \quad (i=1,...,n) \qquad (21)$$

where liquid L and vapor V mole fractions were calculated after solution:

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} y_i = 0 \qquad (22)$$

Equilibrium constants:

$$K_i = \frac{y_i}{x_i} \qquad (23)$$

or using fugacity coefficients

$$K_i = \frac{\phi_i^L}{\phi_i^V} \qquad (24)$$

$$\phi_i^L = \frac{f_i^L}{x_i p}, \quad \phi_i^V = \frac{f_i^V}{y_i p} \quad (i=1,...,n) \qquad (25)$$

were used for solution.

The fugacity coefficients were calculated using the Peng-Robinson equation of state (Peng and Robinson 1976). The equilibrium model was solved by the Quasi-Newton Successive Substitution (QNSS) and Dominant Eigenvalue Method (DEM) (Nagy 1991).

Figures 3 and *4* show solutions of set of equations (9-11) for the near-well zone. It can be stated that the temperature drop is large in the well vicinity (up to 8.6 K/m).

Knowing timewise variation of temperature and pressure at the reservoir radius and performing calculations of Constant Composition Expansion (CCE) or Constant Volume Depletion (CVD) simulation for vapor-liquid phases' distributions, it is possible to determine the rate of condensate saturation in the near-well zone using equation (10).

The pressure-temperature path at the reservoir radius 0.5 m inside the phase envelope and constant liquid fraction (1%) in the system (Case 2) is shown in *Figure 5* and the change of main component (methane, hexane) volumes of liquid condensate in the system at the reservoir radius 0.5 m versus descending depletion pressure is presented in *Figure 6*.

Increase of condensate saturation with time for a selected reservoir radius for the proposed non-equilibrium condensation model is shown in *Figure 7*.

The results of simulation (pressure and temperature) in the vicinity of a well for Case 2 are presented in TABLE I. The composition of a liquid phase is presented in TABLE II for reservoir radius r=0.5 m (Case 2).
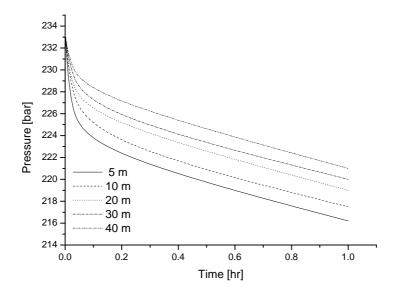

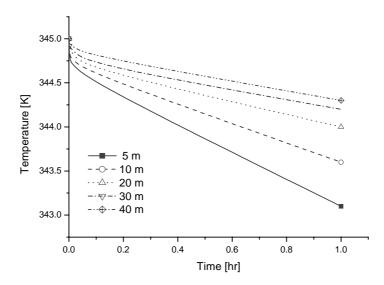*Figure 1. The change of pressure in the near-well zone (Case 1)*

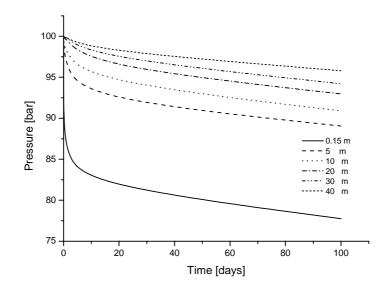*Figure 2. The change of temperature in the near-well zone (Case 1)*

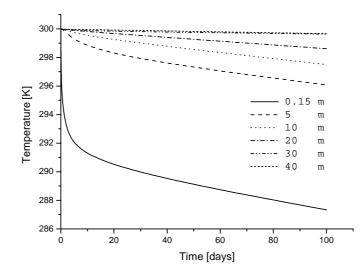*Figure 3. The change of pressure in the near-well zone (Case 2)*

Figure 4. The change of temperature in the near-well zone (Case 2)

TABLE I. THE RESULTS OF SIMULATION (PRESSURE AND TEMPERATURE) IN VICINITY OF
WELL FOR CASE 2.

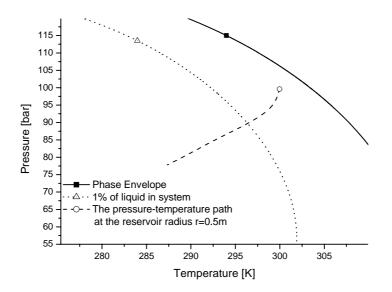| Time [days]	Radius [m]							
	0.15		5		10		20	
	T[K]	p[bar]	T[K]	p[bar]	T[K]	p[bar]	T[K]	p[bar]
$6.0 \cdot 10^{-5}$	299.97	99.6	299.97	99.9	299.97	99.9	299.97	99.9
$3.5 \cdot 10^{-3}$	299.38	94.5	299.97	99.1	299.97	99.5	299.97	99.9
$4.2 \cdot 10^{-2}$	296.79	90.0	299.97	99.0	299.97	99.4	299.97	99.9
1.0	292.80	85.2	299.97	95.8	299.97	97.8	299.97	99.9
10.0	290.45	81.8	298.24	92.3	299.38	94.5	299.79	97.5
100.0	287.33	77.7	296.08	89.1	297.50	90.9	298.61	94.2

Figure 5. The pressure-temperature path at the reservoir radius 0.5 m inside the phase envelope and constant liquid fraction (1%) in the system (Case 2)
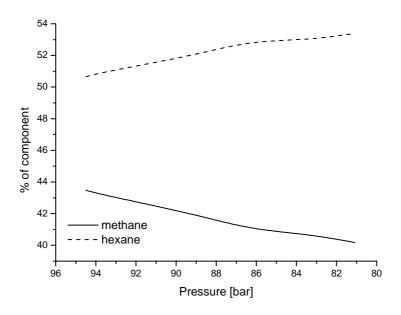

Figure 6. The percentage change of main components (methane, hexane) of liquid condensate in the system at the reservoir radius 0.5 m versus descending depletion pressure (Case 2)

TABLE II. LIQUID COMPOSITION (MOLE FRACTIONS) FOR RESERVOIR RADIUS R=0.5 M AS THE FUNCTION OF TIME: 1.5 HR, 1.5 DAY, 10 DAYS, 40 DAYS, 1 YEAR, 2 YEARS (CASE 2).

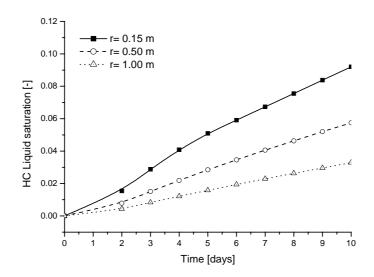| Time	1.5 hr	1.5 day	10 days	40 days	1 year	2 years
T[K]	299.33	299.10	296.46	296.00	291.58	289.90
P[MPa]	9.45	9.34	8.90	8.64	8.325	8.11
Methane	0.4348	0.4311	0.4195	0.4102	0.4068	0.4017
Ethane	0.0279	0.0279	0.0282	0.0282	0.0291	0.0294
Propane	0.0307	0.0309	0.0320	0.0324	0.0342	0.0350
N-hexane	0.5066	0.5101	0.5203	0.5292	0.5299	0.5340

Figure 7 Condensate saturation increases versus time for a selected reservoir radius for proposed non-equilibrium condensation model (Case 2).

5. Conclusions

The temperature effect in the near-well zone may be considered as an additional skin-effect in the early-time analysis of transient gas well drawdown (or gas injection) tests with high constant rates at a wellhead. In the case of thin, non-isolated upper- and underburden reservoirs, equation (4) must be modified considering heat losses in a vertical direction.

The Joule-Thomson effect is a main factor which influences temperature changes of flowing fluid.

In the case of condensate reservoirs the high temperature drop may cause the additional retrograde condensation of heavier components of a hydrocarbon mixture. The effect of retrograde condensation has been regarded in case of an increase of condensate saturation up to the critical value. If the value of liquid saturation is greater than critical, the flow must be considered as the two-phase flow.

Nomenclature

a, b	coefficients in EOS
A1	coefficient in equation (8)
c	compressibility
c_p	specific heat at constant pressure
F_b	turbulence coefficient
h	reservoir thickness
H	specific enthalpy
k	permeability coefficient
L,L'	mole fraction of liquid phase
M_g	molecular mass of gas phase
M_L	molecular mass of condensate
p	pressure
$p_1(x,t)$	function defined by equation (8)
r	radial distance
r_w	well radius
R	universal gas constant
S	condensate saturation of porous media
T	temperature
v	specific volume
$\bar{u}$	velocity vector
Z_g	compressibility factor of gas phase
Z_L	compressibility factor of condensate
$Y_0(u), Y_1(u), J_0(u), J_1(u)$	Bessel function in equation (8)

Greek Letters

β_u	correction factor for non-equilibrium condensation
ϕ	porosity coefficient
η_s	parameter defined by equation (10)
λ	heat conductivity coefficient
μ	viscosity coefficient
μ_h	Joule-Thomson coefficient
ρ	density
$(\rho c)_o$	specific heat of volume unit
$(\rho H)_o$	enthalpy of porous medium volume unit
τ	time
ξ	dimensionless parameter
ψ	dissipation energy function
ν	volume

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