

# **Quantal Diffusion in Heavy-Ion Collisions**

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### Abstract

We investigate the quasi-fission reactions in the basis on the Stochastic Mean-Filed (SMF) approach that provides a microscopic and quantal description of the multi-nucleon exchange mechanism. In deep-inelastic heavy-ion collisions, colliding ions stick and move together for a long time. During this contact time many nucleons exchange between projectile and target nuclei, and the composite system then separate in two main primary fragments without forming a compound nucleus. Quasi-fission is a non-compound nuclear process in deep-inelastic heavy-ion collisions and the multi-nucleon exchange mechanism in the quasi-fission reactions is important. We calculate the quantal transport coefficients for heavy-ion collisions at bombarding energies below their fusion barriers and determine the primary fragment mass distributions. Quantal calculations are compared with the experimental data.

Keywords: Multinucleon transfer, quasi-fission reaction, stochastic mean-field theory.

## **1. INTRODUCTION AND THEORY**

The different reaction mechanisms at low energy heavy-ion reactions can be classified according to the initial orbital angular momentum of the system. In deep-inelastic heavy-ion collisions at low energy near the Coulomb barrier, the initial di-nuclear system resaparetes in two fragments without fusing a compound nucleus formation. This process is called quasi-fission that is characterized by a long contact time and large number of nucleon transfer. The contact time in quasi-fission process is much shorter than the fusion-fission time. During this long contact time, a large number of mass and charge transfer take place from the heavy partner to the light one.

Transport coefficients (nucleon drift and diffusion coefficients) for multinucleon exchange in quasifission process are related to the macroscopic variables such as mass distributions and cross sections of target-like and projectile-like fragments. For a given impact parameter b or the initial orbital angular momentum  $\ell$ , mass distribution of the primary fragment is given by the average of the Gaussian functions as

$$P(A) = \frac{\eta}{\sum_{\ell} (2\ell+1)} \sum_{\ell} (2\ell+1) \frac{1}{2\pi \sigma_{AA}(\ell)} \exp\left\{-\frac{1}{2} \left[\left(\frac{A-A(\ell)}{\sigma_{AA}(\ell)}\right)^2\right]\right\}$$
(1)

On the other hand, cross section for production of nuclei with neutron and proton number is calculated by integrating the probability distribution of the macroscopic variables over the range of the impact parameters corresponding to the experimental data

$$\sigma(\mathbf{N}, \mathbf{Z}) = \int_{b_1}^{b_2} 2\pi b \frac{1}{2\pi \sigma_{NN}(b)\sigma_{ZZ}(b)\sqrt{1-\rho_b^2}}$$

$$\exp\left\{-\frac{1}{2\left(1-\rho_b^2\right)} \left[\left(\frac{Z-Z_b}{\sigma_{ZZ}(b)}\right)^2 - 2\rho_b\left(\frac{Z-Z_b}{\sigma_{ZZ}(b)}\right)\left(\frac{N-N_b}{\sigma_{NN}(b)}\right) + \left(\frac{N-N_b}{\sigma_{NN}(b)}\right)^2\right]\right\} db$$

$$(2)$$

where  $\rho_b = \sigma_{NZ}^2(b) / \sigma_{ZZ}(b) \sigma_{NN}(b)$ . The coupled differential equations for the covariances  $\sigma_{\alpha\alpha}(b)$  are given by Eqs. (17)–(19) in Ref. [6], which involve the neutron  $D_{NN}(t)$ , proton  $D_{ZZ}(t)$  and neutron/proton  $D_{NZ}(t)$  diffusion coefficients and the derivatives of drift coefficients. The calculation of diffusion coefficients plays a central role to obtain macroscopic variables.

In recent years the TDHF approach has been extensively used to study quasi-fission reactions [1-3]. In TDHF studies it is possible to calculate only the mean values of macroscopic variables. In the SMF, the standard description is extended beyond the mean-field by incorporating quantal and thermal fluctuations at the initial state [4]. The SMF approach provides a microscopic framework for diffusion mechanism. An expression of the diffusion coefficient is derived in a quantal framework of SMF [6]. The relevant macroscopic variables evolve according to the generalized Langevin description characterized by a set of quantal transport coefficients that are determined by the occupied TDHF wave functions [5-7]. They include quantal effects, memory effects, the full collision geometry, and involve no adjustable parameters.

In this work, we study multinucleon transfer in the off-central deep-inelastic collisions of heavy-ions in the basis of a stochastic mean-field approach [5-7]. We calculate fragment mass distribution and cross sections in a fully quantal framework including memory effects and compare our results with experimental data.

#### 2. RESULTS AND DISCUSSION

We firstly calculate the dispersion of fragment mass distribution in  ${}^{40}Ca + {}^{40}Ca$  reaction at the center energy  $E_{cm}$ =115 MeV in TDHF and SMF approaches. There is data available for  ${}^{40}Ca + {}^{40}Ca$  at a slightly higher bombarding energy  $E_{cm}$ =128 MeV and over a different angular range [8]. Dispersion of the fragment mass distributions over the measured angular range is found between  $\sigma_{AA}$ =2.8 and 4.6. While the standard mean-field approximation (TDHF) predicts much smaller values, SMF improves an agreement with experiment.

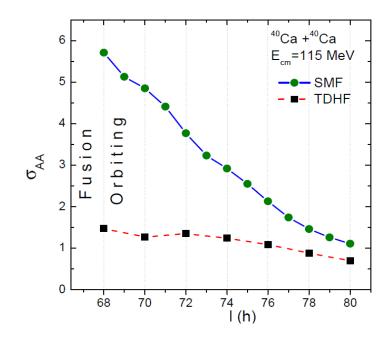


Fig 1. Asymptotic values of  $\sigma_{AA}$  as a function of the orbital angular momentum in SMF and TDHF approaches.

We then calculate neutron and proton diffusion coefficients and covariances in  ${}^{58}\text{Ni}+{}^{60}\text{Ni}$  reaction at the center energy  $\text{E}_{cm}$ =135.6 MeV. Fig. 2 and Fig 3 show nucleon diffusion coefficients and covariances for the impact parameter b=5.36 fm, respectively. The fluctuations in the diffusion coefficients are appeared due to the effect of the shell structure and the effect of the Pauli blocking of the occupied single particle states. The contact time in the collision is found about 800 fm/c that is larger than the average memory time. As expected, neutron exchange becomes larger than proton.

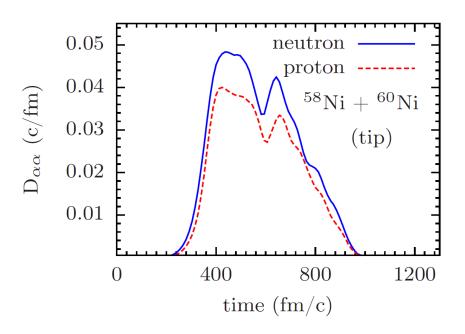


Fig. 2. Neutron and proton diffusion coefficients and covariances in  ${}^{58}\text{Ni}+{}^{60}\text{Ni}$  reaction at the center energy  $E_{cm}$ =135.6 MeV energy and the impact parameter b=5.36 fm

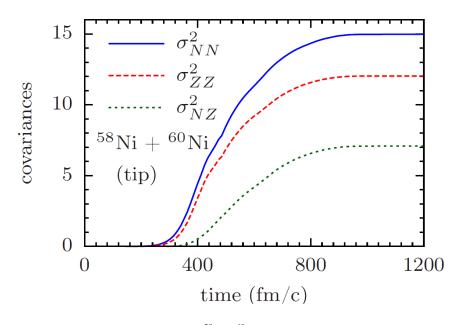


Fig. 3. Covariances in the collision of  ${}^{58}Ni+{}^{60}Ni$  reaction in the tip geometry at the center energy  $E_{cm}=135.6$  MeV and the impact parameter b=5.36 fm

We also calculate the mass number dispersion per unit nucleon  $\sigma_{MR} = \sigma_{AA}$  /A for the side plus tip configurations as a function of the impact parameters that is presented in Fig. 4. The result of SMF calculations is compared with data. There are four data points that are reported in [9] including experimental error bars. The SMF calculations with the side plus tip configurations provide a good fit to the data.

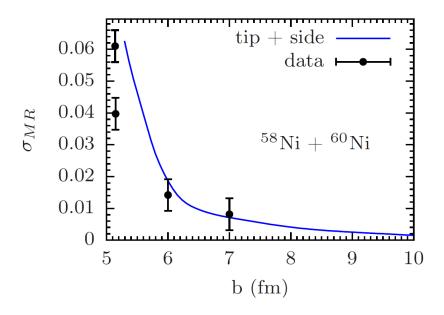


Fig. 4. Dispersion per unit nucleon  $\sigma_{MR} = \sigma_{AA} / A$  for the side plus tip configurations as a function of the impact.

We finally applied the quantal diffusion approach in the collision of  ${}^{48}Ca + {}^{238}U$  at the central energy  $E_{cm} = 193$  MeV and calculated primary fragment yield that is given in Fig. 5. Results of SMF [6] provide a good description of the measured fragment mass distribution [10].

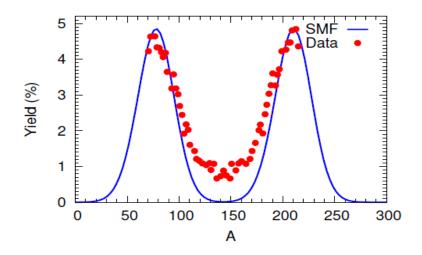


Fig 5. Primary fragment yield in  ${}^{48}Ca+{}^{238}U$  collisions at  $E_{cm} = 193$  MeV and comparison with data.

Fig. 6 shows the cross sections for the primary fragments as a function of mass number. Experimental data is not available to compare with the calculated cross sections.

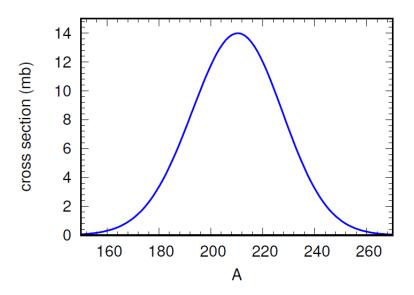


Fig 6. Cross section in  ${}^{48}Ca + {}^{238}U$  collisions at  $E_{cm} = 193$  MeV as a function of mass number.

We present a quantal diffusion description for the multinucleon exchange mechanism for off-central and symmetric/asymmetric heavy-ion systems in the di-nuclear regime. Quantal diffusion description includes the full geometry of the collision dynamics and does not involve any adjustable parameter other than the Skyrme parameters of the TDHF. We find that the quantal diffusion description deduced from the SMF approach provide a good description for the fragment mass distribution observed in <sup>58</sup>Ni+<sup>60</sup>Ni and <sup>48</sup>Ca+<sup>238</sup>U collisions.

#### 3. ACKNOWLEDGMENTS

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