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Excited Doubly Heavy Baryons Spectrum in QCD

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Abstract

The spectral parameters of $J=\frac{1}{2}$ doubly heavy baryons (containing b or c quarks) are studied in the framework of the QCD sum rules. Our predictions on the mass and residues of doubly heavy baryons are compared with the existing ones in the literature.

Key Words: Doubly Heavy Baryons, QCD Sum Rule

1. INTRODUCTION

Quark model is very powerful in studying the hadron physics. The hadrons with single heavy quark predicted by quark model has already been observed in the experiments. The first observation of hadrons with doubly heavy quarks (Ξ_{cc}^{++}) had been reported by SELEX Collaboration [1] in the decay $\Xi_c^{++} \to \Lambda_c^+ K^- \pi^+$ with mass $(3519 \pm 1)\ MeV$ and LHCb Collaboration [2] observed the state of Ξ_{cc}^{++} in the $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ decay with mass $(3624.40 \pm 0.72 \pm 0.14)\ MeV$.

This observation opened a new window in theoretical investigations. The mass of doubly heavy baryons are studied within various models like lattice QCD [3–5], Hamiltonian Model [6], hyper-central method [7], and QCD sum rules [8–12]. Mainly in these works, only the mass of ground state baryons is calculated. Moreover, in the framework of QCD sum rules [13,14], the first radial excitations of heavy-light mesons [15], octet [16] and decuplet baryons [17] are studied.

2. QCD SUM RULES FOR DOUBLY HEAVY BARYONS

In the present work, we calculate the spectroscopic parameters of excited doubly heavy baryons within the QCD sum rules method. In calculations, we used the most general form of the interpolating currents of the doubly heavy baryons with $J=\frac{1}{2}$. In order to calculate the spectroscopic parameters of doubly heavy baryons in QCD sum rules, we start by considering the two-point correlation function

$$\begin{aligned}\Pi(q^2) &= i \int d^4x\, e^{iqx} \langle 0 \,|\, T\,\{\,\eta(x)\,\bar{\eta}\,(0)\}|0\,\rangle \\ &= \Pi_1\,(q^2)\, \not{q} + \Pi_2(q^2),\end{aligned} \tag{1}$$

where η is the current for the relevant baryon. In our calculations, we used the following general form of the interpolating currents for the $J=\frac{1}{2}$ doubly heavy baryons.

$$\eta^S = \frac{1}{\sqrt{2}}\epsilon^{abc}\,\{\left(Q^{a^T} C q^b\right)\gamma_5\, Q'^c + \left(Q'^a C q^b\right)\gamma_5\, Q^c + \beta\,[\,\left(Q^{a^T} C\gamma_5 q^b\,\right)Q'^c + \left(Q'^c\, C\gamma_5 q^b\right)Q^c\,]\,\},$$

$$\eta^A = \frac{1}{\sqrt{6}} \epsilon^{abc} \{ 2\left(Q^{a^T} C Q'^b\right)\gamma_5 \, q^c + \left(Q^a C q^b\right)\gamma_5 \, Q'^c + 2\,\beta \left(Q^{a^T} C\gamma_5 Q'^b\right) q^c + \beta\left(Q^{a^T} C\gamma_5 q^b\right) Q'^c - (Q'^{a^T}) C\gamma_5 q^b) Q^c \}, \quad (2)$$

where a, b, c are color indices, Q and q are the heavy and light quarks, C is the charge conjugation operator, β is an arbitrary parameter, and T is the transposition. Note that η^S (η^A) is the symmetric and anti-symmetric with respect to the exchange of two heavy quarks. Obviously, in η^S both heavy quarks may be identical or different while in η^A, the two heavy quarks must be different.

The main idea of the QCD sum rules is that the correlation function is calculated in two different kinematical regions, namely in terms of hadrons and in terms of quark and gluons in the deep Euclidean region. Then matching the coefficients of the relevant Lorentz structures and using quark-hadron duality ansatz we get the sum rules for quantity under study.

The phenomenological part of the correlation function is obtained by saturating it with the baryons carrying the same quantum numbers as the interpolating current. Then ground and first radially excited states are isolated and we get,

$$\Pi^{S,A} = \frac{\lambda^2 (\not{q} + m)}{-q^2 + m^2} + \frac{\lambda_1^2 (\not{q} + m_1)}{m_1^2 - q^2} + \ldots \quad (3)$$

where $\lambda(\lambda_1)$, $m, (m_1)$ are the residue and the mass of ground (radially excited) state.

The expressions for the correlation function containing orbitally excited state is obtained from Eq. (3) with the replacements;

$$\begin{aligned} &\lambda_1 \to \tilde{\lambda} \\ &m_1 \to -\tilde{m} \end{aligned} \quad (4)$$

The correlation function in terms of the quarks and gluons in the Euclidean region is calculated with the help of the operator product expansion (OPE). Using the Wick theorem and performing contraction of the heavy and light quark field for symmetric and anti-symmetric currents cases, one can obtain the expression of the correlation function. These expressions are obtained in [10] therefore we do not present them here.

In order to obtain the mass sum rules for the mass and residues the coefficients in $\not{q}$ or I structures, QCD and phenomenological sides are matched. As a final step the Borel transformation over $-q^2$ is performed to suppress the higher states continuum contributions.

Using the quark-hadron duality ansatz, we get the following sum rules:

$$\begin{aligned} \Pi_1^{S(A)(B)}(M^2) &= \int_{(m_Q + m_Q')^2}^{s_0} \rho_1^{S(A)} \, e^{-\frac{s}{M^2}} \, ds \\ \Pi_2^{S(A)(B)}(M^2) &= \int_{(m_Q + m_Q')^2}^{s_0} \rho_2^{S(A)} \, e^{-\frac{s}{M^2}} \, ds \end{aligned} \quad (5)$$

$$\begin{aligned} \lambda^2 \, e^{-m^2/M^2} + \lambda_1^2 \, e^{-m_1^2/M^2} &= \int_{(m_Q + m_{Q'})^2}^{s_0} \rho_1^{S(A)} \, e^{-\frac{s}{M^2}} \, ds \\ m\,\lambda^2 \, e^{-m^2/M^2} + m_1 \, \lambda_1^2 \, e^{-m_1^2/M^2} &= \int_{(m_Q + m_Q')^2}^{s_0} \rho_2^{S(A)} \, e^{-\frac{s}{M^2}} \, ds \end{aligned} \quad (6)$$

where Π_1 and Π_2 are invariant functions in the coefficient of the Lorentz structures $\not{p}$ and I correspondingly. The expressions of $\rho_i^{S(A)}$ can be found in [10,11]. Eq. (6) contains four unknowns, namely masses of ground and excited states and their residues. To obtain the values of four unknowns, we need two extra equations which can be obtained by taking derivatives with respect to $\frac{-1}{M^2}$ from the two equations in Eq. (6). Solving these four equations for mass and residue of radially excited state we find

$$m_1^2 = \frac{\Pi_2'^{(B)} - m\,\Pi_1'^{(B)}}{\Pi_2^{(B)} - m\,\Pi_1^{(B)}}$$
$$\lambda_1^2 = \frac{1}{m_1^2 - m^2}\left(\Pi_1'^{(B)} - m^2\,\Pi_1^{(B)}\right)e^{m'^2/M^2} \tag{7}$$

For orbitally excited state residue, we get

$$\widetilde{\lambda^2} = \frac{1}{m + m_1}\left(\Pi_1^{(B)}\,m^2 - \Pi_2^{(B)}\right)e^{m'^2/M^2} \tag{8}$$

The mass of the orbitally excited state is determined from Eq. (7) by replacing $m_1 \rightarrow -\widetilde{m}$. To determine the mass and residue of the excited state doubly heavy baryons, the mass and residue of the ground states are taken as input parameters.

3. NUMERICAL RESULTS

Now, we present our numerical analysis on mass and residues of doubly heavy baryons. Masses have been used in the $\overline{MS}$ scheme for the c and b quarks,

$$\overline{m}_c(\overline{m}_c) = (1.28 \pm 0.03)\ GeV$$
$$\overline{m}_b(\overline{m}_b) = (4.16 \pm 0.03) GeV\,. \tag{9}$$

The values of other input parameters are:

$$m_s(2\,GeV) = (95^{+9}_{-3})\,MeV$$
$$m_0^2 = (0.8 \pm 0.2)\,(GeV^2)$$
$$\langle\,\bar{q}q\,\rangle(1\,GeV) = -(0.246^{+0.028}_{-0.019})^3 GeV^3$$
$$\langle\,\bar{s}s\,\rangle\,(1\,GeV) = (0.8 \pm 0.2\,)\,\langle\bar{q}q\rangle\,(1\,GeV) \tag{10}$$

The sum rules for the spectroscopic parameters of doubly heavy baryons contains three auxiliary parameters: the Borel mass M^2, continuum threshold s_0 and parameter β. Obviously, mass should be independent of these parameters. Therefore, the working regions of these parameters should be determined in such a way that the mass value exhibits independence of them. The continuum threshold s_0 usually is related to the energy of the first excited state. In our calculations we used $s_0 = \left(m_{ground} + 0.8\right)^2$. For ground state mass, we used the results of [10,11]. These values of s_0 includes only ground and first excited states. The lower bound of M^2 is determined by requiring the convergence of the OPE, the upper bound of M^2 is decided from the condition of pole dominance contribution. Under these conditions for s_0 and M^2, we get following working regions that are presented in Table (1).

Our final results on mass and residues for the considered doubly heavy baryons are presented in Tables (2), (3) and (4). For completeness, in this table, we also present the predictions for the mass of doubly heavy baryons obtained from other theoretical approaches.

Table 1. The working domains of continuum threshold and Borel parameters M^2.

Baryon	$\sqrt{s_0}(GeV)$	$M^2(GeV^2)$
Ξ_{cc}	(4.3 ± 0.1)	$4 \div 7$
Ξ_{bb}	(11.1 ± 0.1)	$10 \div 16$
Ξ_{cb}	(7.7 ± 0.1)	$7 \div 11$
Ξ'_{cb}	(7.9 ± 0.1)	$7 \div 11$
Ω_{cc}	(4.5 ± 0.1)	$4 \div 7$
Ω_{bb}	(11.2 ± 0.1)	$10 \div 16$
Ω_{cb}	(7.9 ± 0.1)	$7 \div 11$
Ω'_{cb}	$(\mathbf{7.9 \pm 0.1})$	$\mathbf{7 \div 11}$

Table 2. The mass of radial and orbital excitations of Ξ_{QQ} doubly heavy baryons in GeV unit.

	Our Results	Ref. [18]	Ref. [6]	Ref. [19]	Ref. [20]	Ref. [21]	Ref. [22]	Ref. [23]	Ref. [24]
$\Xi_{cc}(\mathbf{1/2})^+$	4.05 ± 0.05	3.920	4.079	4.029	4.183	3.976	3.910	4.030	–
$\Xi_{cc}(\mathbf{1/2})^-$	4.05 ± 0.05	3.861	3.947	3.910	–	3.880	3.838	4.073	3.77 ± 0.18
$\Xi_{bb}(\mathbf{1/2})^+$	10.30 ± 0.10	10.609	10.571	10.576	10.751	10.482	10.441	10.551	
$\Xi_{bb}(\mathbf{1/2})^-$	10.30 ± 0.10	10.551	10.476	10.493	–	10.406	10.368	10.691	10.38 ± 0.15
$\Xi_{bc}(\mathbf{1/2})^+$	7.05 ± 0.05	7.263	–	–	7.495	–	–	7.353	–
$\Xi_{bc}(\mathbf{1/2})^-$	7.05 ± 0.05	7.156	–	–	–	–	–	7.390	–
$\Xi'_{bc}(\mathbf{1/2})^+$	7.00 ± 0.04	–	–	–	–	–	–	–	–
$\Xi'_{bc}(\mathbf{1/2})^-$	$\mathbf{7.00 \pm 0.04}$	–	–	–	–	–	–	–	–

Table 3. The mass of radial and orbital excitations of Ω_{QQ} doubly heavy baryons in GeV unit.

	Our Results	Ref. [7]	Ref. [6]	Ref. [19]	Ref. [21]	Ref. [20]	Ref. [22]	Ref. [24]
$\mathbf{\Omega}_{cc}(\mathbf{1/2})^+$	4.10 ± 0.10	4.041	4.079	4.227	4.180	4.112	4.268	–
$\mathbf{\Omega}_{cc}(\mathbf{1/2})^-$	4.10 ± 0.10	3.989	3.947	4.086	4.046	–	4.002	3.91 ± 0.14
$\mathbf{\Omega}_{bb}(\mathbf{1/2})^+$	10.40 ± 0.10	10.736	10.571	10.707	10.693	10.604	10.830	–
$\mathbf{\Omega}_{bb}(\mathbf{1/2})^-$	10.40 ± 0.10	10.646	10.476	10.607	10.616	–	–	10.38 ± 0.15
$\mathbf{\Omega}_{bc}(\mathbf{1/2})^+$	7.10 ± 0.10	7.480	–	–	–	–	7.559	–
$\mathbf{\Omega}_{bc}(\mathbf{1/2})^-$	7.10 ± 0.10	7.386	–	–	–	–	–	–
$\mathbf{\Omega}'_{bc}(\mathbf{1/2})^-$	7.01 ± 0.05	–	–	–	–	–	–	–
$\mathbf{\Omega}'_{bc}(\mathbf{1/2})^+$	$\mathbf{7.01 \pm 0.05}$	–	–	–	–	–	–	–

4. CONCLUSION

In conclusion, we estimate the masses and residues of the first radial and orbital excitations of doubly heavy baryons. We compare our results with the predictions of other theoretical approaches. Our predictions on the spectroscopic parameters of excited states of doubly heavy baryons will be useful looking for these states in future experiments.

Table 4. The residue of radial and orbital excitations of doubly heavy baryons.

	λ_1 (GeV^3)	$\tilde{\lambda}$ (GeV^3)	$\tilde{\lambda}$ (GeV^3) Ref. [24]
Ξ_{cc}	0.138 ± 0.025	0.101 ± 0.002	0.159 ± 0.037
Ξ_{bb}	0.596 ± 0.056	0.568 ± 0.037	0.365 ± 0.089
Ξ_{cb}	0.346 ± 0.085	0.257 ± 0.011	–
Ξ'_{cb}	0.199 ± 0.073	0.183 ± 0.083	–
Ω_{cc}	0.174 ± 0.042	0.117 ± 0.005	0.192 ± 0.041
Ω_{bb}	0.708 ± 0.015	0.618 ± 0.042	0.444 ± 0.101
Ω_{cb}	0.257 ± 0.022	0.219 ± 0.006	–
Ω'_{cb}	$\mathbf{0.141 \pm 0.033}$	$\mathbf{0.157 \pm 0.071}$	–

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