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Tutulu-tutulu sınır şartları altında eksenel yönde tabakalı fonksiyonel derecelendirilmiş ince kirişlerin termal burkulma analizi

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Thermal Buckling Analysis of Axially Layered Functionally Graded Thin Beams under Clamped-Clamped Boundary Conditions

Araştırma Makalesi / Research Article

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ABSTRACT

In the present article, the critical buckling temperature of axially layered functionally graded thin beams for the first mode was studied under clamped-clamped boundary conditions. The beams were made to be three layers using functionally graded materials with ceramic and metal systems in the axial direction. Analyses were performed using finite element and Taguchi methods. The beam configurations were designed based on Taguchi L_9 orthogonal array in order to detect the maximum critical buckling temperature and were analyzed using finite element software ANSYS. Analysis of signal-to-noise ratio was utilized to determine the layers with optimum levels and the influence of ceramic and metal materials in each layer. Analysis of Variance at the 95 % confidence level was employed in order to select the most significant layers and their percent contribution on response characteristic. The optimum result of the critical buckling temperature was predicted based on the 95 % confidence intervals of confirmation analysis and population.

Keywords: Thermal buckling, functionally graded materials, finite element analysis, beam.

Tutulu-Tutulu Sınır Şartları Altında Eksenel Yönde Tabakalı Fonksiyonel Derecelendirilmiş İnce Kirişlerin Termal Burkulma Analizi

ÖZ

Sunulan makalede, birinci mod için eksenel yönde tabakalı olarak fonksiyonel derecelendirilmiş ince kirişlerin kritik burkulma sıcaklığı tutulu-tutulu sınır şartları altında çalışılmıştır. Kirişler eksenel yönde seramik ve metal sistemli fonksiyonel derecelendirilmiş malzemeler kullanılarak üç tabakalı olarak yapılmıştır. Analizler sonlu elemanlar ve Taguchi metotları kullanılarak gerçekleştirilmiştir. Maksimum kritik burkulma sıcaklığı tespit etmek için kiriş konfigürasyonları Taguchi L_9 ortogonal diziye bağlı tasarlanmıştır. Sinyal gürültü oran analizi her tabakadaki seramik ve metal malzemelerin etkisini ve optimum seviyeli tabakaları değerlendirmek için kullanıldı. % 95 güven seviyesinde Varyans analizi yanıtlar üzerinde en etkili tabakalar ve onların yüzde katkılarını seçmek için uygulanmıştır. Kritik burkulma sıcaklığının optimum sonucu doğrulama analizinin ve popülasyonun % 95 güven aralığına dayanılarak tahmin edilmiştir.

Anahtar Kelimeler: Termal burkulma, fonksiyonel derecelendirilmiş malzemeler, sonlu elemanlar analizi, kiriş.

1. INTRODUCTION

Temperature can be the important factor for buckling analyses of the beams for mechanical engineering. The high increase in the temperature may affect the buckling behavior directly. This situation can explain the mechanical and thermal properties of the materials. The materials with temperature resistant and high strength may be required in order to resolve this situation. In the literature, there are many studies about thermal buckling and post-buckling of the beams and plates made of functionally graded materials. Fu et al. [1] analyzed the thermal buckling behavior of functionally graded beams having longitudinal crack and compare results obtained from classical Euler beam theory and Timoshenko beam theory. In addition, they used analytical and numerical solutions based on thermal load. Kiani and Eslami [2] carried out the buckling behavior of functionally graded beams subjected to different types of thermal loading under different boundary conditions and they used Euler–Bernoulli beam theory for the derivation of equations. Sun et al. [3] evaluated the thermal buckling and post-buckling of Timoshenko beams designed using functionally graded materials on nonlinear elastic foundation. Majumdar and Das [4] presented a study including the thermal buckling load of functionally graded beams with clamped-clamped boundary conditions and they used linear and nonlinear thermal gradient in thickness direction for analyses. Eslami et al. [5] investigated the thermal buckling behavior of

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functionally graded beams under different boundary conditions. Li et al. [6] studied the thermal post-buckling behavior of functionally graded Timoshenko beams and they used transversely non-uniform temperature rise for analysis. Wattanasakulpong et al. [7] presented an improved third-order shear deformation theory in order to analyze the thermal buckling and vibration behaviors of the functionally graded beams and they used different boundary conditions. Ghannadpour et al. [8] investigated the buckling behavior of rectangular plates designed using functionally graded materials with ceramic and metal materials under different thermal loadings and they used different boundary conditions. Javaheri and Eslami [9] studied the thermal buckling characteristics of functionally graded rectangular plates based on various thermal loading. It can be seen from mentioned literature that there are many studies with thermal buckling analysis functionally graded beams and plates. In this study, the critical buckling temperature of axially layered functionally graded thin beams for the first mode was evaluated under clamped-clamped boundary conditions using finite element and Taguchi methods.

2. MATERIAL and METHOD

Analysis of critical buckling temperature of the axially layered functionally graded thin beams was conducted based on Aluminum (metal) and Zirconia (ceramic) materials. The mechanical and thermal properties of the both materials are listed in Table 1.

Table 1. Mechanical and thermal properties of Aluminum and Zirconia [10]

Materials	Young's Modulus (GPa)	Poisson's Ratio (-)	Thermal Expansion Coefficient ($10^{-6}/^0C$)
Aluminum	70	0.3	23.0
Zirconia	151	0.3	10.0

Each layer of the axially layered functionally graded thin beams was modelled using various percent volume fractions of Aluminum and Zirconia materials. The mechanical and thermal properties of the layers were achieved based on a simple rule of mixture of composite materials. Poisson's ratios for both materials were constant and were used to be 0.3 as shown in Table 1. The effective material properties P_f of each layer of axially layered functionally graded thin beams, such as Young's modulus, and thermal expansion coefficient, may be solved as shown in Equation 1 [11].

$$P_{ef} = \sum_{j=1} P_j V_{f_j} \tag{1}$$

According to Equation 1, P_j and V_{fj} are utilized to be the material properties and volume fraction for the constituent material j respectively and then the sum of the volume fractions for all the constituent materials can be solved to be 1, with regard to Equation 2 [11].

$$\sum_{j=1} V_{f_j} = 1 \tag{2}$$

After the effective material properties of the layers were calculated based on Equation 1 and 2, the beam configurations were made using Taguchi's L9 orthogonal array. This array has three levels for each control factor. Thus the beams configurations with different levels and various layers is made. Ceramic volume fractions in layers were carried out based on 7 % increase. The layer and their levels were given in Table 2.

Table 2. Layers and their levels

Layer	Symbol	Level		
		Level 1	Level 2	Level 3
Layer 1	L1	7 % Aluminum 93 % Zirconia	14 % Aluminum 86 % Zirconia	21 % Aluminum 79 % Zirconia
Layer 2	L2	28 % Aluminum 72 % Zirconia	35 % Aluminum 65 % Zirconia	42 % Aluminum 58 % Zirconia
Layer 3	L3	49 % Aluminum 51 % Zirconia	56 % Aluminum 44 % Zirconia	63 % Aluminum 37 % Zirconia

The optimum response characteristics of the beam configurations for maximum temperature were carried out using "higher is better" quality characteristic as given in Equation 3 [12].

$$(S/N)_{HB} \text{ for } \Delta T = -10.\log\left(n^{-1}\sum_{i=1}^{n}(y_i^2)^{-1}\right) \tag{3}$$

In Equation 3, n is taken to be the number of numerical analyses in a trial and y_i also considered to be ith data studied. The statistical analyses were carried out using Minitab 15 software [15].

3. FINITE ELEMENT SOLUTION

Numerical analyses were performed using finite element software ANSYS Mechanical APDL [14]. Structure analysis was used based on temperature. The axially layered functionally graded thin beams were modelled according to BEAM189 element type. This element is described to be quadratic 3-node beam element for three dimension and also it is related to Timoshenko beam theory [13]. In modelling, each layer was performed according to 70 mesh and so the mesh operations of the beam configurations were carried out based on 210 mesh totally. In the finite element analysis, rectangular beams were used. Each beam has clamped-clamped boundary conditions. The length, cross section base, and cross section height of the beams are 210, 8, and 2 in mm, respectively. In additions, beam configurations are made using three layers with ceramic and metal systems. The axially layered FG thin beam type is presented in Figure 1.

Figure 1. Axially layered functionally graded thin beam with clamped ends

4. RESULTS AND DISCUSSIONS

Analysis The beam configurations were created according to Taguchi's L9 orthogonal array and numerical results of the thermal buckling temperatures were calculated. In order to study the critical buckling temperature of the beams, numerical data analyzed were convert to S/N ratio values based on "higher is better" characteristic. The numerical temperatures for first mode and S/N ratio results were given in Table 3.

Table 3. Critical buckling temperatures and S/N ratio data

Trial No.	Control Factors			Results	
	Layer 1	Layer 2	Layer 3	Temperature ΔT (°C)	S/N Ratio η (dB)
1	$L1_1$	$L2_1$	$L3_1$	15.5848	23.8540
2	$L1_1$	$L2_2$	$L3_2$	16.2064	24.1937
3	$L1_1$	$L2_3$	$L3_3$	16.8684	24.5415
4	$L1_2$	$L2_1$	$L3_2$	15.9467	24.0534
5	$L1_2$	$L2_2$	$L3_3$	16.5943	24.3992
6	$L1_2$	$L2_3$	$L3_1$	16.4856	24.3421
7	$L1_3$	$L2_1$	$L3_3$	16.3557	24.2734
8	$L1_3$	$L2_2$	$L3_1$	16.2352	24.2092
9	$L1_3$	$L2_3$	$L3_2$	16.8892	24.5522
Overall Mean $(\bar{\bar{T}})$				16.3518	

In order to see the effect of the temperature based on first mode on the axially layered FG thin beams, the numerical results in Table 3 were presented in Figure 2 visually. According to Figure 2, the maximum affected layers were monitored on middle layers

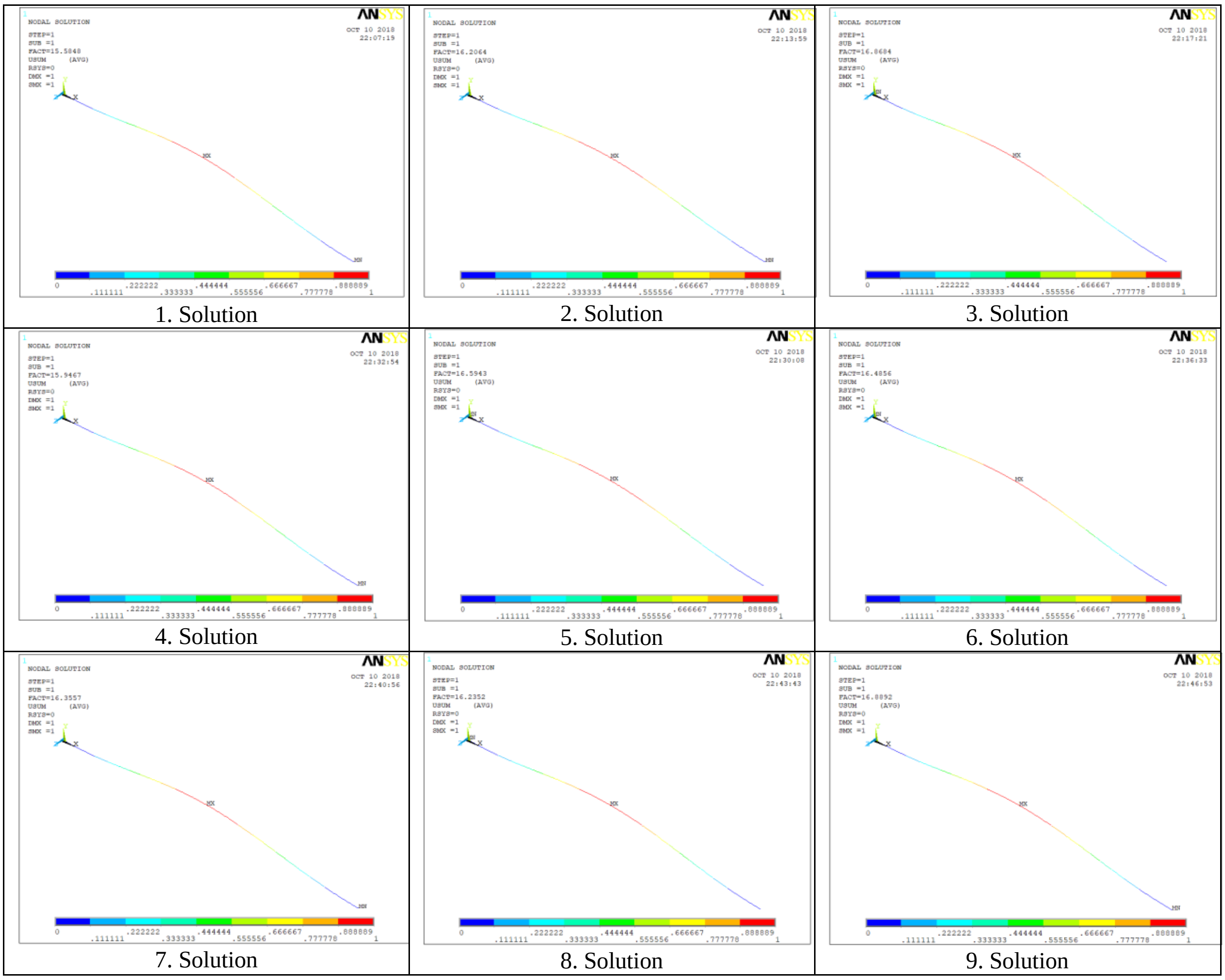
4.1 Identification of Layers with Optimum Levels and Their Effects

In order to observe the influence of layers with different levels on the thermal critical buckling response, finite element analyses were conducted based on Taguchi's L9 orthogonal array. The average results of critical buckling temperatures for each layer at each level based on finite element data and S/N data were calculated using Minitab 15 software. The results solved are given in Table 4.

Table 4. Response table for S/N ratio and raw values

Level	S/N ratio in dB			Means in ^{0}C		
	L1	L2	L3	L1	L2	L3
1	24.20	24.06	24.14	16.22	15.96	16.1
2	24.26	24.27	24.27	16.34	16.35	16.35
3	24.34	24.48	24.40	16.49	16.75	16.61
Delta	0.15	0.42	0.27	0.27	0.79	0.5
Rank	3	1	2	3	1	2

According to Table 4, the optimum levels for the maximum thermal buckling temperature of the axially layered FG thin beams were obtained using third levels of layers. The increase of the percent ceramic volume fraction in layer causes the increase of the critical buckling temperature of axially layered functionally graded thin beams according to first mode.



1. Solution, 2. Solution, 3. Solution, 4. Solution, 5. Solution, 6. Solution, 7. Solution, 8. Solution, 9. Solution

Figure 2. First mode shapes of the axially layered FG thin beams with clamped ends

4.2 Analysis of Variance for Critical Buckling Temperature

Analysis of variance (ANOVA) was assessed in order to investigate the significance of the control parameter such as L1, L2, and L3 towards critical buckling temperature of axially layered FG thin beams. Analyses was carried out using finite element results at 95 % confidence level and ANOVA results obtained were demonstrated in Table 5. From this table, it is clear that layers affect the mean and then layers are significant parameters on response due to $P < 0.05$. In addition, L1 with 7.93 %, L2 with 65.18 %, and L3 with 26.88 % contribution ratios are the most affected parameters, respectively. In analysis, percent error data was found as 0.01 %.

Table 5. ANOVA results

Source	DF	Seq SS	Adj SS	Adj Ms	F	P	% Effect
L1	2	0.11262	0.11262	0.05631	1342.27	0.001	7.93
L2	2	0.92531	0.92531	0.46266	11028.47	0.000	65.18
L3	2	0.38151	0.38151	0.19076	4547.12	0.000	26.88
Error	2	0.00008	0.00008	0.00004			0.01
Total	8	1.41953					100
S = 0.00647697, R-Sq = 99.99%, R-Sq(adj) = 99.98%							

4.3 Estimation of Optimum Results

The optimal result for maximum critical buckling temperature is predicted using significant layers based on P < 0.05 and then layers with optimum levels such as $L1_3$, $L2_3$, and $L3_3$ were used. Estimated mean of the critical buckling temperature can be expressed using Equation 4 [12].

$$\mu_{\Delta T} = \overline{L1}_3 + \overline{L2}_3 + \overline{L3}_3 - 2\overline{T} \quad (4)$$

In Equation 4, $\overline{T}$ = 16.3518 specifies overall mean of the critical buckling temperature based on Taguchi's L9 orthogonal array and it is taken from Table 3. Also, $\overline{L1}_3$ = 16.49, $\overline{L2}_3$ = 16.75, and $\overline{L3}_3$ = 16.61 denotes the average results of critical buckling temperature for finite element raw data at third level of L1, L2, and L3 respectively and these values are taken from Table 4. According to Equation 4, $\mu_{\Delta T}$ is calculated to be 17.1464 ºC and then the 95 % confidence intervals of confirmation analyses (CI_{CA}) and population (CI_{POP}) are solved by using Equation 5 [12] and Equation 6 [12], respectively.

$$CI_{CA} = \left(F_{\alpha;1;n_2} V_{error} \left[\frac{1}{n_{eff}} + \frac{1}{R} \right] \right)^{1/2} \quad (5)$$

$$CI_{POP} = \left(\frac{F_{\alpha;1;n_2} V_{error}}{n_{eff}} \right)^{1/2} \quad (6)$$

$$n_{eff} = \frac{N}{(1 + T_{DOF})} \quad (7)$$

where, α = 0.05 denotes risk and n_2 = 2 typifies the error data for degree of freedom in Table 5. Error data for variance is typified as V_{error} and is used as 0.00004 from Table 5. R = 1 is sample size of confirmation analyses. $F_{\alpha;1;n_2}$ is presented as $F_{0.05;1;2}$ and is determined to be 18.5 [12] according to F ratio results for the 95 % CI (α=0.05). Sum of number of the results is typified as N and is take to be 9 from Table 3. Sum of degree of freedom for significant control parameters is presented as T_{DOF} and is calculated to be 6. n_{eff} is solved to be 1.286 so CI_{CA} and CI_{POP} are found to be ± 0.0363 and ± 0.0240 respectively. Thus, the estimated confidence interval based on confirmation analyses [12] is:

$$Mean\ \mu_{\Delta T} - CI_{CA} < \mu_{\Delta T} < CI_{CA} + Mean\ \mu_{\Delta T}$$

The population for the 95 % confidence interval [12] is:

$$Mean\ \mu_{\Delta T} - CI_{POP} < \mu_{\Delta T} < CI_{POP} + Mean\ \mu_{\Delta T}$$

Finite element and predicted results for optimal critical buckling temperature according to confidence intervals were listed in Table 6. In addition, optimum result based on finite element software ANSYS was given in Figure 3 visually.

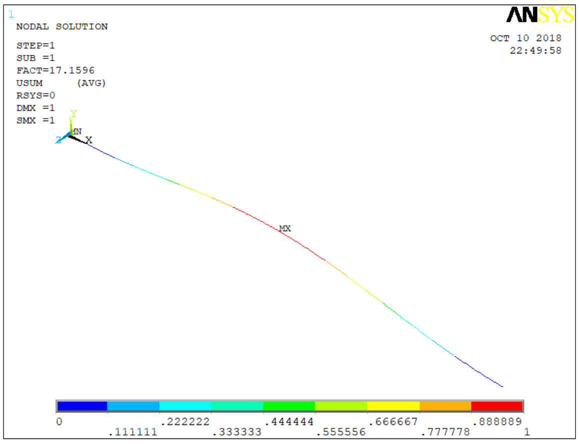


Figure 3. The optimum numerical result

Table 6. Finite element and predicted results

Optimal Layers	Finite Element Result (ºC)	Predicted Result (ºC)	Estimated Confidence Intervals for 95% Confidence Level
$L1_3$-$L2_3$-$L3_3$	17.1596	17.1464	17.1101< $\mu_{\Delta T}$ <17.1827 for CI_{CA}
			17.1224 < $\mu_{\Delta T}$ < 17.1704for CI_{POP}

5. CONCLUSIONS

In this study, the critical buckling temperature of axially layered functionally graded thin beams for first mode was investigated according to clamped-clamped boundary conditions. The beams were made from the functionally graded materials containing ceramic and metal materials in the axial direction. Analyses were carried out based on finite element and Taguchi methods. The beam configurations were determined based on Taguchi L_9 orthogonal array in order to select the maximum critical buckling temperature and numerical analysis was performed using finite element software ANSYS. Analysis of signal-to-noise ratio was utilized to study the layers having the optimum levels and the effects of ceramic and metal materials in each layer. Analysis of Variance for the 95 % confidence level was carried out in order to determine the most significant layers and their percent contribution on responses. This study concludes that the increase of the percent ceramic volume fraction in layer provides the increase of the critical buckling temperature of axially layered functionally graded thin beams based on first mode and thus optimum result for maximum critical buckling temperature was obtained using layers having the third levels. The layers were carried out to be control parameters with significant effects since P data is smaller than 0.05 value according to ANOVA at 95 % confidence level. The maximum affected layers were monitored on middle layers. In addition, effect ratios of Layer 1, Layer 2, and Layer 3 on responses were found to be 7.93 %, 65.18 %, and 26.88

%, respectively. The optimum critical buckling temperatures based on finite element and predicted results were solved to be 17.1596 and 17.1464 respectively. In addition, 95 % confidence intervals of confirmation analysis and population were determined to be 17.1101< $\mu_{\Delta T}$ <17.1827 for CI_{CA} and 17.1224 < $\mu_{\Delta T}$ < 17.1704for CI_{POP} respectively.

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