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An Exergy-based MILP algorithm for Heat Pumps Integration in industrial processes

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Abstract

Industrial heat pumps are efficient thermodynamic systems able to recover low grade heat and deliver it at higher temperatures (up to 120°C for the current available solutions). They are identified as a very efficient way to reduce primary energy consumption in processes, especially in food & drink or pulp & paper industries. Nevertheless, the optimal integration of multiple heat pumps in a large process with numerous heat fluxes is challenging. The present paper aims at describing an algorithm that was developed for this purpose, based on the GCC (Grand Composite Curve) of Pinch Analysis and on Exergy Theory. The temperature scale of the GCC is divided in areas defined by the Main Pinch Point and Potential local Pinch Points. Then, every potential heat pump is evaluated, absorbing heat in any area for delivering in an upper one. The corresponding heat load and COP are calculated. Exergy cost of remaining cold utilities is calculated with a Carnot based-efficiency, exergy cost of hot utilities according to their nature and temperature. The global exergy cost is used as criteria. Thanks to its formulation, the algorithm may suggest heat pumps solutions in non-obvious areas. The algorithm is tested on a literature case and shows equivalent or better exergy costs in a satisfying calculation time

Keywords: *Energy integration; exergy; heat pumps; MILP optimization.*

1. Introduction

As energy gets more and more expensive, many countries are setting policies to reduce energy consumption in housing, transport and industry. Designing Heat Exchangers Network (HEN) for heat recovery in industrial systems is a major issue. A preliminary step is proposed: the algorithm described hereafter aims at proposing sets of various number of Heat Pumps (HP). This automatic preselection of relevant utilities will help design the fittest and most efficient HEN including both heat exchangers and utilities.

Designing relevant heat pumps inside a large process and identifying the heat streams to be associated with via these heat pumps is a challenge which still has to be tackled. For domestic heating, Schiffmann and Favrat [1] created a 1D-compressor model and used a multi-objective optimizer to trade off efficiency and wide range of pressures. In industry, as thermal needs and process temperature ranges are more diversified, the number of possibilities is soaring. Becker [2] and Murr [3] also used a multi-objective optimizer to calculate the best temperature levels of refrigerant. Based on Energy Integration, the latter adapts existing heat pumps models to the process thermal needs by using Grand Composite Curve (GCC). Even if optimization of heat pumps' existing models operating conditions is quite common in literature, few references relate to the design of one or more heat pumps for a large process. Testing many temperature levels for evaporator and condenser among a wide range of temperatures implies combinatorial analysis and the use of sequential optimization techniques leads to skyrocketing time calculation.

A new mathematical formulation for heat pumps design, based on Exergy and Energy Integration, has been developed and is presented in this paper. Physical models of heat pumps are not necessary. This method allows the user to skip the heat pumps pre-design step which consists in using existing models of HP or create new ones to provide data to the HEN designing tool. Only GCC data are needed. Results consist in providing the characteristics of optimal HP, namely evaporation and condensation temperature levels, electrical compressor power and transferred heat load. The main advantage of this formulation is to combine short calculation time and accurate results. For instance, one run rarely outstands 10 seconds for a set of 5 HP's. As this algorithm is aimed at being included in a metaheuristic one, it may run hundreds of time. Such short calculation time guarantees the feasibility of the metaheuristic algorithm. The final aim is to run many HEN design steps with a large number of different heat pumps combinations in a reasonable time. That is why heat pumps preselection needs to be an automatic step before HEN design.

2. Mathematical formulation of pre-selection problem

2.1. Thermodynamics and Energy Integration

Even though GCC seems uncorrelated from real processes, it remains very useful to seek HP location. The first common rule is that HP must cross the Main Pinch Point (MPP) to be efficient (Fig. 1, c), the two remaining cases lead to an increase of Cold Minimum Energy Requirement (MER_{Cold}) (Fig. 1,b) or a decrease of MER by degrading electricity in the heat pump compressor (Fig. 1,a).

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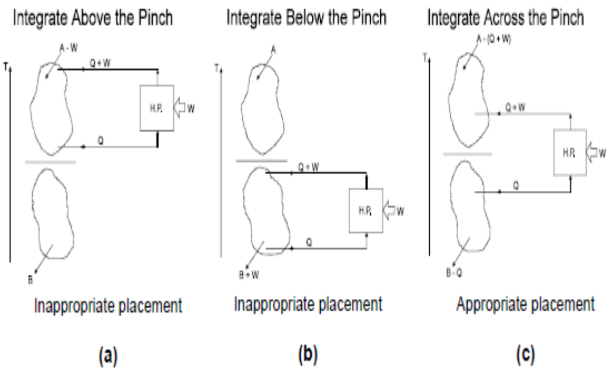

Figure 1. Placement of Heat Pumps ([4]).

In Fig. 2, a full GCC is presented and used to introduce the concept of Potential Pinch Point (PPP) in addition to the MPP. PPP corresponds to local or relative heat load minima on the GCC. They also delimitate self-sufficient pockets (areas in green). A HP which crosses not only the MPP but also a PPP can have an appropriate placement in some cases.

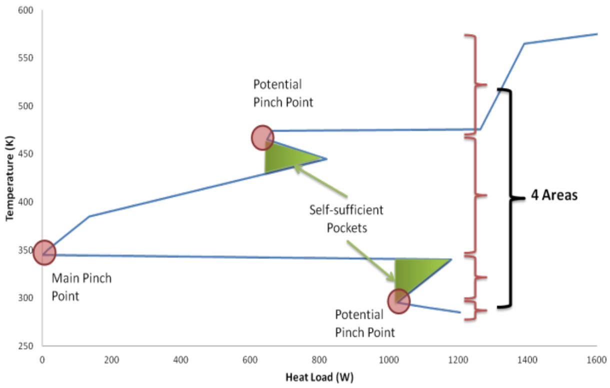

Figure 2. Potential Pinch Points and Self-sufficient pockets on GCC.

Figure 3 illustrates the case of a too large heat load transfer from a self-sufficient pocket to a temperature level higher than MPP's one. As the bottom of the GCC crosses the ordinate axis, the whole GCC has to be translated to the right to avoid negative values (Fig. 4).

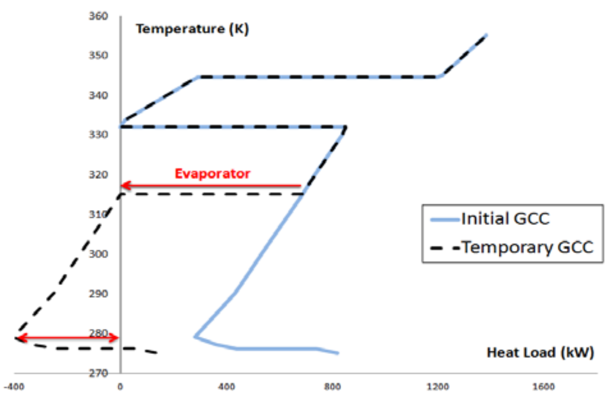

Figure 3. Impact of Self-sufficient Pockets on Heat Load Transfer (1).

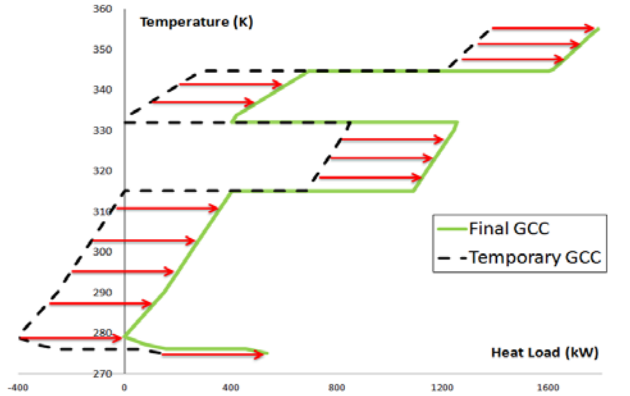

Figure 4. Impact of Self-sufficient Pockets on Heat Load Transfer (2).

Consequently, the maximum negative value is added to the MER. To remove heat load inside a self sufficient pocket, HP has to be coupled with another HP crossing at least the PPP involved. The heat load transfered from the self-sufficient pocket is thus larger. Self-sufficient pockets over the MPP cannot be exploited: a HP with evaporator temperature equal or higher than MPP temperature is equivalent to heating, but in a more exergy-expensive way.

Defining these rules generates forbidden configurations, which lead to one or more PPP becoming the new MPP. A PPP could reach the ordinate axe like the MPP, but do not cross it. These constraints are taken into account in the problem formulation described hereafter, thus reducing the number of possible HP's to be tested. As a consequence, the algorithm performances are improved.

2.2. Data pre-processing

Let the temperature scale be divided into temperature intervals bounded by PPP. The example in Fig. 2 gives 4 areas. Temperatures and Heat Loads will be identified by two subscripts; the first stands for the area, the second for the position inside the area. For instance: $T_{y,j}$ or $Q_{z,k}$. For the y area, S_y stands for the size of this area i.e. the number of temperature levels. The condition z > y means that evaporation and condensation levels are separated by at least one PPP.

To calculate HP efficiency, Carnot's COP and exergy efficiency are required. On one hand, Exergy efficiency is the ratio between Carnot's COP and the real COP. For most HP, exergy efficiency between 0.55 and 0.60 is considered as feasible. So, in the algorithm, exergy efficiency $RdtEx_{y,j,z,k}$ will be considered constant and alike for all HP's, and is taken into account with this formula:

$$\begin{aligned}&\forall (y,z) \in [1, Area]^2, \forall j \in [1, S_y], \forall k \in [1, S_z] \\ &RdtEx_{y,j,z,k} = RDTEX\end{aligned} \tag{1}$$

On the other hand, Carnot's COP is given by the Carnot formula: $COP_{Carnot} = \frac{T_h}{T_h - T_c}$ with T_h the temperature at the condenser and T_c at the evaporator. When reading the GCC temperatures, these temperatures values are corrected with heat exchanger pinches. In our case, the pinches of all

evaporators (resp. all condensers) are fixed, equal, and set as a simulation parameter, EvaP (resp. CondP).
Heat loads and temperature levels at evaporator and condenser depend on the studied process. In this model, every possible HP will be tested. The only constraints are that the evaporation level cannot be in the highest temperature interval (over Main Pinch) and its corresponding condensation level in a temperature interval above the evaporator's.
To calculate the heat load exchanged between the HP and the process, a variable $F_{y,j,z,k}$ is used. It corresponds to the ratio between the heat load extracted by the HP and the heat load available on GCC at this temperature level. According to previous definition, the HP with the variable $F_{y,j,z,k}$ extracts heat at $T_{y,j}$, delivers heat at $T_{z,k}$ and its COP is defined by :

$$COP_{y,j,z,k} = RDTEX * \frac{T_{z,k} + CondP}{(T_{z,k} + CondP) - (T_{y,j} - EvaP)} \tag{2}$$

2.3. Constraints & Objective functions

For all GCC points, heat load extracted by all HP at $T_{y,j}$ is defined by:

$$Q_{prel\ y,j} = \sum_{z=y+1}^{Area} \sum_{k=1}^{S_z} (F_{y,j,z,k} * Q_{y,j}) \tag{3}$$

Likewise, the total heat load delivered at $T_{z,k}$ is defined by:

$$Q_{app\ z,k} = \sum_{y=1}^{z-1} \sum_{j=1}^{S_y} \left(F_{y,j,z,k} * Q_{y,j} * \frac{COP_{y,j,z,k}}{COP_{y,j,z,k} - 1} \right) \tag{4}$$

Finally, the electric consumption for heat pump's compressor is:

$$P_{elec\ y,j} = \sum_{z=y+1}^{Area} \sum_{k=1}^{S_z} F_{y,j,z,k} Q_{y,j} \frac{1}{COP_{y,j,z,k} - 1} \tag{5}$$

Then, the net heat load, NHL is expressed, respecting GCC building method which goes from Main Pinch to the extreme temperature level:

$$\forall y \in [1, Area-1], \forall j \in y, NHL_{y,j} = Q_{y,j} - \sum_{k=j+1}^{S_y} (Q_{app,y,k} - Q_{prel,y,k}) + \sum_{z=y}^{Area-1} \sum_{k=1}^{S_z} (Q_{app,z,k} - Q_{prel,z,k}) \tag{6}$$

For the upper area, i.e. above MPP:

$$\forall k \in Area, NHL_{Area,k} = Q_{Area,k} + \sum_{i=1}^{k} (Q_{app,Area,i} - Q_{prel,Area,i}) \tag{7}$$

NHL aims at building new composites curves that include HP's. The key constraint is then to get $NHL_{y,j} \geq 0$ at any temperature to avoid MPP shifting.

As this problem formulation is linear, a first LP algorithm was used. It was not efficient enough because a large number of HP, sometimes with very small heat load, were proposed in the result's file. To be more realistic, a Mixed Integer Linear Programming (MILP) Algorithm formulation has been chosen, inspired by the Minimum Utilities Cost Problem of Papoulias and Grossmann [5]. For each HP, the continuous variable $F_{y,j,z,k}$ is majored by a Boolean variable $Ut_{y,j,z,k}$, indicating whether the the HP is used or not. Thus, $F_{y,j,z,k}$ can range from 0 to 1 if the HP is installed or equal to 0 if not. Moreover, the sum of the Boolean variables $Ut_{y,j,z,k}$ has an upper limit set by the user, allowing to constraint the number of HP's which result in eliminating the less interesting ones, and making data post-processing easier.
The last point is the objective function definition. The global exergy consumption is the overall exergy consumed by cold utility (exergy needed to satisfy MER_{Cold}), hot utility (exergy needed to satisfy MER), and the electric consumption of all HP's. Exergy conversion is done like Feng and Zhu [6]: Electricity is considered as pure exergy, so conversion factor is 1. For MER, as fuels are burned out, conversion factor is about 0.7, corresponding to exergy level for thermal source at high temperature level. For MER_{Cold}, exergy cost is either equal to the power of cooling systems working between ambient temperature and the coldest process temperature, if the cooling needs are below ambiance, or null.
The algorithm will select HP characteristics to minimize the global exergy function by implementing HP at the fittest temperature levels.

3. Results

3.1. Case Study

The algorithm was tested on a dairy case described in the literature [2] which includes many singularities: with PPP over and below the MPP and multiple phase transitions. Simulations are run for 2 and 3 HP's. The choice of these numbers will be validated by a sensitivity analysis in the next part.

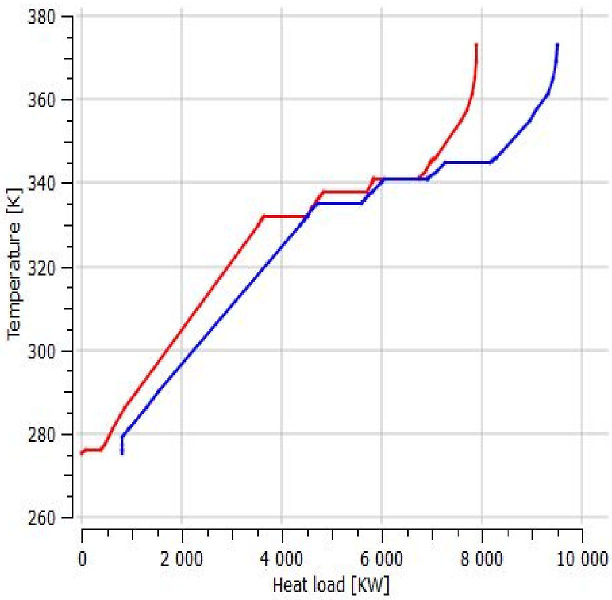

Figure 5. Dairy process Composites Curves (source Becker [2])

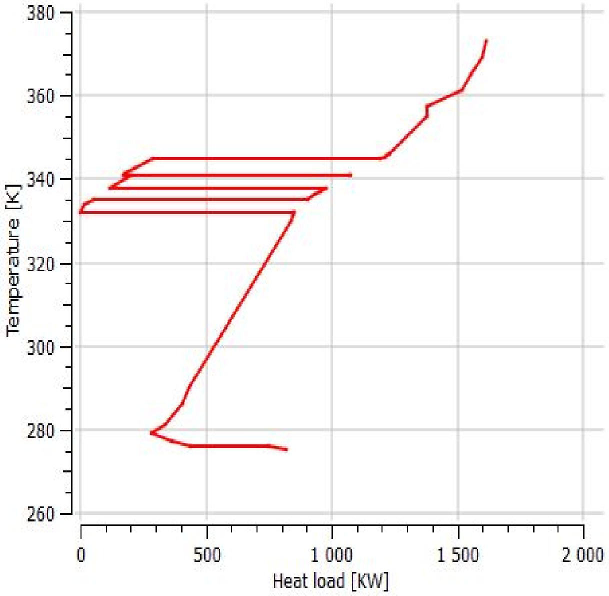

Figure 6. Dairy process Grand Composite Curve (Source Becker [2]).

3.2. Simulations Parameters

This algorithm has been implemented in the software CERES, developed by Mines ParisTech in C++ language. This is a platform for multi objective optimization of waste heat recovery in industry.
From the table of hot and cold streams of a process, extracted from simulation models, CERES first plots Composite Curves and Grand Composite Curves. Then, data are extracted from this curve to feed our algorithm written with GNU Linear Programming Kit (GLPK) and integrated in C++ within CERES.

Ambient temperature is set at 15°C, corresponding to 288.15K. ΔTmax, which is the maximum temperature difference between two discrete points of the GCC, is set to 4K. As there is a potential HP between every couple of temperature, a small temperature step leads to an exponential number of HP's to be tested, according to combinatorial analysis. The impact of ΔTmax parameter will be studied in the first sensitivity analysis of part 4. Exergy Efficiency RDTEX is set at 0.6, pinches at evaporators EvaP and at condensers CondP are set at 2K.
Two calculations are tested: the first one has to preselect 2 HP's, and the second one 3. All the parameters are summarized in Table 1.

Table 1. Algorithm parameters.

Parameter	*Value*
T0 (K)	288.15
ΔTmax (K)	4
PACMAX	[2-3]
Exergy Efficiency	0.60
EvaP (K)	2
CondP (K)	2
Solver	GLPK

3.3. Results

It is important to notice the main difference between this study and the reference case study [2].In the latter, HP's are consistent models picked up among a set of 16 distinct existing HP's (4 different refrigerants combined with 4 different compressors technologies) whereas in this paper, several hundred of HP's implementation are evaluated, but they are simplified models.
Fig. 7 presents the result of MILP algorithm calculation for 3 HPs. It highlights that PAC 3 is implemented to take advantage of the Self-sufficient Pocket by transferring additional heat into it, which enables PAC 2 to efficiently transfer twice more energy from 332K to 344K. In other terms, PAC 3 fills with energy the Self-sufficient Pocket and allows PAC 2 to transfer out more energy. Because of its lower COP, MILP algorithm designs a smaller capacity for PAC 1 (compared to the case with 2 HP's) which reduces the overall electricity consumption.

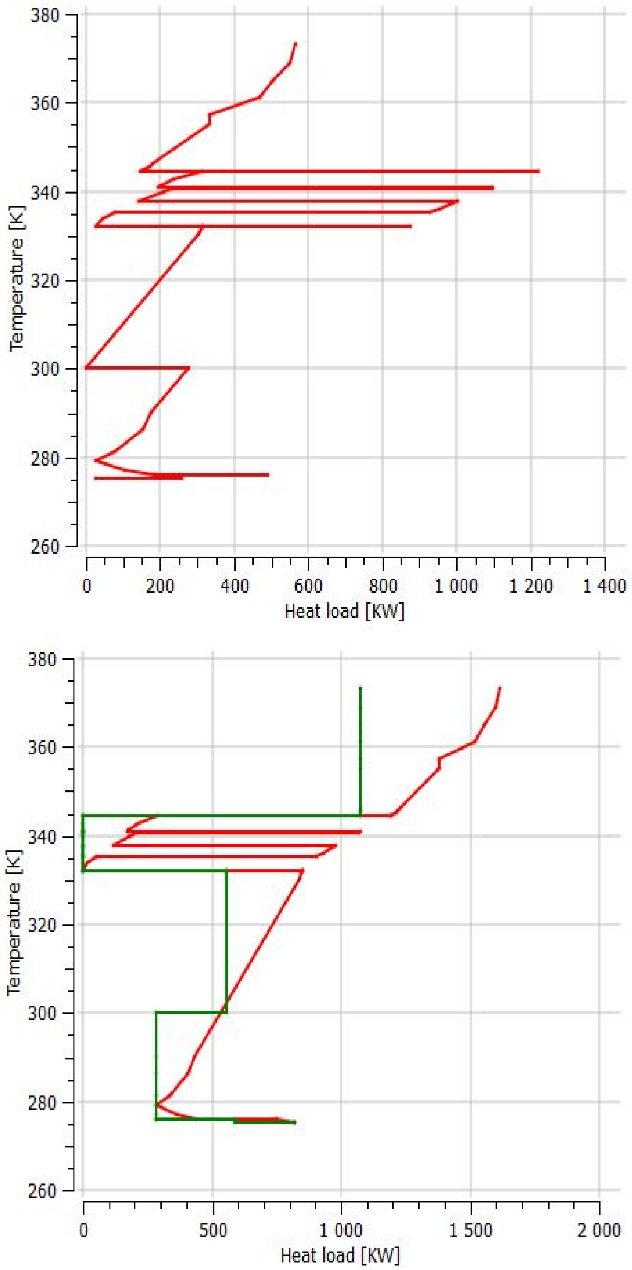

Figure 7. GCC (above) and Integrated Composite Curve (ICC)(below) solutions (3 HP's case).

Table 2 and 3 summarize the results obtained for the two simulated cases and compares them to the best configuration obtained in Becker's study.
In 2 HP's solution, the delivery temperature level for PAC1 is quite different from Becker's study [2], in which HP1 is

an existing chiller, so it can be modified. However, its low COP is compensated by a higher heat load transfer and a large decrease of MER. In spite of higher electricity consumption, the total exergy consumption remains smaller than in Becker's case as summarized in Table 3.

Table 2. HP characteristics.

	Evaporator		*Condenser*		
Solution	*Temperature (K)*	*Heat Load (kW)*	*Temperature (K)*	*Heat Load (kW)*	*COP*
2HP's					
PAC 1	273.2	537	346.7	830	2.8
PAC 2	330	282	346.7	306	12.6
3HP's					
PAC 1	274.2	304	346.7	467	2.9
PAC 2	330	559	346.7	607	12.6
PAC 3	273.2	233	302.2	277	6.3
Becker's					
HP 1	271.2	537	308.2	650	5.8
HP 2	329.2	615	349.2	668	12.6

When implementing 3 HP's, temperature levels of PAC 2 & 3 are very close to Becker's HP's. On the other hand, Becker's algorithm is based on an economic criterion that certainly would eliminate PAC 1. Even if the MER remains higher, implementing a third HP causes a decrease in exergy consumption and a drastically fall of electricity consumption, due to higher COP's.

In more complex processes, a large range of HP's sets could be found by changing parameters. They are all optimal from an exergy point of view and are proposed as candidate utilities to the HEN algorithm implemented in CERES. It finds the best group of Heat Pumps according to economic criterion.

Table 3.Energy and Exergy consumption.

Final State	*2 HP's*	*3 HP's*	*Becker's, 2012*
MER (kW)	478	545	975
MER_{Cold} (kW)	0	5	50
Electric Consumption (kW)	317	255	165
Exergy Consumption (kW)	652	637	852.5

4. Sensitivity Analysis

4.1. Temperature step

A study on the impact of temperature step is carried out. One of the advantages of the algorithm is to rely only on the use of GCC data (i.e. angular points). They may appear insufficient in some particular cases. Indeed, an angular point highlights a new available fluid or the depleted one, but with raw data, there can be a large temperature interval without such point (and so missing data for algorithm) on a GCC, like between 290.2K and 330.2K on *Fig. 6*.

. A maximum temperature step ΔTmax is introduced: Any temperature range which exceeds ΔTmax value is halved until it respects temperature step. If a moderate step improves the results accuracy, a too small step can overcome the GLPK solver's capacity. The impact of ΔTmax on total exergy results and time calculation is evaluated for the previous dairy case.

Table 4. Impact of Temperature Step (solver: CPLEX, IBM CPLEX Optimizer).

ΔTmax (K)	*Total Exergy (kW)*	*Electric Consumption (kW)*	*Calculation Time*
None	677	151	< 1s
40 → 20	679	211	< 1s
19 → 10	685	240	< 1s
9 → 5	687	250	< 5s
4 → 3	688	256	<10s
2	689	258	**676 s**
1	689	255	**2633 s**

As Total Exergy is similar for all temperature steps, the main differences between simulations lay in temperature levels of HPs which respect GCC and Main Pinch for small temperature step. A temperature step of 3 or 4K appears optimal regarding both accuracy and calculation speed. However, if time calculation is not a limiting factor, the algorithm is able to tackle a problem with a very good accuracy.

4.2. Impact of HP number

Before using economic criteria to determine the optimal number of HPs, a sensitivity analysis is carried out. Any extra-HP reduces exergy consumption by improving COP of other HP's and fits better the heat load transfer to the GCC. Is there a maximal number of HP to optimize the process? What is the effect of each extra HP implementation?

Based on the previous case, new simulations are carried out, with number of HP varying from 1 to 15 and other parameters remaining unchanged (except ΔTmax risen to 5K). Results are focused on the total exergy consumption but also on the total electric consumption of HPs and displayed in Figure 8.

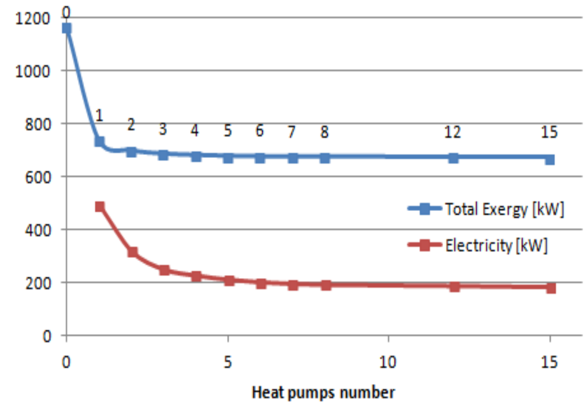

Figure 8. Impact of HP number on Exergy and Electric Consumption.

The results show a constant exergy cost for 3 or more HP, whereas electricity consumption stops decreasing significantly with more than 5 HP. The use of more HP according to these criteria appears useless: simulation time and temperature step are the criteria to be challenged to improve accuracy. To reduce the solution panel, an economic criterion including energy prices and policies is necessary.

Nevertheless, with the actual formulation, it is possible to set a maximum heat pumps number rather high without overcoming solver capabilities. In addition to the work described in this article, the methodology can be used to identify new technological developments: use of gliding temperature heat extraction and release (such as HP's with non azeotropic blends or transcritical HP's). For instance, with $PACMAX = 20$, a solution is found and a HP with a condensation temperature which ranges from 328.2 to about 293.2K appears. Such technological answer could not be found if number of standards HP, with constant phase transition levels, were drastically limited.

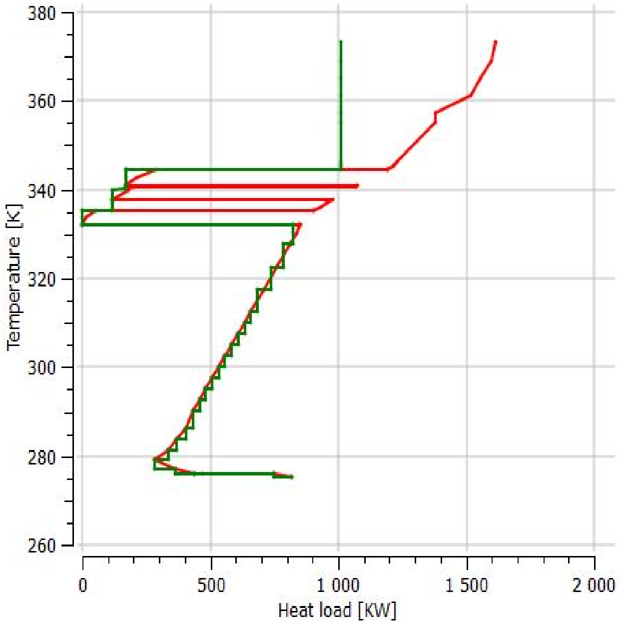

Figure 9. Illustration of new technological heat pump solution.

5. Conclusion and perspectives

This paper presents a new and powerful mathematical formulation for heat pumps design. Its accuracy and the possibility to identify breakthrough solutions, such as heat pumps with gliding temperature refrigerant widen the solutions panel. With few necessary data and a short calculation time, this tool can be used on a large process in order to efficiently identify the best utilities candidates. Indeed, the sets of solutions for any process optimized with exergy criteria are input utilities candidates for HEN design which distinguishes the most economical solution among them.

Moreover, working with GCC erases heat flows designation. The proposed HP's do not match nominative heat flow which leads to some technical unfeasibility: 2 or more heat flows to plug to a condenser, a solid heat flow that exchanges heat with a HP ... Such problems are solved by post-processing the HEN design's result.

The next step will be to propose a methodology to automatically design the HP cycles and choose the best refrigerant for each HP.

Aknowledgments:

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Nomenclature

CondP Pinch at Condenser Heat Exchanger [K]

EvaP Pinch at Evaporator Heat Exchanger [K]

P Electric Power [W]

Q Heat Load [W]

RDTEX Exergy Efficiency of a heat pump

T Temperature [K]

Subscripts

app heat load transferred from below temperature

j point position on evaporator area

k point position on condenser area

prel heat load transferred to above temperature

y evaporator area

z condenser area

Acronyms

COP Coefficient of Performance

GCC Grand Composite Curve

HEN Heat Exchangers Networks

HP Heat Pump

MER (hot) Minimal Energy Requirement [W]

MER_{Cold} Cold Minimal Energy Requirement [W]

MPP Main Pinch Point of a GCC

NHL Net Heat Load [W]

PPP Potential Pinch Point, relative heat load minima on GCC

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