

A Status-transition Model for CO₂ Heat Pump Water Heater Based on Modified Lorentz cycle

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Abstract

Energy management is a systematic activity for improving energy performances of a target system, and an energy management system is expected to solve operational planning problems and report or suggest opportunities for performance improvement. An equipment model is required to reflect the characteristics of the actual equipment's performance and to have a simple structure to apply to operational planning problems. The model should be able to diagnose changes with performance degradation over time. In this study, we proposed a thermodynamically-sound model of a CO₂ heat pump water heater, suitable for solving operational planning problems and diagnosing degradation of equipment. The proposed model consists of a heat pump unit (HP) and a hot water storage tank (ST). The HP model is a status-transition model, constructed based on the Lorentz efficiency, which is identified by experimental values and a theoretical maximum coefficient of performance (COP) for a trans-critical heat pump cycle. The ST model is simplified and can describe temperature distribution in the ST because the unit COP of the HP influences the thermal stratification of the ST. The proposed model is preferable in its simplicity and robust performance for a wide temperature range by comparison with a conventional statistical regression model.

Keywords: Energy management system; heat pump water heater; Lorentz cycle; operational planning.

1. Introduction

1.1 Background

Energy management is a systematic activity to improve energy performances of a target system, and it has already been introduced in the industrial field. Here, an energy performance represents a "measurable performance" of the use of energy, which is a broad concept concerning energy, such as energy usage rate, energy efficiency, and energy consumption by purpose. Energy management is based on a continuous improvement framework. Specifically, an energy performance indicator (EnPI), which is an index for quantifying energy performance through energy review, was identified, objectives and plans were established, improvement and deterioration in performance were measured, and verified appropriate countermeasures were implemented [1]. In addition, a function was implemented to predict the energy demand of the day ahead and to derive an optimal operational plan.

Hot water demand accounts for about 30 % of domestic energy consumption in Japan [2]. A residential CO₂ heat pump water heater (HPWH) is widely spreading as a high-efficiency device. From the long-term energy supply and demand forecast in the Japanese market established in 2015, 14 million units of HPWH might be installed in Japan by 2030 [3]. Nagai et al. [4] suggested that the HPWH can effectively consume surplus electric power from surplus photovoltaic power generation. Therefore, it is expected that the HPWH is a key apparatus in residential energy systems. The precise model of the HPWH for an energy management system, especially for residential use, is crucial for rational use of energy by residents.

1.2 Conventional Modelling Method of CO₂ Heat Pump Water Heater

Conventionally, the HPWH model was derived as a statistical model based on the experimental data, justifying the efficiency of the apparatus with industrial standards. The most reliable and certified model for a HPWH in Japan might be the statistical model based on a Japanese Industrial Standard (JIS C 9220). The manufacturer presented the certified test results based on the standard with the typical demand profile. The seasonal averaged performance was determined by the standard.

Regarding the HP model, for an operational planning problem, it was common to treat the coefficient of performance (COP) as a constant value [5]. However, it has been confirmed by measurements on actual equipment that the COP varies with several factors including ambient temperature, HP unit inlet water temperature from the storage tank, hot water temperature at the outlet of the HP unit, and defrosting mode operation. Therefore, models that express input/output relationships of energy incorporating some of these characteristics have been used. For example, as a statistical model, a model that explains the COP by a regression equation of ambient temperature and hot water temperature is widely used [6]. In addition, an example has been reported of using a simplified model for calculating the COP based on the theoretical heat pump cycle. To properly express the trans-critical heat pump cycle, J. Stene [7] proposed a model based on a theoretical Lorentz cycle. Here, the Lorentz efficiency, which is the ratio of the COP of the theoretical cycle to the actual COP, is defined. Poul et al. [8] assumed a constant Lorentz efficiency throughout the year, derived empirically. However, in practice, the

Lorentz efficiency varies with conditions. Although evaluating annual energy consumption by the statistical model is sufficient, it is not good enough to resolve a more precise time-interval in a daily operational planning. However, the models utilized for energy management have not, so far, been well-studied.

Regarding the hot water storage tank (ST), conventionally, only a relation of energy input/output has been described [9] for daily operational planning. However, the inlet water temperature entering the gas-cooler of the HP varies with the temperature distribution in the ST. When the HPWH is operating in a state when the ST is close to full capacity, the gas cooler inlet water temperature rises. This decreases the unit COP. Therefore, to incorporate the comprehensive behavior of the whole HPWH system into the operational planning problem, the ST model needs to express not only the conventional input/output relation of energy, but also the temperature distribution in the ST. Several calculation methods for simulating the temperature distribution in the ST have been reported [10-12]; however, the computational load is too large to apply to the operational planning problem. From the above, almost no models have been reported to express the heat storage amount and the temperature distribution in the ST assuming application to the operation planning problem.

As a reference model to evaluate the annual energy consumption of a HPWH installed in Japan, the so-called energy conservation law (ECL) model [13] is used. The ECL model reproduces the performance regarding the influence of typical climate conditions and product types. However, because this model is a statistical model, when energy consumption is calculated extending from acquired test conditions, for example under a condition that the ambient air temperature is very high in the summer season and the hot water demand is small, the model sometimes outputs unreasonable outliers (Figure 1). In addition, as the time resolution of the ECL model is one day, it can't be used for a daily operation planning problem, which is preferably set at shorter time intervals. From the above, the ECL model can't be used for the energy management system.

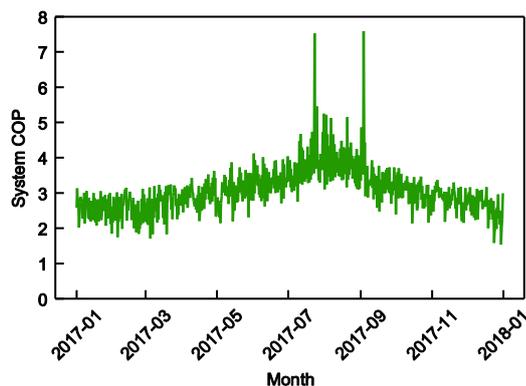


Figure 1. Annual system COP by ECL model.

1.3 Objective

In this study, the objective is to propose a model suitable for solving a day-ahead operational planning problem. The main parameter of the proposed model, Lorentz efficiency, is suitable as a key performance index for diagnosing aging deterioration or degradation of equipment in an energy management system.

2. Methodology

First, we will present a model of the HPWH as a combination of a HP unit model and a hot water ST model, based on a thermodynamically sound trans-critical heat pump cycle model for a CO₂ HPWH. Then, we will clarify the effectiveness of the model as applied to the daily operational planning problem. Next, we reveal the relationship of the inlet water temperature to the HP with the increase of the heat storage level in the ST and reflect it in the proposed model.

The model of the residential CO₂ heat pump unit (HP) based on the physical cycle model is proposed for each operation status. Next, validation by experiment is carried out, and the accuracy of the model is confirmed. Finally, the relationship between the unit COP and the storage level of the ST, the HP unit, which is a gas cooler, and the inlet water temperature is confirmed by experiment.

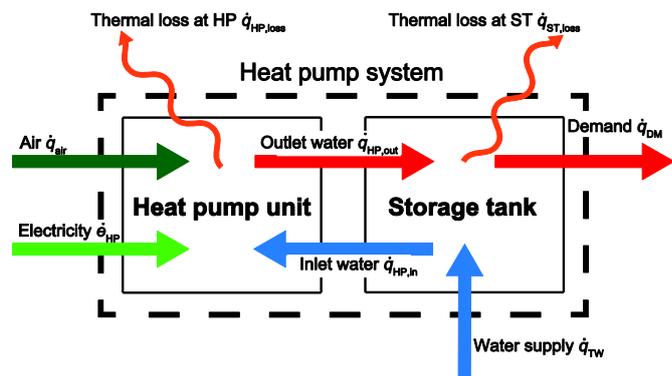


Figure 2. Energy flow of heat pump water heater.

3. Construction and Evaluation of Heat Pump Water Heater Model

The schematic energy flow diagram of the residential HPWH to be covered in this research is shown in Figure 2. The system is regarded as a connecting system with a HP and an ST. Heat from the air, heat from the water supply, and electricity power are inputs to the system, considering the thermal loss from the HP and the ST, and then, hot water is supplied to meet the hot water demand.

The proposed model is based on the experimental results and is composed of the HP model and the ST model. First, we summarize the measurement system and the experimental conditions. Next, a status-transition model is constructed from the experimental data. The HP model is constructed from each of the three operation statuses identified from the experimental results. The ST model describes the energy input/output relation reflecting the heat loss performance of the ST from experiments and expresses the relation between the heat storage level amount and the inlet water temperature. Finally, the values calculated by the ECL model and the proposed model are compared with the experimental value to verify the accuracy of the model.

3.1 Target System

This section describes the target system and experimental conditions. Table 1 shows the specification of the target system for the experiment. Here, an annual performance factor (APF) is the annual hot water demand divided by the annual energy consumption to meet the demand. In the experiment, the ambient temperature, that is, the temperature of the air flowing into the outdoor unit, is varied in the range of 5 to 25 °C, and the HPWH is operated to acquire energy input/output data. Specifically,

the time series data of ambient temperature T_{amb} °C, HP unit inlet water temperature $T_{HP,in}$ °C, outlet water temperature $T_{HP,out}$ °C, outlet water flow rate F_{HP} L/min, and electricity consumption of HP \dot{e}_{HP} kW are measured. Figure 3 shows the measurement system diagram. As experimental conditions, all hot water in the ST was removed before experiments, the ambient temperature was changed within the range shown in Figure 4 and operated, and data was acquired for 29 days. The heating rate \dot{q}_{HP} kW is calculated by Eq. (1):

$$\dot{q}_{HP} = c_p^{water} F_{HP} (T_{HP,out} - T_{HP,in}) \quad (1)$$

where c_p^{water} is the specific heat of water kJ/(kg K). The COP of HP, COP_{HP} , is calculated by the following Eq. (2).

$$COP_{HP} = \frac{\dot{q}_{HP}}{\dot{e}_{HP}} \quad (2)$$

Here, the thermocouple uncertainty is ± 1 °C, the flowmeter uncertainty is ± 0.001 L/min, and the wattmeter uncertainty is ± 3 W. When the outlet water temperature is 90 °C, the inlet water temperature is 20 °C, heating rate of the HP is 4.5 kW and the case of the coverage factor is 2, the uncertainty of the measuring system for COP_{HP} is 3.00 ± 0.08 .

Table 1. Specification of CO₂ heat pump water heater on test-bed.

Specification	Value
Electricity consumption of heat pump unit (Rated)	Winter Intermediate $\dot{e}_{HP, rated}$ kW
Heating rate of heat pump unit (Rated)	$\dot{q}_{HP, rated}$ kW
Hot water outlet temperature	$T_{HP, out}$ °C
Annual performance factor (APF)	η_{HP}^{APF}
Volume of storage tank	L

3.2 Proposed Model of Heat Pump Unit

In the experiment of the HP, the ambient temperature was changed, the HP was operated at midnight, and the experiment simulated a situation where the HP unit stopped operating when the equipment is determined to have accumulated the amount of hot water demanded for the next day. In this section, those with a water outlet temperature of 90 °C are shown. From the experimental results, it is confirmed that the operation status of the HP is classified into three types: starting-up status, steady-state status, and shutting-down status. The HP model is constructed for each status. The steady-state status model is constructed based on the theoretical trans-critical heat pump cycle, and the starting-up status and shutting-down status models are constructed by simplifying and approximating the transient characteristics from experimental results.

3.2.1 Starting-up Status Model

As examples of the experimental results of the starting-up status of the HP unit, the data of electricity consumption of the HPWH and the heating rate from the start to 30 minutes later are shown in Figures 5 and 6, respectively. The sampling interval is 1 second. Then, the results are averaged for 1 minute and normalized with the electricity consumption and heating rate based on the value after 30 minutes of

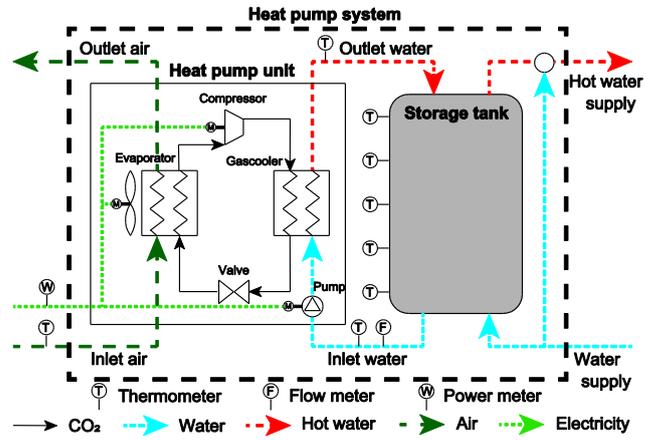


Figure 3. Measuring system.

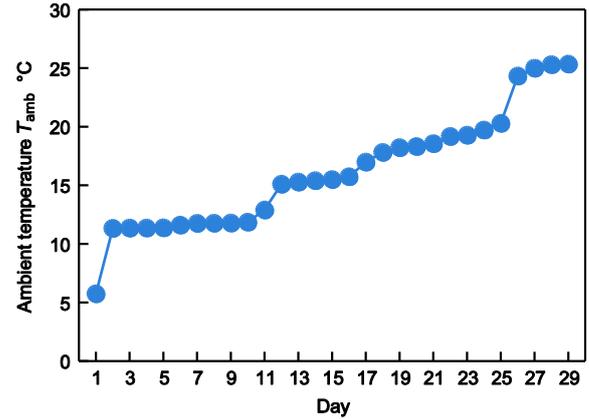


Figure 4. Experimental condition.

operation in the steady-state. The distribution of data on multiple experimental data is shown in Figures 5 and 6. Figure 6 shows that the heating rate reaches the steady-state within 30 minutes after starting-up status, although the electricity consumption has some overshoot as shown in Figure 5. Figure 6 also shows that the heating rate rises slightly behind the timing of power consumption. From Figures 5 and 6, it was confirmed that the electricity consumption and the heating rate of the HP unit can be approximated by constant transient characteristics. Therefore, the starting-up status model is based on a constant-speed approximation in which the integrated value for 30 minutes from the start-up is equal to the actual measurement. The start-up rates for electricity consumption and the heating rate are identified by experiments.

3.2.2 Steady-state Status Model

Secondly, the results of the steady-state status are shown in Figure 7. The 15-minute average values of the steady-state status are plotted, and the relationship between ambient temperature and unit COP is shown in Figure 7. It shows that unit COP and ambient temperature have a positive linear correlation.

The trans-critical CO₂ cycle is accompanied by continuous temperature glide in the heat exchange process in the gas cooler. Therefore, in this study, the modified Lorentz cycle is adopted as the theoretical reference cycle [7]. Figure 8 shows the T - s diagram of the modified Lorentz cycle. When assuming an ideal heat exchange with no temperature

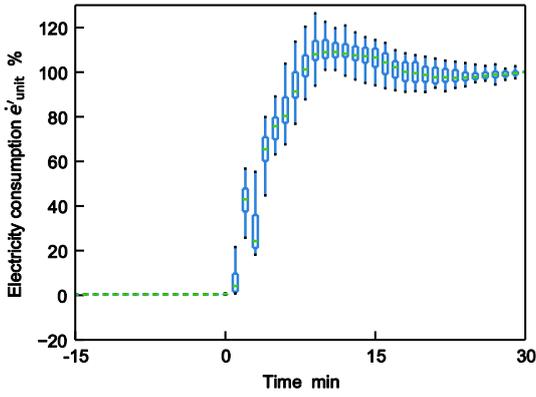


Figure 5. Electricity consumption on starting-up status.

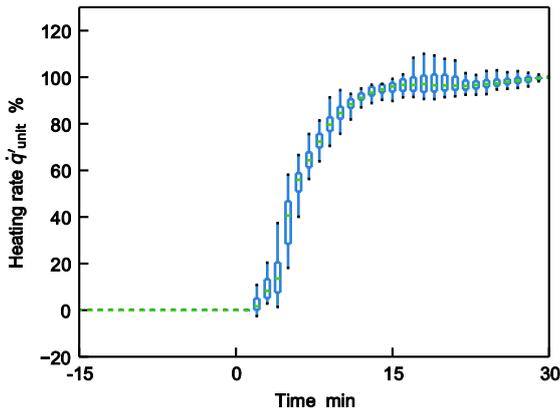


Figure 6. Heating rate on starting-up status.

difference, the evaporator outlet temperature, the compressor outlet temperature, and the gas cooler outlet temperature correspond to the ambient temperature, the outlet water temperature, and the inlet water temperature. At this time, the coefficient of performance of the modified Lorentz cycle COP_{LZ} is expressed by the Eq. (3).

$$COP_{LZ} = \frac{(T_{HP,out} - T_{HP,in})}{(T_{HP,out} - T_{HP,in}) - (T_{amb}) \ln \left(\frac{T_{HP,out}}{T_{HP,in}} \right)} \quad (3)$$

Also, the Lorentz efficiency η_{LZ} [7] is given by

$$\eta_{LZ} = \frac{COP_{HP}}{COP_{LZ}} \quad (4)$$

where COP_{HP} is the unit COP of the HP calculated from experimental data.

3.2.3 Shutting-down Status Model

Finally, as experimental results of the shutting-down status, data of electricity consumption and heating capacity of the last 30 minutes are shown in Figures 9 and 10. Data sampled at intervals of 1 second is averaged for 1 minute and normalized with electricity consumption and heating capacity with the values in the last 30 minutes of operation in the steady state. In Figures 9 and 10, 0 minutes is the stop time. The distribution of data on multiple experiment data is shown in Figures 9 and 10. Figure 9 shows that it is almost constant until the stop time, and that the electricity consumption immediately becomes 0% after the stop time.

In the shutting-down status model, it is assumed that the electricity consumption and the heating rate immediately become 0% after stopping the operation of the HP. The

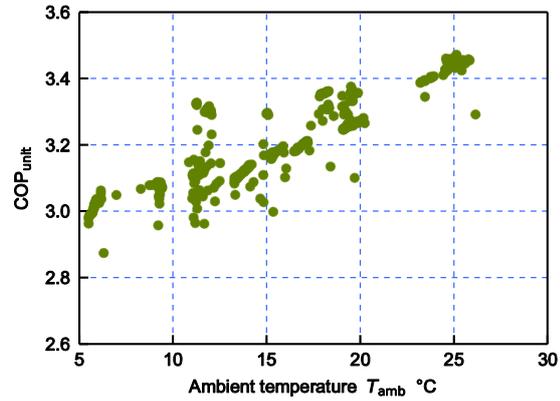


Figure 7. Relationship between unit COP and ambient temperature.

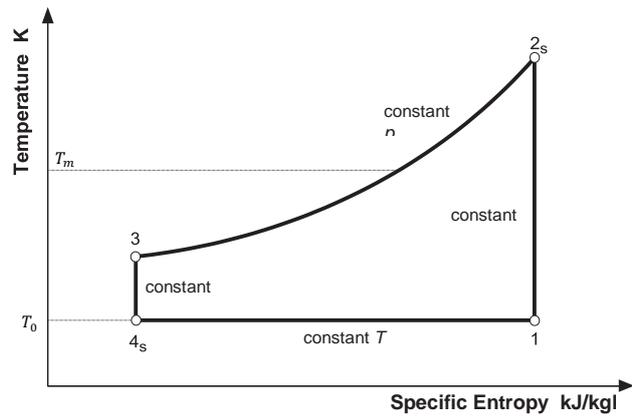


Figure 8. T-s diagram of the modified Lorentz cycle.

variation of the last several minutes of the degrading heating rate will be discussed in Sections 3.3.2 and 4.

3.3 Proposed Model of Storage Tank

The ST model describes an energy input/output relationship reflecting the heat loss performance of the ST from experiments and expresses the relationship between the heat storage level amount and the inlet water temperature. In addition, relationships of the inlet water temperature with an increase in the heat storage level of the ST are clarified and reflected in the model, based on the existing model.

3.3.1 Heat Balance in Consideration of Heat Loss Characteristics at ST

For the ST model based on the energy input/output relationship reflecting the thermal insulation performance of the ST, the heat storage level of the ST at the time t , $q_{ST}(t)$, is expected as follows:

$$\frac{q_{ST}(t) - q_{ST}(t - \delta t)}{\delta t} = \dot{q}_{ST,in}(t) - \dot{q}_{ST,out}(t) - \eta_{ST,loss} q_{ST}(t - \delta t) \quad (5)$$

where δt is the sampling time interval and $\eta_{ST,loss} \text{ h}^{-1}$ is the heat loss factor of the ST.

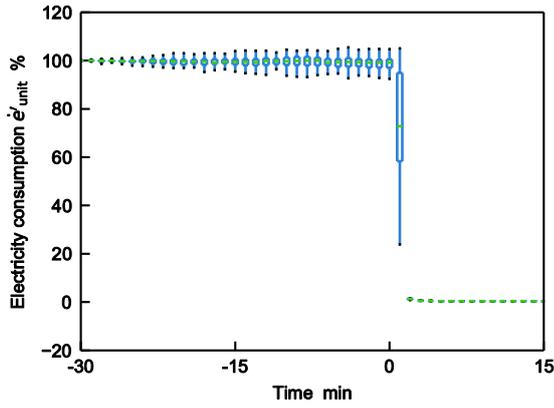


Figure 9. Electricity consumption on shutting-down status.

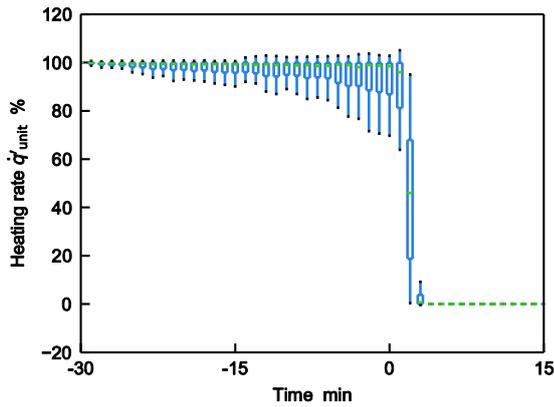


Figure 10. Heating rate on shutting-down status.

Here, the heat loss factor is identified from the heat loss characteristic experiment. The heat loss factor is given by Eq. (6).

$$\eta_{ST,loss} \delta t = \left(1 - \frac{q_{ST}(t)}{q_{ST}(t-1)} \right) \quad (6)$$

Then, $\eta_{ST,loss}$ was identified as the heat loss rate per hour. From experimental data, $\eta_{ST,loss}$ is identified as $0.92 \times 10^{-2} \text{ h}^{-1}$.

3.3.2 Relations Between Heat Storage Level and Inlet Water Temperature

On multiple experimental days, a decline in unit COP before shutdown was confirmed. Figure 11 shows that the unit COP falls down to about half the maximum at the time when the heat storage amount exceeds 90%. The reason is that increasing the inlet water temperature to the gas cooler brings a temperature mismatch with CO₂ and hot water. Then, the heating rate at the gas cooler rapidly decreased. Similarly, on other experiments, it is confirmed that the inlet temperature increases rapidly from 20 to 60 °C in the case of 90% of the heat storage level. (Exp. No.1 and 2 in Figure 12).

Yokoyama et al. proposed a hot water storage tank model as a simple one-dimensional model [10]. In the model, the ST is divided into numbers of control volumes, and energy balance and mass balance are formulated for each control volume. Here, Table 2 shows the parameters used for this model.

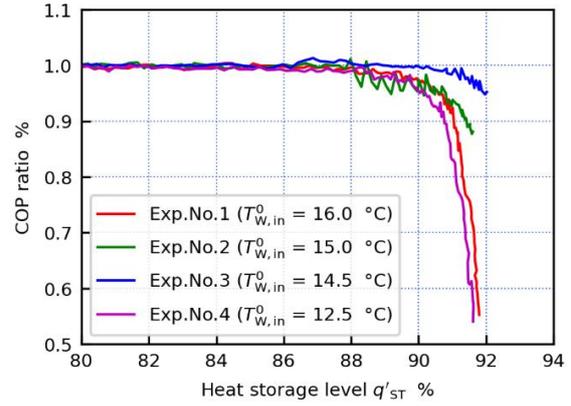


Figure 11. Relationship between heat storage level and COP of the HP.

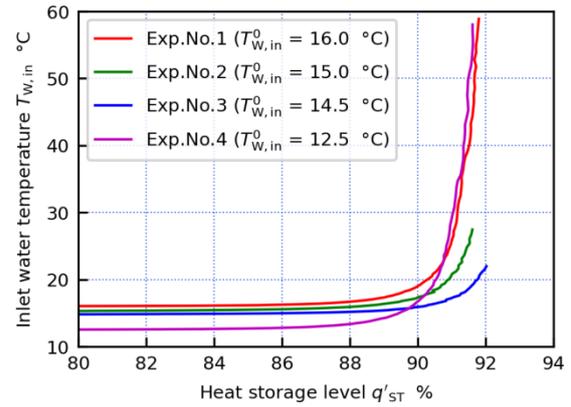


Figure 12. Relationship between heat storage level and inlet water temperature (Experimental data).

Table 2. Input parameters of the storage tank model.

Items			Value
Density	ρ	kg/m ³	1000
Diameter	D_{ST}	m	0.52
Height	H_{ST}	m	1.76
Volume	V_{ST}	m ³	0.37
Cross sectional area	S_{ST}	m ²	0.21
Side surface area	A_{ST}	m ²	2.88
Overall heat transfer coefficient	U_{ST}	W/(m ² K)	1.4
Number of control volumes	J		120
Sampling time interval		s	1

Figure 13 shows a comparison of calculated results simulating driving conditions with experimental values, given the initial value of temperature distribution of the ST on one representative day using the ST model. Here, $J = 1, 25, 52, 74, 97$ is the representative point of the six control volumes, and $J = 120$ corresponds to the gas cooler inlet water temperature. It can be seen from Figure 13 that the temperature distribution of the ST is fully reproducible by the model. In particular, the rise in water temperature can be well-reproduced.

Using this model, the relations between the heat storage level and the inlet water temperature at several presumed initial values of inlet water temperature $T_{W,in}^0$ are calculated. However, in the case of no hot water being left in the ST, the ST is operated at a constant flow rate and heating rate from

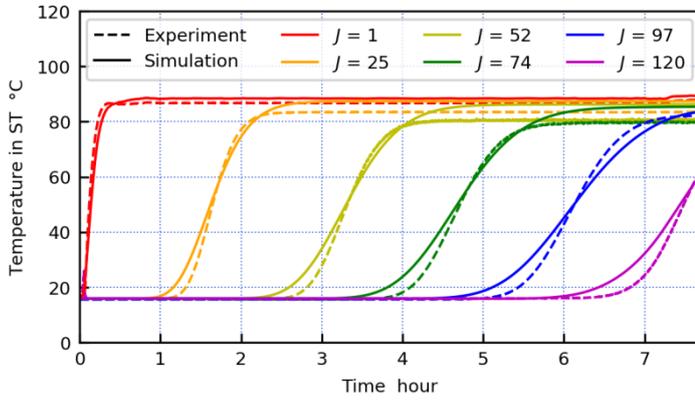


Figure 13. Temperature distribution in the ST during the HP operation.

the HP starting conditions, and no hot water is discharged to the demand during the operation.

Figure 14 shows that the point at which the inlet water temperature rising point depends on the $T_{W,in}^0$, and it can be seen that temperature rise starts earlier as the $T_{W,in}^0$ is lower. In the case of $T_{W,in}^0 = 5^\circ\text{C}$, the inlet water temperature rises from the heat storage amount of about 80%. From the above, it became clear that this model should be used to estimate the change of the inlet water temperature to calculate the precise unit COP.

3.4 Validation of the Proposed Model

In this section, using the model proposed above, we will calculate the power consumption of the whole system necessary to meet the hot water demand of the day, with the ambient air temperature, the inlet water temperature, the hot water discharge temperature, and the unit heating rate measured in the experiment as input, and we will verify the accuracy of the model. In addition, for the same experimental values, we will calculate values for the existing energy saving method model and compare and examine the results.

As a reference, the ECL model calculates the daily electricity consumption for the hot water demand [13]. The electricity consumption at a day d , $E_{\text{sys}}^{\text{ECL}}(d)$ kWh/day, is calculated by Eq. (7).

$$E_{\text{sys}}^{\text{ECL}}(d) = \left\{ f_{\text{ex}} T_{\text{ex,ave}}(d) + f_L L''_{\text{DM}} + f_{\text{e,rated}} \eta_{\text{rated}} + f_{\text{etc}} \right\} (1 + f_{\text{mode2}} r_{\text{mode2}}) C_{\text{LT}} C_{\text{def}} \times 10^3 / 3600 \quad (7)$$

The system COP, $\text{COP}_{\text{sys}}^{\text{ECL}}(d)$, in the ECL model is calculated by

$$\text{COP}_{\text{sys}}^{\text{ECL}}(d) = \frac{Q_{\text{DM}}^{\text{HW}}(d)}{E_{\text{sys}}^{\text{ECL}}(d)} \quad (8)$$

where $Q_{\text{DM}}^{\text{HW}}(d)$ is the hot water demand at the day d , in kWh/day. To validate the implementation of the ECL model, we reproduced the Figure 7.1.48 in the literature [13] (at Figure 15).

Regarding the proposed model, it is confirmed by the experiment that the Lorentz efficiency varies with the ambient temperature and the incoming water temperature. In the experiment, as a result of changing the ambient temperature between 5°C and 25°C , the Lorentz efficiency varied within the range shown in Figure 16. Figure 17 shows the comparison of the values calculated by the ECL

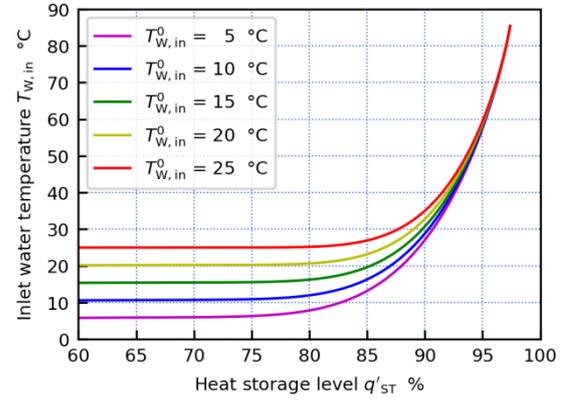


Figure 14. Relationship between heat storage level and inlet water temperature (Calculated data).

model and the proposed model, and we confirmed the effect of reflecting the seasonal change in the Lorentz efficiency. We set the Lorentz efficiency as a constant value, which corresponds to an annual average of 0.33, and Figure 17(a) shows that the proposed model reproduces with appropriate accuracy as compared with the ECL model. In Figure 17(b), one can see that from the proposed model considering the ambient temperature dependency of the Lorentz efficiency, we can expect more accurate results on electricity consumption of the HPWH. Table 3 shows the accuracy of each model.

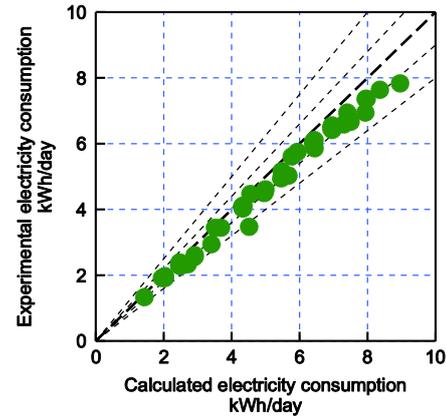


Figure 15. Validation of the ECL model.

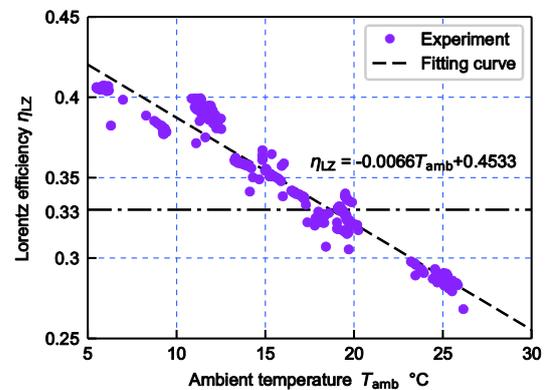
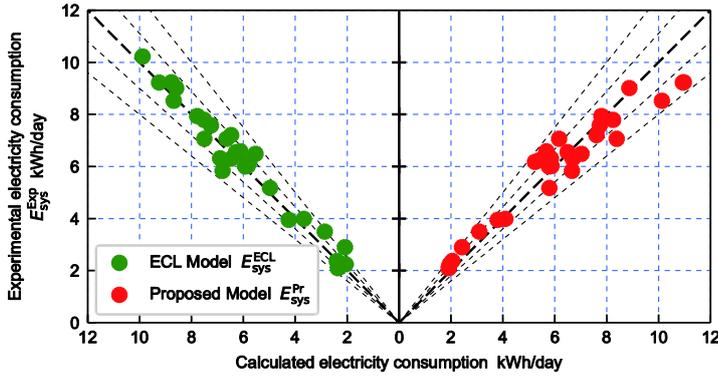
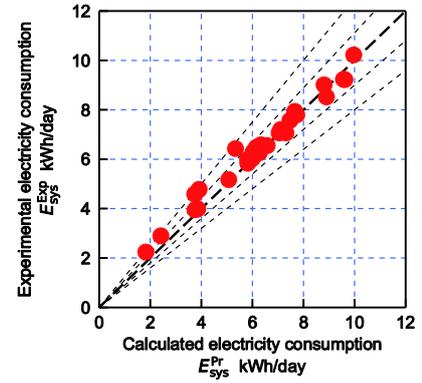


Figure 16. Relationship between Lorentz efficiency and ambient temperature.



(a) ECL model and Proposed model ($\eta_{LZ} = 0.33$)



(b) Proposed model ($\eta_{LZ} = f(T_{amb})$)

Figure 17. Comparison between calculated value and experimental value.

Table 3. Accuracy of the models.

Model	Accuracy %
ECL model	92.9
Proposed model ($\eta_{LZ} = 0.33$)	89.9
Proposed model ($\eta_{LZ} = f(T_{amb})$)	94.9

4. System COP Degradation due to Increased Heat Storage Level

From the previous chapter, it was confirmed that when the inlet water temperature to the gas cooler rises, the unit COP falls owing to the mismatch of the temperature profile in the gas cooler. In this chapter, the impact of degrading the COP on daily energy consumption is investigated.

From the experimental data reproducing the operation process from a certain heat storage amount state to the full storage state, the influence of the system COP of the day decreased owing to the rising inlet water temperature. As a comparative example, suppose a case where the inlet water temperature of the same experiment data does not rise, the heating rate of the HP does not decrease, and the hot water supply is demanded. This condition is defined as a base-case. Here, the ratio of the system COP, which is the ratio of the daily hot water demand and the power consumption, is defined as k_{sys} , and is defined by the following Eq. (9):

$$k_{sys} = \frac{COP_{sys}^{exp}}{COP_{sys}^{base}} \quad (9)$$

where COP_{sys}^{exp} is the system COP of the experiment, and COP_{sys}^{base} is the system COP of the base-case. In addition, the ratio of hot water demand to tank maximum heat storage level is Q_{DM}^{HW} , and the relationship between Q_{DM}^{HW} and k_{sys} is shown in the Figure 18. It shows that Q_{DM}^{HW} and k_{sys} have a positive correlation. When the hot water demand is large, because the ratio of the time period during which the decrease of the system COP is small, there is almost no degradation in the system COP. In contrast, when the demand for hot water supply is small, as the inlet water temperature rises immediately after the start of operation, it can be seen that the system COP decreases significantly, because of the ratio of operating in the state of a low COP.

From the above, when applying this study to operation planning of estimating the energy consumption per day, it can be predicted whether or not the system COP declines as long as the heat storage level in the ST is known at the beginning of the day, and, accompanying an increase in the heat storage amount, to what specific extent the system

COP decreases can be fully reproduced by correcting for the hot water demand.

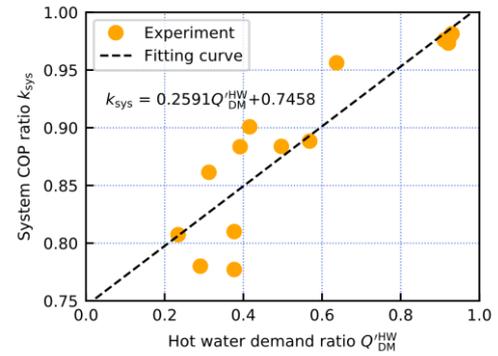


Figure 18. Relationship between hot water demand ratio and system COP ratio.

5. Conclusions

The objective of this study was to propose a model suitable for solving operational planning problems and diagnosing aging deterioration and/or degradation of equipment in an energy management system. The following results were obtained:

(1) A HPWH model, which is composed of the HP unit and the storage unit, was proposed. The HP unit model was expressed with a status-transition model with three operating statuses. The main model parameter for each operating status is identified from time series data obtained by experiment. The model accuracy concerning the seasonal change in Lorentz efficiency, which can be traced by the energy management system, was 94.9 %.

(2) From the actual equipment operation, it was confirmed that the unit COP declines as the heat storage level increases. This was due to an increase in the inlet water temperature to the gas-cooler. From the experimental values and the existing model, it was confirmed that when the heat storage amount exceeded about 80%, the incoming water temperature rose, and then the unit COP decreased by 50% at the maximum.

(3) It is confirmed by experimental data that the system COP of the HPWH is strongly influenced by the initial heat storage level a day before. Comparing with a base-case in which the inlet water temperature is assumed to be a constant value, it is confirmed that the system COP will be degraded by 20% at the maximum as compared to the low-hot water demand case, when the gas cooler inlet water temperature from the storage unit is high.

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Nomenclature

A	side surface area, m^2
C	correction factor
COP	coefficient of performance
c_p	specific heat capacity, $kJ/(kg\ K)$
D	diameter, m
d	date, day
E	electricity consumption, kWh
\dot{e}	electricity flow rate, kWh/h
F	water flow rate, L/min
f	correction factor of ECL model
H	height, m
J	number of control volumes
k	the ratio of the system COP
L	load, MJ
P	pressure, Pa
Q	heat storage level (fixed), kWh
\dot{Q}	heat flow rate (fixed), kWh/h
q	heat storage level, kWh
\dot{q}	heat flow rate, kWh/h
r	proportion of second mode
T	temperature, $^{\circ}C$
t	time index
S	cross sectional area, m^2
U	overall heat transfer coefficient, $W/(m^2\ ^{\circ}C)$
V	volume, m^3

Greek symbols

δ	differential
ρ	density, kg/m^3
η	efficiency

Subscripts and superscripts

amb	ambient
APF	annual performance factor
ave	average
calc	calculated
def	defrost
DM	demand
ECL	energy conservation law
etc	etcetera
ex	external
Exp	experiment
HP	heat pump unit
HW	hot water
in	inlet
L	load
LT	low temperature
LZ	Lorentz
mode	mode
out	outlet
Pr	proposed
rated	rated
ST	storage tank
sys	system
TW	tap water

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