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AN ARTIFICIAL NEURAL NETWORK (ANN) APPROACH FOR SOLUTION OF THE TRANSCENDENTAL EQUATION OF LONGITUDINAL VIBRATION

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Abstract: Study of mechanical vibration is one of the major issues in engineering applications. Especially, during the design and test stages of a mechanical component or system, vibration must be considered. When a vibration issue is studied theoretically, a differential equation called characteristics equation or equation of motion (EOM) is obtained. A solution of EOM gives vibrational behavior of an object considered. When vibration of a continuous system is studied, a transcendental equation is finally obtained, whose solution by classical methods is not possible. In this study, the solution of the transcendental equation which is derived from the longitudinal vibration of a bar with one end fixed and a mass at the other end was presented. For this purpose, an ANN model was constructed and the datasets were created for the ANN model. The effects of the number of neurons, input data, and training function on the model were examined. In addition, multiple regression models were developed using the ANN data also natural frequency formulation was obtained by ANN analysis for each mode. A finite element modal analysis was performed by ANSYS software. The results obtained by ANN and ANSYS were compared with analytic calculation and it was shown that they were in enough agreement.

Keywords: Artificial Neural Network (ANN), Longitudinal Vibration, Finite Element Method(FEM)

Yapay Sinir Ağları Yaklaşımı ile Boylamasına Titreşimin Transendental Denklemin Çözümü

Öz: Mekanik titreşim konusu mühendislik uygulamalarındaki en önemli konulardan biridir. Özellikle, mekanik bileşenin veya sistemin dizayn ve test aşamaları boyunca titreşim dikkate alınmalıdır. Bir titreşim konusu teorik olarak çalışıldığında, özdenklem ya da diferansiyel denklem olarak isimlendirilen hareket denklemi elde edilir. Hareket denklemi, çözümü düşünülen cismin titreşim davranışını verir. Sürekli bir sistemin titreşimi çalışıldığında, sonuç olarak 'transcendental denklem' elde edilir. Transcendental denklemin çözümü klasik yöntemlerle mümkün değildir. Bu çalışmada, bir ucu sabitlenmiş ve diğer ucunda kütle taşıyan bir çubuğun boyuna titreşiminden elde edilen transcendental denklemin çözümü sunuldu. Bu amaçla, bir YSA modeli oluşturuldu ve bu modelde kullanılmak üzere veriler hazırlandı. Modeldeki nöronlar, giriş verileri, eğitim fonksiyonlarının etkileri incelendi. Bununla birlikte, YSA verileri kullanılarak çoklu regresyon modelleri geliştirildi ayrıca her mod için ANN analizi ile doğal frekans formülasyonları elde edildi. Sonlu elemanlar modal analizi ANSYS kullanılarak yapıldı. YSA ve ANSYS'ten elde edilen sonuçlar analitik çözümle karşılaştırıldı ve hepsinin sonuçlarının oldukça yakın oldugu ortaya çıktı.

Anahtar Kelimeler: Yapay Sinir Ağları (YSA), Boyuna Titreşim, Sonlu Elemanlar Yöntemi

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1. INTRODUCTION

Today's engineering systems are becoming increasingly complex in terms of design and materials. As a result, new methods are being developed to determine more accurately and practically the dynamic behavior of a multiplicity of degrees of freedom or large structures. One of the most basic features of engineering constructions is the vibrations in these structures. Actually, all structures in nature have an infinite number of vibration frequencies and mode shapes. Calculation of frequencies of these structures and their mode shapes are significant to solve the vibration induced engineering problems(Timoshenko,1937; Magrab,1979; Leissa and Qatu,2011;Avcar and Saplıoğlu,2015). Vibration analysis of structural systems has been performed using different methods (Hsieh and Plaut,1990; Ece at al, 2007; Şimşek and Kocatürk,2007; Civalek and Gürses,2009; Avcar,2010; Akgöz and Civalek,2013; Avcar,2014; Yeşilce,2015). Some researchers were used approximate and exact solution method to calculate natural frequencies. William F. Stokey (2002) studied the natural frequencies of the longitudinal vibration of a uniform rod having a rigid mass attached to it. He obtained approximate natural frequencies by Rayleigh's Method. These methods contain differential equations that dynamic behavior of a structure is defined by these equations, and solved with the aid of them. There are various difficulties encountered while solving with differential equations. The reason is that the differential equations are interdependent (Çalışkan,1993). Natural frequencies of longitudinal vibration of a beam that attached a mass can be also solved by using commercial finite element packages such as ANSYS, ABAQUS etc. As for the solution process with the help of computer software increases the time loss when compared to the solution process with the help of artificial neural networks(ANN). Because of these difficulties, in this study, it was tried to be found the Natural Frequencies for a beam that carry a mass with the aid of ANN. Artificial neural networks have been used to solve a wide variety of problems in many areas of engineering, and they continue to be used increasingly today. Gates et al. presented a method of using artificial neural networks to stabilize large flexible space structures. In this study, they showed the neural controller learns the dynamics of the structure to be controlled and constructs a control signal to stabilize structural vibrations (Gates et al.,1993). Ding et al., (2014) used artificial neural networks with the aim of structural dynamics-guided for locating and quantifying damage in beam-type structures (Ding et al., 2014). Bağdatlı et al. investigated nonlinear vibration of stepped beams having different boundary conditions with ANN (Bagdatli,2009). Lazarevska et al.(2014) presented a study that used some of the positive aspects of the neural network's model that used for determination of fire resistance of construction elements (Lazarevska et al., 2014). Flood and Christopilos studied construction processes using artificial neural networks (ANN). Their goal was to evaluate a neural network approach to modeling the dynamics of construction processes that exhibit both discrete and stochastic behavior, providing an to the more conventional method of discrete-event simulation (Flood and Christophilos,1996). Jeng et al., In their study, an ANNwere applied to several civil engineering problems, which had difficulty to solve or interrupt through conventional approaches of engineering mechanics (Jeng et al., 2014). Furthermore, in a series of research on the artificial neural network by Toktas et al. (Özdemir and Toktaş,2008;Nalbant et al., 2009; Toktaş and Başak,2009) were presented. They used ANN for solving some problems. Such as, experimental investigation of the effects of uncoated, PVD- and CVD-coated cemented carbide inserts and cutting parameters on surface roughness in CNC turning and its prediction, contemporary analyses in assessing residual stress topographic images enclosing a cold expanded hole and chain gear design. They showed that ANN is an emerging research field. In addition, ANN techniques have been used to solve the complex problems of various engineering branches, such as mechanical, electrical, computer and other engineering fields. This method is increasingly used in the engineering field as it facilitates solving complex engineering problems.

2. STATEMENT OF THE PROBLEM

When the vibrations of continuous systems, especially bars and beams have been studying, a characteristic equation in the type of a transcendental equation is obtained. These characteristic equations have no exact analytical solution. In this study, an artificial neural network approach was proposed for the solution of the transcendental equationderived from the longitudinal vibration of a bar with one end fixed and a mass at the other end, as shown in Fig.1. If the solution procedures of a continuous system are applied to the longitudinal vibration of a bar with one end fixed and a mass at the other end, the following transcendental characteristic equation is obtained (Rao,2007).

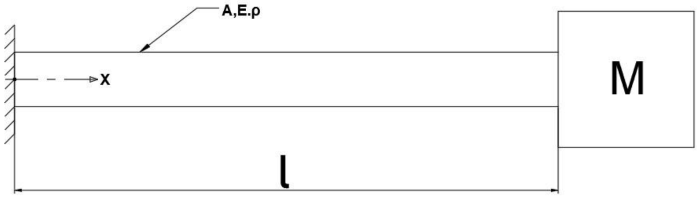

Figure 1:
Geometry of a bar carrying a mass

The equation states the transcendental characteristic equation for the longitudinal vibration of a bar with one end fixed and a mass at the other end.

$$\alpha_n tan\, \alpha_n = \beta, \qquad n = 1,2, \ldots \tag{1}$$

$$with\ \alpha_n = \frac{\omega_n l}{c}\ or\ \omega_n = \frac{\alpha_n c}{l} \tag{2}$$

where $\beta = \frac{m}{M}$; and m is the mass of the bar.

$$\beta = \frac{A\rho l}{M} = \frac{m}{M} \tag{3}$$

As it is seen, the transcendental equation depends on the mass ratio *(* β*)*. βis known, the neural frequency is unknown.

The main goal is to find the natural frequencies of the bar. To achieve this, an artificial neural network approach was applied to this problem. Since the bar is a continuous system, it has an infinite number of natural frequencies. In this study, only the first six natural frequency is considered. The reason why the beta values are chosen in the range of 0.5 to 3 is that the FE analysis is not working properly. Since the rod is a continuous system, it has an infinite number of natural frequencies. The first six natural frequencies are discussed to demonstrate the validity of the ANN model. If necessary, all frequencies of the bar can be found with this method. Here, fifty mass ratio (β) is considered as β=0.5; 0.55; 0.6; 0.65; 0.7; 0.75---2.75; 2.8; 2.85; 2.9; 2.95; 3 and the corresponding natural frequencies were obtained. This solution will be obtained by an ANN method in the subsequent parts.

3. ANALYSIS OF SYSTEM

3.1 FINITE ELEMENT ANALYSIS

There is a need to get natural frequencies and corresponding mode shapes of structures for a convenient design and avoiding resonance. Modal analysis is a process in which natural frequencies and corresponding mode shapes of a considered structure are determined. In other words, the process is a free vibration analysis. For simple systems, a theoretical modal analysis is easy but for complex structures or continuous systems, it is difficult compared to simple systems. Nowadays, finite element programs are very useful to analyze any kind of engineering problems. ANSYS program is widely used in engineering problems. In this study, modal analysis of the bar with one end fixed and carrying a mass at the other end was carried out by ANSYS. The goal of modal analysis in this section is to confirm whether ANN and analytically calculated natural frequencies are correct or not. Firstly, 3D model of the bar was created then a Finite Element(FE) model was obtained. Secondly, boundary conditions were applied, analysis options were defined as it is shown Fig.2 through Fig.4. Later, the first six natural frequencies and the corresponding mode shapes were determined for longitudinal vibration as shown Fig.5.

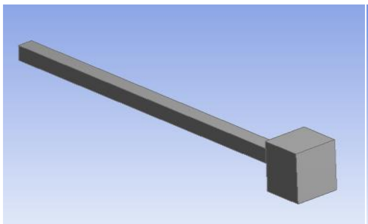

Figure 2:
3D model

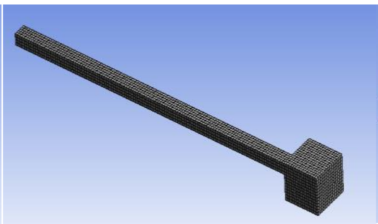

Figure 3:
FE model

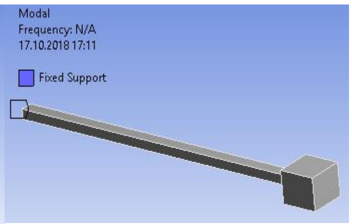

Figure 4:
Boundary condition

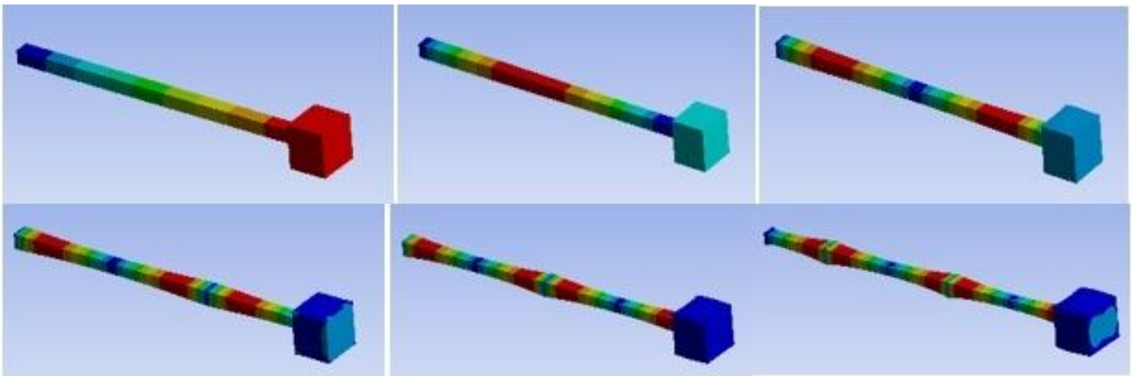

Figure 5:
Mode shapes of longitudinal vibration of a bar carrying a mass

In this study, the main goal is to obtain the natural frequencies easily using equations developed with the ANN approach. For this purpose, the steps given below were followed.

- The transcendental equation was solved by a numerical method (Wolfram).
- Longitudinal vibration of a bar with one end fixed and a mass at the other end was analyzed by Finite Element software ANSYS and the first six natural frequencies were obtained.
- The first six natural frequencies are obtained by using ANN prediction and modeling system for a bar with one end fixed and a mass at the other end.
- The obtained values by using the numerical method, the finite element method using Ansys software and ANN estimation method were compared with each other.
- The values obtained were very close to each other. From β (m/M) versus longitudinal natural frequency graph, a function was created for each mode by using the values of ANN. In this way; longitudinal natural frequency can be found easily by using the rod mass and an end mass.
- Numerical, FE and ANN results were compared. And the average errors were given in Table 1. and Table 2.

Table 1. Comparison of Ansys and Analytical values

	Ansys	Analytical	%Error
ω_1	1042.4	1012.198	2.983
ω_2	5191.6	5101.202	1.772
ω_3	10011	9856.881	1.563
ω_4	14876	14684.694	1.302
ω_5	19666	19532.130	0.685
ω_6	23734	24387.580	2.679

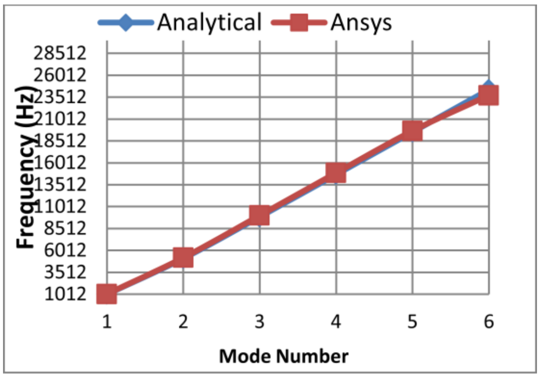

Figure 6:
Comparsion of Ansys and Analytical values

3.2. ARTIFICIAL NEURAL NETWORK(ANN)

With the development of technology; analysis programs, simulation software,engineering simulation programs etc. have been used frequently. One of the main reasons why these programs are frequently preferred is the fact that the analyzes and the predictions are quite realistic. In this study, artificial neural networks called estimation program used. The term ANN stand for an artificial neural network. ANN is a system which is modeling of a human biological brain with the simplest description. Also, they use defined rules to achieve appropriate results for a problem(Nalbant et al., 2009). Similarly; ANN is a way that to make a smart program with using the model of the human brain that simulate the working network of the neurons (Avcar and Saplıoğlu,2015).To explain briefly, biological neural networks consist of many neurons. A neuron has different shape and size depend on its function. It is important to examine the way neurons work and their activities to construct artificial neural networks. ANN is a set of neuron clusters that work for a specific purpose and can be seen as a black box. A set of neuron clusters work as processing elements. The process of collecting data, processing the collected data and sending the results to the relevant element is made through each processing element (Nalbant et al., 2009). There are three main neural network structure elements which are the input element, the output element, and a hidden layer. The hidden layer can be at least one or more. Firstly, input signals are accepted at the input layer. It passes through the hidden layer. Finally, it reaches the output layer of the artificial neural network model. Also, the backpropagation algorithm is the most useable algorithm for the multi-layer network because the mathematical program has complex, nonlinear relations. MSE stands for Mean Square Error. Mse is a performance index for Backpropagation algorithm. In this algorithm, the error is a difference between a target value and network value. Mse should be minimum (Hakim et al.,2014). Shortly, the ANN algorithm has some steps to predict the data. Artificial neural network method and working principle can be explained as follows. These steps are written below respectively.

- Data collection
- Training and testing data separation
- Select Network architecture
- Parameter tuning and weight initialization
- Data transformation

The artificial neural network method works with the MATLAB software program. In the ANNs, input values, test function and training function are used to obtain output values. In this study, the analytical calculation is made firstly. Education and test data are prepared for learning and good estimation. Hidden layer, the number of repetitions, and the number of neurons are determined before running the program and these values are entered when running the code in the MATLAB program. Training ends in two ways. First is an error level has been reached the target. Second, iteration is repeated until the end. Also, the purpose of the test data is to understand whether the artificial neural network method makes a good estimate. Test data is data that has never been used in education. The results are compared and checked with the test data. At the same time, some statistical methods are used to make comparisons like R2, RMSE, MEP (Nalbant et all, 2009). In figure 7, the ANN model is shown basically. In this study, three hidden layers are used. As the table illustrates the network with three hidden layers of [2+9+11+1] neurons at each layer has provided the best results (Fig. 7).

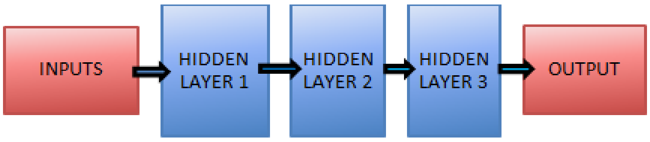

Figure 7:
The basic artificial neural network model

For each layer in fig. 7 there is a function that runs in the bacground. In the firs and second layers, the tan-sigmoid transfer function, in the third layer the log-sigmoid transfer function and finally to the output the lineer transfer function Works.

$$a = \text{tansig}\,(n) = \frac{2}{(1+e^{-2n})} - 1, \qquad a = \text{logsig}\,(n) = \frac{1}{(1+e^{-n})}, \qquad a = \text{purelin}\,(n) \qquad (4)$$

In fig. 8 the multiple-correlation coefficients and comparison between linear regression and ANN for training, validation, and testing were shown.

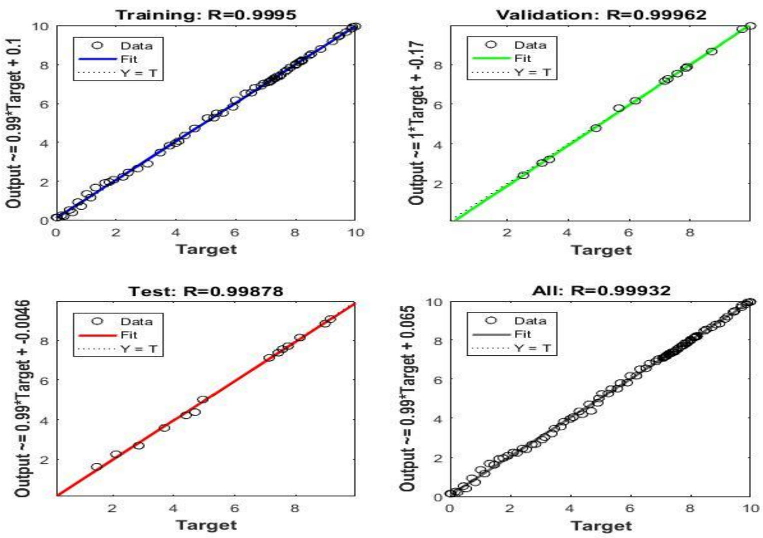

Figure 8:
Training, Validation, Test and Over All plot

Fig. 9, the error histogram in the complete training process is shown. As for the fig. 10, the performance plot shows that MSE becomes small while the number of epochs is increased.

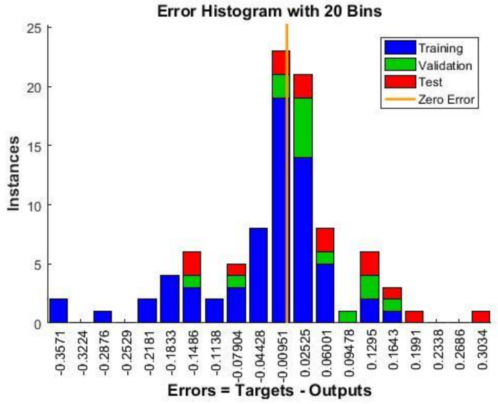

Figure 9:
Error histogram plot for training data

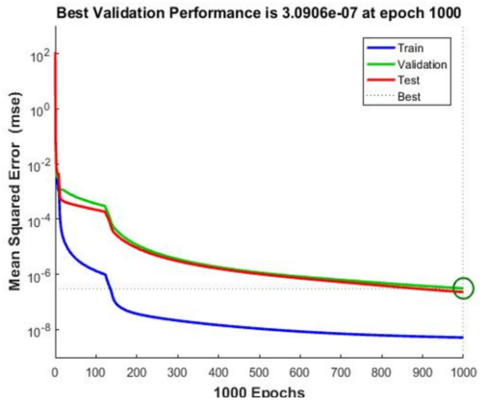

Figure 10:
Best validation performans plot of ANN

Table 2 Comparison of Ansys-Analytical, ANN-Analytical and Ansys-ANN values

	Ansys	Analytical	%Error	ANN	Analytical	%Error	Ansys	ANN	%Error
ω_1	1042.4	1012.198	2.983	1012.3	1012.198	0.010	1042.4	1012.3	2.887
ω_2	5191.6	5101.202	1.772	5101.3	5101.201	0.001	5191.6	5101.3	1.739
ω_3	10011	9856.881	1.563	9866	9856.881	0.092	10011	9866	1.448
ω_4	14876	14684.694	1.302	14736	14684.694	0.349	14876	14736	0.941
ω_5	19666	19532.130	0.685	19532	19532.13	0.0006	19666	19532	0.681
ω_6	23734	24387.580	2.679	24388	24387.58	0.001	23734	24388	2.755

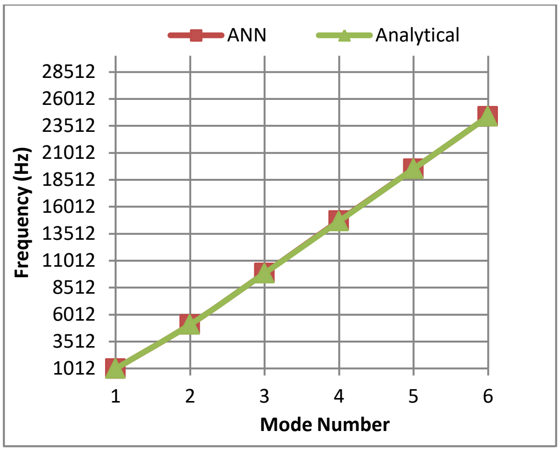

Figure 11:
Comparison of ANN and Analytical values

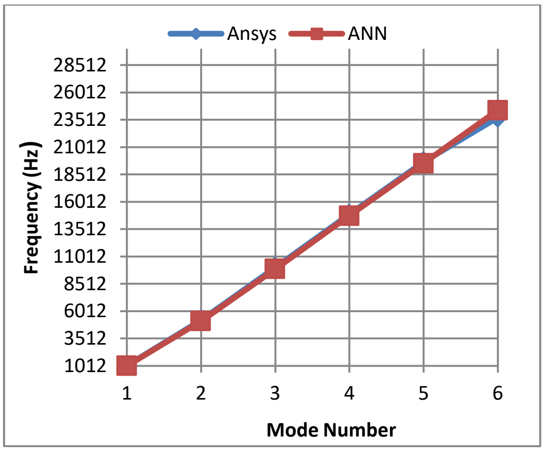

Figure 12:
Comparsion of Ansys and ANN values

ANN'smodeling was used for developing and then train the simulation of a solution of the transcendental equation of longitudinal vibration. Results obtained from, ANSYS, Analytical and ANN prediction was compared by the use of statistical error analysis methods. Here, mean error percentages were significantly small for training and testing. These different approaches were in agreement.

The following figure and equations indicate beta versus longitudinal natural frequency. Beta is the ratio of the mass of the bar to the attached mass. Natural frequencies are the values obtained as a result of artificial neural network method. When the beta values are substituted into the equations, the natural frequencies can be obtained without a need for solving the transcendental equation. The natural frequency values obtained by the artificial neural network method were very close to the results obtained from Ansys and analytical solutions. If the mass of the bar and the value of the attached mass are known, then the natural frequencies of the longitudinal vibration of the structure can be easily calculated by using equations shown in the Fig.13.

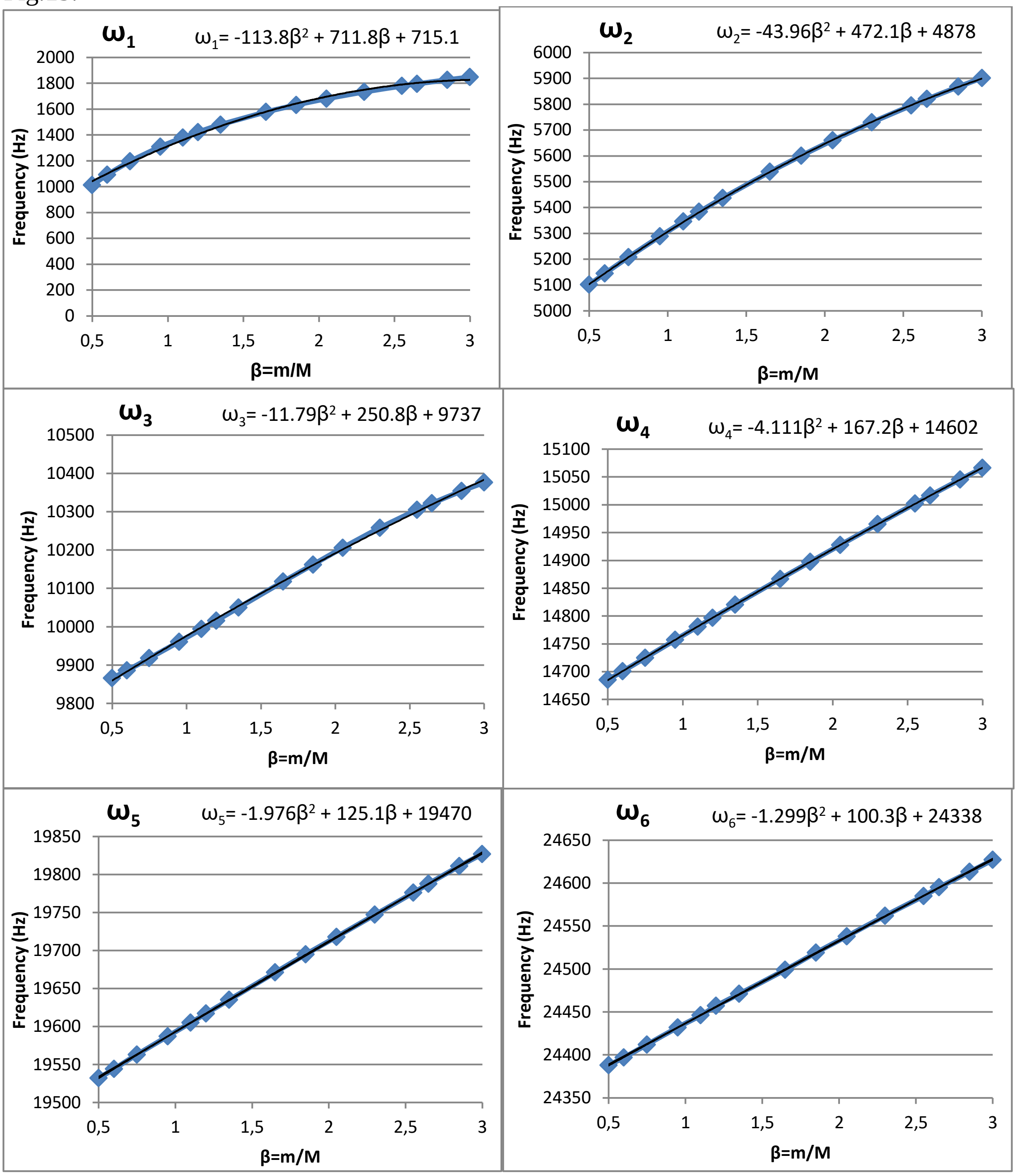

Figure 13:
Equation of frequencies for longitudinal vibration

4. CONCLUSION

ANN method is one of the most suitable learning methods for nonlinear, complex and dynamic tasks, ANN is frequently used and preferred as one of the modeling and prediction methods. One of the main reasons for its preference is due to its quite accurate results. The ANN is widely used to get quick results to suitable problems with obtained data. In this study, an ANN approach has been applied to, the solution of the transcendental equation of longitudinal vibration of a bar for the first six modes. The natural frequency predictions are obtained as equations for the first six natural frequency. ANNs prediction gave a good result with the minimum error. It is seen that ANN results are good agreement with Ansys and analytical, results. Using ANN data, longitudinal natural frequencies were calculated for each mode. If the mass of rod (m) and the end mass (the attached mass) are known, natural frequencies can be found easily from the obtained function for the transcendental equation. (β=0.5; 0.55; 0.6; 0.65; 0.7; 0.75---2.75; 2.8; 2.85; 2.9; 2.95; 3). It is seen that the application of ANN approach to the solution of the transcendental equation is possible and gives accurate results. The method can be applied for more modes than six. It can also be applied to the solution of the similar transcendental equations in different fields.

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