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Araştırma Makalesi / Research Article

Robertson-Walker evreninde kütleli spin-1 parçacıklarının yaratılması

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Anahtar kelimeler
DKP Denklemi;
Akım Yoğunluğu;
Parçacık Yaratma;
Spin-1 parçacıklar.

Özet

Bu çalışmada, ilk adım olarak, zamana bağlı bir dış elektrik alanın varlığında Robertson-Walker(RW) tipi (1+1)-boyutlu uzay-zaman modeli kullanılarak kütleli spin-1 parçacıklarını dinamiğini betimleyen Duffine-Kemmer-Petiau (DKP) denkleminin tam çözümleri elde edilmiştir. Daha sonra, elde edilen çözümler kullanılarak akım yoğunluğu hesaplanarak parçacık yaratma durumları tartışılmıştır. Parçacık yaratma sürecinin doğrudan ölçek parametresi ve elektrik alanın büyüklüğüne bağlı olduğu sonucuna ulaşılmıştır.

Creation of massive spin-1 particles in Robertson-Walker universe

Keywords
DKP Equation;
Current Density;
Particle Creation;
Spin-1 particles.

Abstract

In the present work, as a first step, exact solutions of Duffine-Kemmer-Petiau (DKP) equation which defines dynamics of massive spin-1 particles has been obtained in the presence of a time-dependent external electric field for a Robertson-Walker(RW) type (1+1)-dimensional space-time model. Next, making use of these solutions, particle creation cases have discussed by calculating current densities. It is concluded that particle production process directly depends on the scale parameter and the strength of electric field.

1. Introduction

In 1929, Oscar Klein, a Swedish theoretical physicist, acquired an interesting result which is known as the Klein paradox by using exact solutions of the Dirac equation for the problem of electron scattering from a high-barrier potential(Klein, 1934). This pioneer study has triggered to develop different approaches relating with pair production. Later, Schwinger proposed another interesting approach (Schwinger, 1951). According to this idea, vacuum is filled with the spin-1/2 particles in the absence of an electric field. On the other hand, in the presence of a strong electric field, one electron tunnels from a negative energy state to a positive energy state. The physical interpretation of such process has been evaluated as an electron-positron pair production by Schwinger who formalized the particle creation rate with the method of quantum electrodynamics (QED) as follow (in natural units, $\hbar = c = 1$)

$$w = \frac{(eE)^2}{4\pi^3} \sum_{n=1}^{+\infty} \frac{1}{n^2} exp\left(-\frac{n\pi m^2}{eE}\right), \tag{1}$$

where E denotes the electric field, m and e are the mass and charge of the electron, respectively. The strength of such an electric field would be approximately equivalent to $10^{16} V/cm$ arising from the collisions between the relativistic particles generated at the modern colliders, i.e at CERN.

It is known that Quantum Field Theory(QFT) analyzes the micro-world while the General Relativity(GR) describes the macro-world. In recent years, the theoretical unification of these two theories has been one of the most exciting and attractive problems in contemporary theoretical physics. However, many attempts to unify QFT and

GR into a single theory of quantum gravity have notbeen decisively resolved yet. For this reason, an alternative way of understanding the QFT in curved spacetime came forward to solve the relativistic particle equations associated with the fermions or bosons in curved spacetime and evaluate the physical processes like vacuum polarization or particle creation from Schwinger's perspective. The curvature arising from the contraction or expansion of the spacetime affects instability of the vacuum like time-dependent strong electric fields(Hawking, 1975; Birrell and Davies, 1982; Villalba, 1995; Villalba, 2002; Villalba, 2002; Salti and Havare, 2005; Havare, Korunur, Aydogdu, Salti and Yetkin, 2005; Parker and Toms, 2009). In recent years, the number of papers related to DKP equation in (1+1) dimensions has been observed an increasing trend since it is hard to be solved in (3+1) dimensions(Darroodi, Hassanabadi and Salehi, 2015; Sogut and Havare, 2010; Boumali, 2007; Sogut and Havare, 2006; Unal, 2005; Villalba and Greiner, 2001; Villalba, 1999; Unal, 1998; Unal, 1997; Garriga, 1994).

In the present study, the Schwinger mechanism in the DKP theory for the relativistic massive spin-1 bosons by considering the co-existence of time-dependent electrical and gravitational fields has been investigated. The outline of the article will be as follows: in section 2, the DKP equation has been solved for a time-dependent electric field in (1+1)-dimensional RW type spacetime model which is representing the early universe and defined by

$$ds^2 = \frac{1}{a^2(t)}dt^2 - a^2(t)dx^2. \quad (2)$$

Here $a(t)$ indicates the cosmic expansion parameter. In section 3, creation process of the massive $W^{\pm}$ bosons by making use of the Gordon decomposition of current density has been investigated. Finally, results has been discussed in section 4.

2. Solution of DKP equation

General form of the DKP equation in the presence of electromagnetic fields for a curved space-time model is given by(Petiau, 1936; Duffin, 1938; Kemmer, 1939)

$$\left[i\beta^{\mu}(\partial_{\mu} - \Sigma_{\mu} + ieA_{\mu}) - m\right]\Psi(t,x) = 0 \quad (3)$$

where m shows mass of the particle, A_{μ} is the 4-vector electromagnetic potential, e describes the charge, and $\beta^{\mu} = \gamma^{\mu} \otimes I + I \otimes \gamma^{\mu}$ are the Kemmer matrices relating with the usual Dirac γ matrices in curved space-time. These matrices satisfy the following commutation relations (Duffin, 1938)

$$\beta^a\beta^b\beta^c + \beta^c\beta^b\beta^a = \beta^a\delta^{bc} + \beta^c\delta^{ba}. \quad (4)$$

It is also known that β matrices in curved spacetime are connected with those ones defined in flat Minkowski spacetime by

$$\beta^{\mu}(x) = e^{\mu}_{i}\tilde{\beta}^{i} \ . \quad (5)$$

Mote that the tilde represents the Minkowski space-time,$e^{\mu}_{i}(x)$ are the tetrads satisfying the following condition

$$g_{\mu\nu} = e^{i}_{\mu}e^{j}_{\nu}\eta_{ij} \quad (6)$$

and the Greek and Latin indices correspond to the curved and flat space-time, respectively.

Next, the spin connections for the spin-1 particles written in the Eq.(3) are given by

$$\Sigma_{\mu} = \Gamma_{\mu} \otimes I + I \otimes \Gamma_{\mu}, \quad (7)$$

where Γ_{μ} are known as the spin connections for the spin-1/2 particles and defined by

$$\Gamma_{\lambda} = -\frac{1}{8}g_{\mu\alpha}\Gamma^{\alpha}_{\nu\lambda}[\gamma^{\mu}(x), \gamma^{\nu}(x)]. \quad (8)$$

Here, $\Gamma^{\alpha}_{\nu\lambda}$indicates the Christoffel symbols:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta}(\partial_{\mu}g_{\beta\nu} + \partial_{\nu}g_{\beta\mu} - \partial_{\beta}g_{\mu\nu}) \ . \quad (9)$$

Since the DKP equation is a first order sixteen-component wave equation in (3+1)-dimensional framework, it is very hard to find its exact solutions. For this reason, a way of analyzing general features of the vector bosons is to investigate in lower dimensional cases. In (1+1)-dimensional space-time, the wave function is expressed by a symmetric spinor form of rank 2. The Kemmer wave function ($\psi_K(x)$) defines a system of two-identical spin-1/2 particles and is derived from the direct product of two Dirac spinors as $\psi_K(x) = \psi_D(x) \otimes \psi_D(x)$ which is still remaining the invariance of local Lorentz transformations(Unal, 2005; Unal, 1998; Unal, 1997):

$$\psi_K = \psi_D \otimes \psi_D = \binom{\rho}{\phi} \otimes \binom{\rho}{\phi} = \begin{bmatrix} \rho\rho \\ \rho\phi \\ \phi\rho \\ \phi\phi \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_0 \\ \psi_{\tilde{0}} \\ \psi_1 \end{bmatrix}, \quad (10)$$

where ρ and ϕ are components of the Dirac wave function. In (1+1)-dimensions, the usual Dirac $\gamma_\mu(x)$ matrices are replaced by the Pauli spin matrices $\sigma^\mu(x)$. Hence, β^μ matrices are written as

$$\beta^\mu(x) = \sigma^\mu(x) \otimes I + I \otimes \sigma^\mu(x), \quad (11)$$

where the flat spacetime Dirac gamma matrices are determined by the Pauli matrices as $\gamma^{(a)} = (\sigma^3, -i\sigma^2)$. For the further calculations, the vector potential is taken in the below form

$$A_\mu = A(0, -E_0 a(t)), \quad (12)$$

where E_0 is the strenght of electric field.

Therefore, considering Eq.(5) and (12), the DKP equation takes the following form:

$$[a\tilde{\beta}^{(0)}(\partial_0 - \Sigma_0) + \frac{1}{a}\tilde{\beta}^{(1)}(\partial_1 - \Sigma_1 - ieE_0 a) - \tilde{m}]\Psi(t,x) = 0. \quad (13)$$

Here $\tilde{m} = (\gamma^{(0)} \otimes \gamma^{(0)})m$. The above equation results in three coupled differential equations as below

$$[2a\partial_0 + \dot{a} + im]h_1 - 2i[\frac{k}{a} - eE_0]h_0 + \dot{a}h_2 = 0$$

$$[2a\partial_0 + \dot{a} - im]h_1 - 2i[\frac{k}{a} - eE_0]h_0 + \dot{a}h_1 = 0 \quad (14)$$

$$h_0 = \frac{1}{m}\frac{k}{a}[\frac{k}{a} - eE_0](h_2 - h_1)$$

where

$$\psi_K = \frac{e^{ikx}}{\sqrt{2\pi}} \begin{bmatrix} h_1 \\ h_0 \\ h_0 \\ h_2 \end{bmatrix}. \quad (15)$$

After some algebra between these coupled equations, the following second order differential equation is obtained

$$[\frac{d^2}{dt^2} + 2\frac{\dot{a}}{a}\frac{d}{dt} + (\frac{m}{2})^2\frac{1}{a^2} + (\frac{k}{a^2} - \frac{eE_0}{a})^2]X(t) = 0 \quad (16)$$

with

$$X(t) = h_1(t) - h_2(t). \quad (17)$$

In order to obtain an analytical solution, the expansion parameter as $a(t) = \sqrt{\Gamma + \Lambda t}$ for which the universe model becomes radiation dominant has been taken. With this choice, Eq.(16) becomes

$$[\frac{d^2}{dt^2} + 2\frac{\Gamma}{\Gamma+\Lambda t}\frac{d}{dt} + (\frac{m}{2})^2\frac{1}{\Gamma+\Lambda t} + (\frac{k}{\Gamma+\Lambda t} - \frac{eE_0}{\sqrt{\Gamma+\Lambda t}})^2]X(t) = 0. \quad (18)$$

If a conformal transformation like $u = \sqrt{\Gamma + \Lambda t}$ is defined and is put into the Eq.(18), the following equations has been obtained

$$[\frac{d^2}{du^2} + (\frac{1}{4} + (\frac{2k}{\Lambda})^2)\frac{1}{u^2} - \frac{8keE_0}{\Lambda^2}\frac{1}{u}(\frac{m}{\Lambda})^2 + (\frac{2eE_0}{\Lambda})^2]\varphi(u) = 0 \quad (19)$$

where$X(u) = u^{-\frac{1}{2}}\varphi(u)$. After setting $u = \beta\eta$, Eq.(19) takes the following form

$$[\frac{d^2}{d\eta^2} + (\frac{1}{4} + (\frac{2k}{\Lambda})^2)\frac{1}{\eta^2} - \frac{8ekE_0}{\Lambda^2}\frac{\beta}{\eta} + ((\frac{m}{\Lambda})^2 + (\frac{2eE_0}{\Lambda})^2)\beta^2]\varphi(\eta) = 0. \quad (20)$$

Exact solution of this equation can be written as a linear combination of the Whittaker functions(Abramowitz and Stegun, 1964):

$$\varphi = N_1 M_{\kappa,\mu}(\rho) + N_2 W_{\kappa,\mu}(\rho) \quad (21)$$

where N_1 and N_2denote arbitrary constants and also ρ, β, κ and μ are given by

$$\rho = \frac{u}{\beta} \quad , \beta = \pm\frac{i}{2}[(\frac{m}{\Lambda})^2 + (\frac{2eE_0}{\Lambda})^2]^{-\frac{1}{2}}, \quad (22)$$

$$\mu = \pm\frac{2ik}{\Lambda}, \ \kappa = \mp\frac{4iekE_0}{\Lambda^2}[(\frac{m}{\Lambda})^2 + (\frac{2eE_0}{\Lambda})^2]^{-\frac{1}{2}}. \quad (23)$$

Other components of the Kemmer wave function can be calculated with the help of the Eq.(14)-(17).

Consequently, total associated Kemmer wave function can be written as given below

$$\psi_K = \frac{\sqrt{2}Ae^{ikx}}{\sqrt{\pi}\sqrt[4]{\Gamma+\Lambda t}} \begin{pmatrix} (1+\frac{i\Lambda}{2m\sqrt{\Gamma+\Lambda t}}-\frac{i\Lambda}{2m\beta}+\frac{i\Lambda\kappa}{m\sqrt{\Gamma+\Lambda t}})W_{\kappa,\mu}+\frac{i\Lambda}{m\sqrt{\Gamma+\Lambda t}}W_{\kappa+1,\mu} \\ \frac{2}{m}[\frac{k}{\sqrt{\Gamma+\Lambda t}}-eE_0] \\ \frac{2}{m}[\frac{k}{\sqrt{\Gamma+\Lambda t}}-eE_0] \\ (1-\frac{i\Lambda}{2m\sqrt{\Gamma+\Lambda t}}+\frac{i\Lambda}{2m\beta}-\frac{i\Lambda\kappa}{m\sqrt{\Gamma+\Lambda t}})W_{\kappa,\mu}-\frac{i\Lambda}{m\sqrt{\Gamma+\Lambda t}}W_{\kappa+1,\mu} \end{pmatrix}$$

where a recurrence relation for the Whittaker functions has been used(Abramowitz and Stegun, 1964):

$$\partial_\mu W_{\kappa,\mu}(z) = \frac{1}{z}\{(\frac{z}{2}-\kappa)W_{\kappa,\mu}(z)-W_{\kappa+1,\mu}(z)\}.$$

Asymptotic behavior of the Whittaker functions is written as follow(Taylor, 1939)

$$W_{\kappa,\mu} \to e^{-\frac{z}{2}}z^{\kappa}$$

which results the following form for the Kemmer wave function

$$\psi_K = \frac{Ae^{ikx}}{\sqrt{2\pi}\sqrt[4]{\Gamma+\Lambda t}}(\frac{\sqrt{\Gamma+\Lambda t}}{\beta}e^{-\frac{\sqrt{\Gamma+\Lambda t}}{2\beta\kappa}})^{\kappa} \begin{pmatrix} -\frac{1}{2}[1+\frac{i\Lambda}{2m\sqrt{\Gamma+\Lambda t}}+\frac{i\Lambda\kappa}{m\sqrt{\Gamma+\Lambda t}}+\frac{i\Lambda}{2m\beta}] \\ \frac{1}{m}[\frac{k}{\sqrt{\Gamma+\Lambda t}}-eE_0] \\ \frac{1}{m}[\frac{k}{\sqrt{\Gamma+\Lambda t}}-eE_0] \\ \frac{1}{2}[1-\frac{i\Lambda}{2m\sqrt{\Gamma+\Lambda t}}-\frac{i\Lambda\kappa}{m\sqrt{\Gamma+\Lambda t}}-\frac{i\Lambda}{2m\beta}] \end{pmatrix}. \quad (24)$$

Here, A is a normalization coefficient and it is defined as

$$A = \sqrt{\pi}[1+(\frac{2eE_0}{m})^2]^{-\frac{1}{4}} \quad (25)$$

3. Calculating components of the Kemmer Current

It is a fact that the vacuum in time oscillates with an expansion parameter like $a(t)$ gives rise to the particle production in curved space-time. In this proposal, particle production or annihilation is defined by Gordon decomposition as follows(Gordon, 1928):

$$J^{\mu} = \bar{\Psi}\beta^{\mu}\Psi, \quad (26)$$

where $\bar{\Psi} = \psi^{\dagger}(\gamma^{(0)} \otimes \gamma^{(0)})$ and $\beta^{\mu} = e^{\mu}_{(i)}\beta^{(i)}$. Now, it is needed to calculate surviving components of the Kemmer current with the help of Eq.(24). However, a particular Kemmer current due to two-dimensional line-element has two components known as charge density and current density.

- Charge Density: Eq.(26) for $\mu = 0$ becomes as follow;

$$j_0 = \frac{i\Lambda}{8m}[1+(\frac{2eE_0}{m})^2]^{-\frac{1}{2}}[(\frac{1}{\beta}-\frac{1}{\beta^*})+(K-K^*)\frac{1}{\sqrt{\Gamma+\Lambda t}}]. \quad (27)$$

From this equation, the following results according to the values of β and κ is written as follows:

$$j_0 = \begin{cases} \frac{1}{2}[1+\frac{2ekE_0}{m^2+4e^2E_0^2}\frac{1}{\sqrt{\Gamma+\Lambda t}}] & \mathrm{Im}\beta > 0 \text{ and } \mathrm{Im}\kappa < 0 \\ -\frac{1}{2}[1+\frac{2ekE_0}{m^2+4e^2E_0^2}\frac{1}{\sqrt{\Gamma+\Lambda t}}] & \mathrm{Im}\beta < 0 \text{ and } \mathrm{Im}\kappa > 0 \end{cases} \quad (28)$$

- Current Density: Eq.(26) for $\mu = 1$is written as

$$j_1 = \frac{2}{m}[1+(\frac{2eE_0}{m})^2]^{-1/2}[\frac{k}{(\Gamma+\Lambda t)^{3/2}}-\frac{eE_0}{\Gamma+\Lambda t}]. \quad (29)$$

The current density defined in Eq.(29) depends on particle charge and momentum while it is totally independent of β and κ. In this case, this equation includes two different possibilities according to the momentum and charge variables like: (a) $k_x > 0$and $e > 0$ (b) $k_x < 0$ and $e < 0$. Thus, the wave function given in Eq.(29) has two different physical situations because of these two possibilities. The first case creates a particle with the positive charge and momentum while the second one yields a particle with negative charge and momentum. It is significant to mention here that the first component of current density has been affected from the sign of k_x and e even though zeroth component does not. Consequently, it can be said easily that there is a relationship between the spatial or the first component of current and particle production(Sogut and Havare, 2006). To understand this relation, it is needed to follow cWKB method(Biswas, Guha and Sarkar, 1993; Biswas, Guha and Sarkar, 1994a; Biswas, Guha and Dasgupta, 1994b; Guha, Biswas, Sarkar and Biswas, 1995; Biswas, Guha and Sarkar, 1995; Sarkar and Biswas, 1998).

4. Conclusion

In this study, exact solutions of the DKP equation in a (1+1)-dimensional curved space-time model in the presence of a time-dependent electric field has been found. Additionally, the surviving components of Gordon decomposition of the vector bosons has been calculated to check whether the particle creation process occurs or not. In order to analyze the evolutionary nature of current component according to the electric field and curved space-time contributions, it is illustrated as a function of time in the following figure;

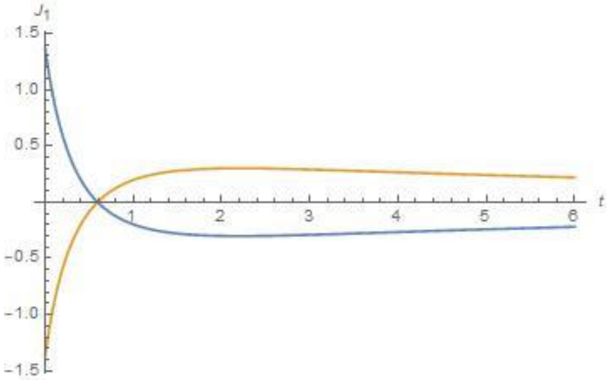

Figure 1: Blue and brown lines correspond to $k_x > 0$ with $e > 0$, $k_x < 0$ with $e < 0$ cases, respectively. The auxiliary parameters have been chosen as $\Lambda = 4.8$, $\Gamma = 2.5$, $E_0 = 1.6$ and $k_x = 3.5$.

It is seen that the curve has a critical turning point since the sign of both current densities has changed around it. Therefore, it has been interpreted as an equilibrium moment from particle-antiparticle rotation perspective. Also, from Eq.(29), it is seen that particle production directly depends on the scale parameter(Λ) and electric strength(E_0), because the turning point does not appear if it is chosen $\Lambda = 0$ and $E_0 = 0$. For this reason, the values of both parameters should be different from zero for mentioning the existence of particle creation in this space-time model.

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