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Effect of Q-Matrix Misspecification on Parameter Estimation in Differing Sample Sizes and Test Length for DINA[1]

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ABSTRACT

In Cognitive Diagnosis Models, every item in the measurement tool has a different effect (which is determined based on the attribute tested) on the classification of individuals in terms of attributes tested. One of the most effective factors that affects the quality of implications and the accuracy of classification, is to develop proper item-attribute relationships, in other words, the correctness of Q-matrix Misspecification of the Q-matrix leads to incorrect decisions about the individuals. The present study, serving as a fundamental research, investigates the effect of the Q-matrix misspecification in the DINA model on parameter estimations in the datasets, which are designed as a simulation and have differing sample sizes (50, 100, 250, 500, and 1,000 participants) and test length (15 and 30 items). The parameter estimations were made by using Markov Chain Monte Carlo method based on Bayesian estimation. The estimations for misspecified Q-matrix have been compared to item parameters regarding the correct Q-matrix appropriate to dataset. In the case of underspecification in Q-matrix, slipping parameters for deficiently specified items and standard error values related to these; in the case of overspecification, guessing parameters related to overestimated items and standard error values related to these were overestimated. The parameter estimation is affected by the Q-matrix misspecification in all of the conditions discussed. Nevertheless, the amount of error in estimation does not show a regular differentiation in accordance with the sample size.

Key Words: Misspecification, Guessing Parameter, Slipping Parameter

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INTRODUCTION

Any measurement tool used in education and psychology or each item in these instruments, is designed to determine whether or not the individuals have a particular attribute or to what extent they have it. Therefore, it is necessary to classify the performance of individuals in each item separately and also in general. Since a measurement tool is not designed for assessing only one attribute, a singular attribute may be inadequate to answer an item. In such cases, it may not be appropriate for any and every item in the tool to have the same effect in terms of the validity of the classification and the correctness of the decisions to be made. In achievement tests, especially, it is effective to integrate what skills are necessary to answer each item correctly in the correct classification the performances in the item and in the whole of the test. This is achievable by using Cognitive Diagnostic Models (CDM) (de la Torre, 2009b; Rupp, & Templin, 2008b; Rupp, Templin, & Henson, 2010).

CDMs reveal whether or not the individual has the required attribute(s) to respond correctly to an item in a measurement tool, and specify his/her attribute as a categorical according to that. In CDMs, when the attributes required to respond to an item correctly are determined properly, the strengths and weaknesses for the scope of the measurement tool can be determined with reference to the individuals' responses. With the DCMs, it is likely to designate in the level of attributes and give feedback by building up the discrete profile (defined through latent variables) of each student for each item and the structure being measured. Besides, rather than making an evaluation based on classification, the students' learning in an educational environment is identified by determining the current situation of the students in the measured areas, and the education process can be lead properly by specifying their levels of readiness and training needs (de la Torre, 2009b; Henson, & Templin, 2006; Rupp, & Templin, 2008a, 2008b; Rupp et al., 2010).

The information quality of CDMs depends on the validity of individuals' classification to the latent classes relevant to their abilities, as well as the accuracy of the inferences to be made and the decisions to be taken, the quality of the items and content validity of the measurement tool. This is directly related to the correct specification of the Q-matrix. The Q-matrix creates a relationship between the structure-related attributes measured by the measurement tool and the items in the measurement tool. In order for an item to be responded to accurately, it shows which attributes are required. If an item inspects an attribute ("1" if not needed) a "0" is written to the cell of that item's line where it intersects with the relevant attribute's column in the matrix. The misspecification of Q-matrix may lead to an individual's faulty profile being formed after the analyses, misjudgments, or wrong decisions (de la Torre, 2009b; Im, & Corter, 2011; Qin et al., 2015; Romero, Ordoñez, Ponsoda, & Revuelta, 2014; Rupp, & Templin, 2008a, 2008b; Rupp et al., 2010; Tatsuoka, 1983). Ensuring the correct specification of the Q-matrix and model data concordance is a key element in determining the quality of diagnostic information obtained through a measurement tool and is of great importance with regards to the validity of the results obtained (de la Torre, & Douglas, 2004; Dogan, & Tatsuoka, 2008; Henson, 2004; Rupp, & Templin, 2008a; Tatsuoka, 1983).

In this present study, research on the DINA (The Deterministic Inputs, Noisy "And" Gate) (de la Torre, 2009b; Haertel, 1989) model -one of the CDMs- was carried out. The DINA model is of the CDMs in which latent variables (attributes) and observed variables (responses) are graded as "double" or "non-compensatory", meaning an individual not possessing the necessary attributes for an item to be responded to cannot be compensated by just having another attribute (de la Torre, 2009a; Henson, & Templin, 2006; Rupp, & Templin, 2008a, 2008b; Rupp et al., 2010). In the DINA model, the probability of improper model responses in the response pattern is estimated in two ways. These are "slipping" (s) which expresses the individual's likelihood of responding incorrectly to the item even if they have all of the attributes assessed, and "guess" (g) which expresses the possibility of responding correctly to the item even though the individual does not have at least one of the measured attributes. The lower the s parameter is, the greater likely that the item and individuals with the measured attributes respond correctly to the item. The lower the g parameter is, the less likely it is for individuals without measured attributes to respond correctly to the item (de la Torre, 2009b; Rupp, & Templin, 2008a; Rupp et al., 2010).

One of the main problems encountered when analyzing the achievement tests developed in accordance with DINA model is that while the s and g parameters should be close to zero as a sign of accuracy (de la Torre, & Douglas, 2004; Li, 2008; Rupp, & Templin, 2008a), it can also be of a high value due to an error in the specification of the Q-matrix. Having analyzed the literature, there are many possible misspecification cases in Q-matrix specification (Baker, 1993; de la Torre, 2008; Im, & Corter, 2011; Kunina-Habenicht, Rupp, & Wilhelm, 2012; MacDonald, 2013; MacDonald, & Kromrey, 2012; Rupp, & Templin, 2008a). The purpose of this present study is to investigate the effect of Q-Matrix misspecification on item parameter estimation in differing sample sizes and test length for the DINA model. With regards to the accuracy of the decisions to be taken at the end of practices, it is crucial to determine how different misspecification may affect parameter estimations and guiding the practitioners according to these specifications. When the theoretical specifications within the scope of the study are reflected to the practice, it is assumed that they will contribute to the accuracy of the results obtained and the decisions based upon them.

METHOD

Research Model

The present study serves as a fundamental research because it investigates the effect of Q-Matrix misspecification on parameter estimation in differing sample sizes and test length for the DINA model.

Data Collection

The data were generated by using the "dina" R package (Culpepper, 2015) in line with the correct Q-matrix and the DINA model attached in Appendix 1. As the data were in the process of production, the parameters g and s were equalized to 0.00. Initially, a respondent attribute profile (the dataset used to reveal whether or not participants have the attributes in the measurement tool [0-1]) was created for samples of 50, 100, 250, 500, and 1,000 participants. It was noted that the number of individuals for each possible attribute were approximately equal. The response patterns were formed for the individuals in line with the respondent attribute profile. 100 replications were made for each case.

The number of attributes is a significant factor in terms of classification consistency. Studies in the relevant literature reveal that the number of attributes ranges from three to eight (Henson, & Douglas, 2005; Kunina-Habenicht et al., 2012; Ömür-Sünbül, & Kan, 2015; Rupp, & Templin, 2008a). The present study was conducted on a case in which four attributes were measured.

The size of sample and test length play a crucial role parameter estimation. The sample sizes in the different studies examined range from 20 to 10,000 participants (Baker, 1993; Cassuto, 1996; de la Torre, Hong, & Deng, 2010; Kunina-Habenicht et al., 2012; MacDonald, & Kromrey, 2011; Ömür-Sünbül, & Kan, 2015), and a test length consisting of 15 to 60 items (Cassuto, 1996; Henson, & Templin, 2006; Kunina-Habenicht et al., 2012; MacDonald, 2013; Ömür-Sünbül, & Kan, 2015; Tatsuoka, 1990; Templin et al., 2009) To estimate parameters truthfully, Orlando and Marshall (2002) suggest a sample size of at least 200, while Tsutakawa and Johnson (1990) suggest 500 or more. The researcher in this present study decided to conduct studies on samples of 50, 100, 250, 500, and 1,000 participants after examining the findings of the relevant studies and the numbers of attributes measured. There are 16 (2^4) different potential attribute patterns for cases in which four attributes are measured. The items were developed to represent 15 possible patterns, except for the "0000" pattern for which no attribute was represented. A 15-item and a 30-item hypothetic measurement tool was examined.

To achieve accuracy of measurement, every attribute was measured with the same number of items, and the items were written to represent all possible attribute combinations while determining the items in the measurement tool and developing Q-matrix (DiBello, & Stout, 2007; Kunina-Habenicht et al., 2012; Rupp, & Templin, 2008a).

In the present study, when there is misspecification of Q-matrix, the misspecification rate is kept stable and two different conditions are considered for the misspecification scheme. When the literature is reviewed, the misspecified Q-matrices with the error rate of 1% and 15% were considered. As the misspecification rate increases, the error amount of the estimation and the number of faulty parameters are found to increase. In addition, as the misspecification rate increases, the estimation bias increases even more (Baker, 1993; MacDonald, 2013). Therefore, it was decided that Q-matrix will be misspecified with a 10% rate. Misspecification method is considered as deficient, balanced, and overspecified in the literature (Baker, 1993; Kunina-Habernicht et al., 2012; MacDonald, 2013; MacDonald, & Kromrey, 2012; Rupp & Templin, 2008a). In the present study, the conditions of deficient and over-specification are analyzed. When the deficient Q-matrix is formed; in the overspecified Q-matrix, 0s are changed to 1s. In the misspecifications of Q-matrix, the items and attributes to be misspecified were determined randomly.

Data Analysis

The analyses were performed with the "dina" R package (Culpepper, 2015). The s and g parameters and their standard errors were calculated from any and every response pattern generated recurrently. 100 iterations were performed in the estimation and the average values obtained were reported. By using the generated response patterns, the analyses were repeated for the conditions in which the Q-matrix was misspecified.

Parameter estimations were made by using Markov Chain Monte Carlo (MCMC) method based on Bayesian estimation (Congdon, 2001; Lee, & Song, 2004; Palomo, Dunson, & Bollen, 2007; Yang, & Dunson, 2010), which provides reliable results in small sample sizes. Gibbs sampling (Gelfand, 2000; Gelfand, Smith, & Lee, 1992; Gill, 2002) was used for MCMC analyses. After analyzing similar studies (Best, Cowles, & Vines, 1995; Culpepper, 2015; de la Torre, & Douglas, 2004; Gelman, & Rubin, 1992) in the related literature, it was determined that the length of chain would be 5,000 and the burn-in would be 1,000 in Gibbs sampling and parameter estimation would be performed according to 4,000 iteration.

FINDINGS

In this section, the findings obtained by using correct and misspecified Q-matrix for hypothetical measurement tools containing 15 and 30 items respectively are presented.

Correct Q-matrix Specification Case for 15 Item Condition

Within the scope of the present study, item parameters and their standard errors of datasets with differing sample sizes were calculated by first using the correct Matrix $Q0_1$. Then, the states of these parameters in the analysis with the misspecified Q-matrix are determined. Table 1 shows the estimated item parameters for differing sample sizes and their standard errors obtained by using the Matrix $Q0_1$.

As can be seen in Table 1, the parameters s and g are generally close to 0.00 as defined in the generation of the data. However, for the 50-participant sample, although the case of parameter values equal to 0.00 is defined, the parameter s estimated for the items (shown in bold) that especially measure three/four attributes, deviate from zero because the sample size is small. As the number of participants in the sample increases, the parameter values approach zero. The standard errors for the s and g parameters estimated by using the correct Q-matrix are also close to zero. Similar to the case of parameter estimation, the standard error values in the 50-participant sample are somewhat higher than the other sample size conditions. The estimates in Table 1 obtained from the correct Q-matrix are assumed to be correct and form the basis for comparison with the parameters estimated with the misspecified Q-matrix.

Underspecified Q-matrix Case for 15 Item Condition

Within the scope of the present study, when $Q0_1$ matrix was specified as 10% deficient (misspecified $Q1_1$ matrix), it was determined how estimated parameters and their standard errors differ from the predicted values. $Q1_1$ matrix was created with a total of six cells with the second attribute measurement of Items 5, 9 and 12, the third attribute of Items 11 and 14, and the fourth attribute of Item 10 by deficiently specified (see Appendix 2). While these items actually measure the attributes related and attributes individuals need to possess these in order to be able to answer the item correctly, the Q-matrix was not correlated with this attribute, did not measure this attribute, and was defined as if required to have this attribute in order to answer correctly. The averages of the parameter estimation made by using $Q1_1$ matrix are presented in Table 2. While the misspecified items are shaded in the tables, the parameter values estimated differently the expected condition are indicated in bold. Since the parameter values are fixed to 0, 00 in the data generation, it is assumed that the values obtained after the analyses using the misspecified Q-matrices differ from the values of the correctly specified Q-matrices if the difference is 05 or more; and the parameters related to these items are shown in bold.

When Table 2 is examined and the relevant values are compared with the values in Table 1, it can be seen that there are small changes in the g parameters of the items (except for Item 2) for the 15-item condition in all sample size conditions. On the other hand, the g parameter value of the correctly specified Item 2 has increased. In the misspecified Q-matrix, there are three misspecifications for three different items when measuring the second attribute. In addition, while the standard error values regarding the g parameters for all items vary slightly, the standard error values concerning the s parameters for the underspecified items have particularly increased. The parameters of Item 11, which was underspecified, are very close to the those of the error-free condition in all sample size conditions. Similarly, the parameter values of Item 14, which was also underspecified, showed an increase in the analyses for only the 50-participant sample, and were close to those of the error-free condition in all other sample size conditions.

Table 1. *Parameters estimated through $Q0_1$ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
Item	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s
1	0.0440	0.0413	0.0563	0.0517	0.0241	0.0240	0.0214	0.0214	0.0101	0.0097	0.0086	0.0084	0.0046	0.0044	0.0045	0.0046	0.0023	0.0022	0.0023	0.0023
2	0.0440	0.0422	0.0512	0.0487	0.0249	0.0239	0.0203	0.0196	0.0089	0.0089	0.0093	0.0093	0.0044	0.0043	0.0045	0.0045	0.0022	0.0022	0.0022	0.0022
3	0.0534	0.0495	0.0410	0.0390	0.0203	0.0198	0.0236	0.0227	0.0085	0.0085	0.0093	0.0094	0.0046	0.0045	0.0045	0.0046	0.0022	0.0022	0.0023	0.0022
4	0.0391	0.0375	0.0581	0.0547	0.0201	0.0195	0.0225	0.0220	0.0080	0.0081	0.0104	0.0101	0.0049	0.0048	0.0044	0.0045	0.0023	0.0023	0.0022	0.0022
5	0.0247	0.0239	0.0954	0.0863	0.0140	0.0138	0.0302	0.0293	0.0053	0.0054	0.0145	0.0146	0.0027	0.0026	0.0079	0.0077	0.0013	0.0013	0.0040	0.0039
6	0.0260	0.0252	0.0699	0.0650	0.0132	0.0128	0.0370	0.0358	0.0055	0.0053	0.0138	0.0136	0.0027	0.0027	0.0072	0.0073	0.0013	0.0013	0.0039	0.0039
7	0.0230	0.0229	0.1155	0.1048	0.0137	0.0131	0.0336	0.0326	0.0052	0.0053	0.0167	0.0166	0.0028	0.0028	0.0069	0.0070	0.0013	0.0013	0.0040	0.0040
8	0.0249	0.0243	0.0680	0.0634	0.0133	0.0128	0.0355	0.0345	0.0051	0.0050	0.0172	0.0170	0.0028	0.0028	0.0073	0.0072	0.0013	0.0013	0.0040	0.0040
9	0.0247	0.0242	0.0791	0.0729	0.0136	0.0134	0.0362	0.0347	0.0050	0.0050	0.0187	0.0179	0.0026	0.0026	0.0079	0.0078	0.0014	0.0014	0.0039	0.0039
10	0.0235	0.0226	0.0777	0.0738	0.0130	0.0127	0.0389	0.0368	0.0049	0.0050	0.0199	0.0198	0.0026	0.0026	0.0080	0.0080	0.0013	0.0013	0.0042	0.0041
11	0.0209	0.0201	**0.1422**	**0.1252**	0.0114	0.0108	0.0581	0.0554	0.0046	0.0044	0.0292	0.0288	0.0023	0.0023	0.0121	0.0122	0.0012	0.0011	0.0071	0.0070
12	0.0206	0.0195	**0.1693**	**0.1421**	0.0118	0.0115	0.0589	0.0565	0.0044	0.0043	0.0354	0.0338	0.0024	0.0024	0.0138	0.0137	0.0011	0.0012	0.0082	0.0082
13	0.0210	0.0199	**0.1622**	**0.1360**	0.0113	0.0110	0.0635	0.0580	0.0046	0.0046	0.0309	0.0305	0.0023	0.0023	0.0148	0.0145	0.0011	0.0011	0.0081	0.0081
14	0.0226	0.0224	**0.1261**	**0.1125**	0.0112	0.0111	0.0659	0.0628	0.0044	0.0043	0.0407	0.0390	0.0023	0.0023	0.0153	0.0144	0.0011	0.0011	0.0080	0.0079
15	0.0199	0.0194	**0.2478**	**0.1921**	0.0104	0.0102	0.0991	0.0909	0.0042	0.0042	0.0679	0.0643	0.0021	0.0021	0.0261	0.0254	0.0011	0.0011	0.0148	0.0146

Table 2. *Parameters estimated through $Q1_1$ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
Item	g	SH_g	s	SH_s	g	SH_g	s	SH_s	g	SH_g	s	SH_s	g	SH_g	s	SH_s	g	SH_g	s	SH_s
1	0.0541	0.0511	0.0511	0.0495	0.0287	0.0275	0.0205	0.0200	0.0107	0.0104	0.0083	0.0080	0.0051	0.0051	0.0042	0.0041	0.0024	0.0024	0.0023	0.0023
2	**0.2257**	0.0829	0.0726	0.0676	**0.3793**	0.0621	0.0372	0.0364	**0.3100**	0.0361	0.0176	0.0167	**0.2676**	0.0260	0.0071	0.0072	**0.3076**	0.0184	0.0041	0.0040
3	0.0686	0.0599	0.0427	0.0410	0.0286	0.0281	0.0224	0.0219	0.0103	0.0102	0.0094	0.0092	0.0057	0.0057	0.0044	0.0044	0.0029	0.0029	0.0023	0.0023
4	0.0745	0.0635	0.0618	0.0581	0.0320	0.0310	0.0234	0.0224	0.0116	0.0118	0.0113	0.0110	0.0076	0.0077	0.0047	0.0046	0.0036	0.0036	0.0025	0.0024
5	0.0354	0.0342	**0.5963**	0.0956	0.0217	0.0208	**0.4348**	0.0648	0.0092	0.0091	**0.5211**	0.0424	0.0041	0.0041	**0.4898**	0.0317	0.0021	0.0020	**0.5026**	0.0224
6	0.0260	0.0246	0.0682	0.0647	0.0136	0.0135	0.0370	0.0350	0.0056	0.0056	0.0144	0.0142	0.0027	0.0028	0.0072	0.0071	0.0013	0.0013	0.0039	0.0040
7	0.0245	0.0243	0.1111	0.0978	0.0139	0.0138	0.0349	0.0337	0.0052	0.0050	0.0162	0.0160	0.0027	0.0026	0.0070	0.0070	0.0013	0.0013	0.0040	0.0040
8	0.0268	0.0258	0.0689	0.0646	0.0140	0.0140	0.0356	0.0336	0.0053	0.0052	0.0175	0.0172	0.0028	0.0027	0.0071	0.0071	0.0014	0.0014	0.0040	0.0039
9	0.0307	0.0301	**0.4178**	0.1208	0.0187	0.0188	**0.4397**	0.0729	0.0070	0.0070	**0.5213**	0.0464	0.0042	0.0043	**0.5288**	0.0300	0.0020	0.0020	**0.4906**	0.0222
10	0.0410	0.0393	**0.5763**	0.0923	0.0183	0.0173	**0.4847**	0.0707	0.0075	0.0076	**0.5920**	0.0442	0.0041	0.0041	**0.5132**	0.0311	0.0020	0.0020	**0.5314**	0.0221
11	0.0217	0.0210	0.1918	0.1545	0.0115	0.0110	0.0640	0.0594	0.0046	0.0046	0.0313	0.0301	0.0024	0.0024	0.0125	0.0124	0.0011	0.0011	0.0077	0.0076
12	0.0221	0.0215	**0.4450**	0.1582	0.0132	0.0128	**0.4452**	0.0902	0.0053	0.0053	**0.5402**	0.0635	0.0027	0.0028	**0.5034**	0.0408	0.0013	0.0013	**0.5177**	0.0307
13	0.0204	0.0198	0.1675	0.1386	0.0116	0.0115	0.0626	0.0589	0.0045	0.0044	0.0309	0.0292	0.0023	0.0023	0.0145	0.0144	0.0011	0.0012	0.0085	0.0083
14	0.0226	0.0218	**0.2063**	0.1603	0.0113	0.0112	0.0767	0.0716	0.0044	0.0044	0.0440	0.0432	0.0023	0.0022	0.0165	0.0160	0.0011	0.0011	0.0082	0.0082
15	0.0200	0.0199	0.2505	0.1907	0.0109	0.0109	0.1005	0.0909	0.0042	0.0043	0.0675	0.0632	0.0022	0.0022	0.0257	0.0242	0.0011	0.0011	0.0149	0.0145

The increases in s parameter do not show a regular variation based on sample size. In other words, as the sample size increases, the s parameter values of the misspecified items remain stable. That is mainly because the number of individuals in the latent classes that represent different levels of competence increases at approximately the same rate, and accordingly the number of individuals who will give wrong answers also increases as the sample size expands. The error values of misspecified items' p parameters approach zero as the sample size expands. Besides, as the sample size expands, the parameters and standard error values of the correctly specified items approach closer to zero. The increase in sample size reduces the error amount of the estimations.

Overspecified Q-matrix Case for 15-item Condition

In the case in which $Q0_1$ matrix is found to be 10% overspecified ($Q2_1$ Matrix), there have been six misspecifications for the following conditions; based on measuring the first attribute for Item 9; based on measuring the second attribute for Item 3; based on measuring the third attribute for Items 2, 9, and 12; and based on measuring the fourth attribute for Item 11 (see Appendix 2). While these items do not measure the relevant attributes, they are associated with these attributes in the Q-matrix and were defined as if the individuals should have these attributes to answer these items correctly. The averages of the parameter estimations using the matrix $Q2_1$ are presented in Table 3.

When Table 3 is examined and the relevant values are compared with the values in Table 1, it is observed that the g parameters of overspecified items and standard error values increase in the sample size cases (except for Item 12 in the 50 participant sample). The increases do not show a regular variation on the basis of sample size. The parameter values increased in the three different sample sizes.

In addition, the s parameters of the items, and standard errors have also increased slightly, usually in overspecified items. The number of items whose s parameters increase, decreases as the sample size increases. Besides, as the sample size increases, the amount of increase in the s parameters and standard errors also decreases.

When the underspecification or overspecification cases for 15-item condition are compared, the s parameters of the underspecified items are generally affected by the underspecifications made through Q-matrix while both the g and s parameters are affected by the overspecifications.

Correct Q-matrix Specification Case for 30 Item Condition

Table 4 presents item parameters and their standard error averages which were calculated when the Q-matrix ($Q0_2$) was correct and the measurement tool consisted of 30 items (see Appendix 1). The $Q0_2$ matrix was formed by adding 15 items with the same attribute pattern as the 15 items in the matrix $Q0_1$.

When Table 4 is examined, it is observed that the parameter values and their standard errors are generally close to zero as defined in the generation of the data. However, the decrease in sample size causes the parameter values to deviate from zero. The s parameter values are affected more by the sample size than the g parameter. Especially in the 50-participant samples, the s parameter values and standard errors fairly deviate from zero.

Table 3. *Parameters estimated through $Q2_1$ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
I	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s
1	0.0594	0.0545	0.0543	0.0510	0.0374	0.0359	0.0207	0.0202	0.0157	0.0158	0.0083	0.0084	0.0068	0.0067	0.0044	0.0044	0.0032	0.0032	0.0023	0.0023
2	**0.2817**	0.0726	0.0708	0.0667	**0.4260**	0.0569	0.0356	0.0354	**0.3423**	0.0339	0.0173	0.0167	**0.3105**	0.0242	0.0074	0.0073	**0.3389**	0.0173	0.0040	0.0040
3	**0.3844**	0.0777	0.0715	0.0649	**0.2802**	0.0506	0.0347	0.0335	**0.3375**	0.0338	0.0174	0.0171	**0.3292**	0.0246	0.0072	0.0072	**0.3335**	0.0174	0.0040	0.0041
4	0.0486	0.0451	0.0697	0.0648	0.0288	0.0274	0.0267	0.0258	0.0103	0.0103	0.0131	0.0128	0.0070	0.0068	0.0050	0.0050	0.0032	0.0032	0.0027	0.0027
5	0.0296	0.0285	**0.1440**	0.1173	0.0185	0.0187	0.0626	0.0565	0.0067	0.0066	0.0327	0.0304	0.0033	0.0034	0.0127	0.0123	0.0016	0.0016	0.0078	0.0078
6	0.0317	0.0310	0.1129	0.0976	0.0149	0.0145	0.0460	0.0437	0.0063	0.0061	0.0204	0.0197	0.0030	0.0029	0.0090	0.0089	0.0015	0.0015	0.0053	0.0053
7	0.0233	0.0226	0.1159	0.1024	0.0134	0.0134	0.0355	0.0345	0.0051	0.0051	0.0167	0.0168	0.0028	0.0028	0.0071	0.0071	0.0013	0.0013	0.0040	0.0040
8	0.0273	0.0271	0.0728	0.0677	0.0139	0.0136	0.0346	0.0332	0.0052	0.0051	0.0180	0.0181	0.0028	0.0027	0.0073	0.0074	0.0014	0.0014	0.0040	0.0040
9	**0.1998**	0.0561	**0.2465**	0.1839	**0.2023**	0.0414	**0.1000**	0.0912	**0.1716**	0.0248	0.0657	0.0626	**0.1911**	0.0180	0.0274	0.0266	**0.2024**	0.0129	0.0145	0.0142
10	0.0303	0.0298	0.1039	0.0912	0.0146	0.0148	0.0473	0.0453	0.0059	0.0059	0.0247	0.0236	0.0032	0.0032	0.0103	0.0103	0.0015	0.0015	0.0055	0.0054
11	**0.0792**	0.0376	**0.2496**	0.1890	**0.0851**	0.0283	0.1021	0.0929	**0.0837**	0.0176	0.0678	0.0628	**0.0921**	0.0133	0.0266	0.0264	**0.0735**	0.0086	0.0150	0.0146
12	0.0600	0.0326	**0.2501**	0.1926	**0.0853**	0.0284	0.0985	0.0890	**0.0633**	0.0156	0.0673	0.0632	**0.0733**	0.0122	0.0262	0.0256	**0.0589**	0.0078	0.0150	0.0150
13	0.0210	0.0211	0.1648	0.1412	0.0111	0.0108	0.0628	0.0594	0.0045	0.0044	0.0316	0.0308	0.0023	0.0022	0.0141	0.0140	0.0011	0.0011	0.0084	0.0085
14	0.0215	0.0212	0.1250	0.1110	0.0112	0.0112	0.0647	0.0612	0.0043	0.0042	0.0410	0.0394	0.0023	0.0022	0.0159	0.0160	0.0011	0.0011	0.0080	0.0077
15	0.0203	0.0197	0.2471	0.1975	0.0108	0.0108	0.1015	0.0908	0.0043	0.0044	0.0655	0.0624	0.0022	0.0022	0.0261	0.0256	0.0010	0.0010	0.0150	0.0144

Table 4. *Parameters estimated through $Q0_2$ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
I	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s
1	0.0330	0.0318	0.0443	0.0426	0.0206	0.0203	0.0181	0.0178	0.0087	0.0086	0.0086	0.0071	0.0087	0.0040	0.0066	0.0055	0.0120	0.0023	0.0069	0.0048
2	0.0345	0.0329	0.0408	0.0395	0.0207	0.0200	0.0184	0.0181	0.0074	0.0074	0.0074	0.0085	0.0091	0.0042	0.0064	0.0053	0.0117	0.0022	0.0071	0.0049
3	0.0415	0.0399	0.0335	0.0321	0.0237	0.0230	0.0170	0.0166	0.0081	0.0080	0.0080	0.0078	0.0091	0.0042	0.0063	0.0053	0.0123	0.0023	0.0068	0.0047
4	0.0407	0.0388	0.0354	0.0335	0.0187	0.0189	0.0195	0.0190	0.0075	0.0075	0.0075	0.0084	0.0088	0.0041	0.0065	0.0055	0.0129	0.0024	0.0064	0.0045
5	0.0237	0.0232	0.0824	0.0748	0.0135	0.0135	0.0337	0.0326	0.0054	0.0053	0.0053	0.0043	0.0051	0.0028	0.0116	0.0100	0.0065	0.0016	0.0112	0.0081
6	0.0248	0.0241	0.0714	0.0679	0.0150	0.0143	0.0280	0.0267	0.0055	0.0054	0.0054	0.0041	0.0053	0.0029	0.0110	0.0094	0.0068	0.0016	0.0108	0.0077
7	0.0274	0.0263	0.0615	0.0572	0.0125	0.0123	0.0378	0.0364	0.0053	0.0053	0.0053	0.0050	0.0050	0.0028	0.0118	0.0102	0.0067	0.0016	0.0110	0.0078
8	0.0248	0.0252	0.0774	0.0714	0.0134	0.0132	0.0325	0.0321	0.0052	0.0052	0.0052	0.0059	0.0049	0.0028	0.0121	0.0104	0.0059	0.0015	0.0119	0.0087
9	0.0242	0.0231	0.0764	0.0701	0.0140	0.0142	0.0327	0.0309	0.0052	0.0052	0.0052	0.0055	0.0049	0.0028	0.0121	0.0103	0.0068	0.0016	0.0108	0.0078
10	0.0252	0.0240	0.0663	0.0612	0.0132	0.0130	0.0355	0.0342	0.0052	0.0051	0.0051	0.0057	0.0051	0.0028	0.0117	0.0100	0.0069	0.0016	0.0109	0.0077
11	0.0207	0.0197	**0.1708**	0.1443	0.0120	0.0122	**0.0498**	0.0478	0.0046	0.0046	0.0046	0.0061	0.0036	0.0024	0.0186	0.0166	0.0037	0.0013	0.0162	0.0125
12	0.0219	0.0211	**0.1254**	0.1105	0.0117	0.0115	**0.0701**	0.0657	0.0046	0.0046	0.0046	0.0067	0.0034	0.0024	0.0215	0.0194	0.0025	0.0012	0.0117	0.0098
13	0.0220	0.0210	**0.1146**	0.1013	0.0113	0.0116	**0.0604**	0.0583	0.0045	0.0045	0.0045	0.0002	0.0036	0.0024	0.0187	0.0166	0.0039	0.0014	0.0156	0.0119
14	0.0218	0.0217	**0.1227**	0.1091	0.0114	0.0114	**0.0596**	0.0551	0.0045	0.0045	0.0045	0.0096	0.0033	0.0023	0.0221	0.0199	0.0036	0.0013	0.0165	0.0127
15	0.0202	0.0193	**0.2475**	0.1919	0.0104	0.0101	**0.1006**	0.0928	0.0043	0.0043	0.0043	0.0055	0.0027	0.0022	0.0346	0.0318	0.0023	0.0012	0.0246	0.0204
16	0.0321	0.0309	0.0435	0.0410	0.0207	0.0199	0.0178	0.0172	0.0087	0.0086	0.0086	0.0069	0.0087	0.0040	0.0066	0.0055	0.0120	0.0023	0.0069	0.0048
17	0.0344	0.0340	0.0406	0.0395	0.0205	0.0206	0.0186	0.0183	0.0074	0.0073	0.0073	0.0085	0.0091	0.0042	0.0063	0.0053	0.0069	0.0021	0.0046	0.0035
18	0.0415	0.0382	0.0334	0.0330	0.0232	0.0230	0.0166	0.0162	0.0081	0.0080	0.0080	0.0076	0.0041	0.0040	0.0039	0.0039	0.0123	0.0023	0.0068	0.0047
19	0.0402	0.0390	0.0347	0.0339	0.0189	0.0188	0.0199	0.0197	0.0075	0.0074	0.0074	0.0086	0.0088	0.0041	0.0066	0.0055	0.0129	0.0025	0.0064	0.0045
20	0.0235	0.0231	0.0835	0.0731	0.0138	0.0138	0.0346	0.0330	0.0054	0.0054	0.0054	0.0043	0.0051	0.0028	0.0117	0.0100	0.0065	0.0016	0.0112	0.0080
21	0.0255	0.0250	0.0711	0.0672	0.0147	0.0146	0.0274	0.0260	0.0055	0.0054	0.0054	0.0036	0.0054	0.0029	0.0110	0.0094	0.0068	0.0016	0.0107	0.0078
22	0.0265	0.0264	0.0616	0.0588	0.0128	0.0127	0.0382	0.0367	0.0053	0.0052	0.0052	0.0047	0.0050	0.0028	0.0118	0.0102	0.0067	0.0016	0.0110	0.0079
23	0.0247	0.0244	0.0766	0.0724	0.0138	0.0136	0.0320	0.0313	0.0052	0.0052	0.0052	0.0058	0.0049	0.0028	0.0121	0.0104	0.0059	0.0015	0.0119	0.0086
24	0.0245	0.0240	0.0746	0.0705	0.0138	0.0137	0.0331	0.0315	0.0052	0.0052	0.0052	0.0057	0.0049	0.0028	0.0121	0.0104	0.0068	0.0016	0.0109	0.0078
25	0.0258	0.0248	0.0651	0.0615	0.0129	0.0127	0.0345	0.0327	0.0052	0.0052	0.0052	0.0060	0.0051	0.0028	0.0117	0.0101	0.0069	0.0016	0.0107	0.0077
26	0.0205	0.0199	**0.1650**	0.1381	0.0124	0.0121	**0.0493**	0.0466	0.0046	0.0046	0.0046	0.0062	0.0036	0.0024	0.0187	0.0167	0.0037	0.0013	0.0163	0.0124
27	0.0215	0.0211	**0.1240**	0.1103	0.0111	0.0111	**0.0706**	0.0654	0.0046	0.0046	0.0046	0.0071	0.0034	0.0024	0.0215	0.0194	0.0038	0.0013	0.0159	0.0123
28	0.0224	0.0215	**0.1085**	0.0974	0.0114	0.0112	**0.0582**	0.0548	0.0045	0.0045	0.0045	0.0005	0.0036	0.0024	0.0186	0.0166	0.0039	0.0013	0.0156	0.0119
29	0.0222	0.0223	**0.1292**	0.1125	0.0116	0.0113	**0.0600**	0.0558	0.0045	0.0045	0.0045	0.0082	0.0033	0.0023	0.0221	0.0199	0.0036	0.0013	0.0164	0.0127
30	0.0196	0.0194	**0.2510**	0.1932	0.0105	0.0102	**0.0981**	0.0882	0.0043	0.0043	0.0043	0.0056	0.0027	0.0022	0.0346	0.0318	0.0023	0.0012	0.0247	0.0205

The s parameter values of the items measuring three to four attributes are further away from zero than others (shown in the table as bold.) for the samples sizes of 50 and 100. For these two samples, the parameter values deviated from zero when the number of attributes measured by item increased.

Underspecified Q-matrix Case for 30 Item Condition

The other underspecified Q-matrix in this study is the $Q1_2$ matrix. In order to facilitate comparability with the 15-item condition, while this matrix is being constructed, the cells to be made misspecification were randomly determined from the cells in the first 15 items. There were no misspecifications between the first 15-item group and the second 15-item group having same attribute pattern. The $Q1_2$ matrix was constructed by making misspecification in 12 cells for the following cases; based on measuring the first attribute for Items 7, 12, 13, and 15; based on measuring the second attribute for Items 5, 8, and 14; based on measuring the third attribute for Items 6, 11, 13, and 15; and based on measuring the fourth attribute for Item 9 (see Appendix 2). The averages of the parameters estimated by using matrix $Q1_2$ are presented in Table 5.

When Table 5 is examined and the related values are compared with the values in Table 4, the s parameters values of the underspecified items increase highly in all sample size conditions when 30 items are included in the measurement tool. The standard error values concerning the s parameters for these items have also increased somewhat. Nevertheless, the increases in the s parameters and standard error values do not show a regular variation on the basis of sample size. In other words, it cannot be said that the increase in the parameter values increases or decreases as the sample size expands. For this 30-item condition, there have been seen very small variations in the g parameters of all items, and also in the s parameters of the correctly specified items compared to the estimations made with the correct Q-matrix.

Overspecified Q-matrix Case for 30 Item Condition

The other overspecified Q-matrix in this study is the $Q2_2$ matrix. $Q2_2$ matrix 2 was constructed by making misspecification in 12 cells for the following cases; based on measuring the first attribute for Items 4, 10, and 14; based on measuring the second attribute for Item 13; based on measuring the third attribute for Items 4, 5, 7, and 12; and based on measuring the fourth attribute for Items 1, 2, and 11 (see Appendix 2). The averages of the parameters estimated by using matrix $Q2_2$ are presented in Table 6.

When Table 6 is examined and the related values are compared with the values in Table 4 the g parameters of the overspecified items increased in all sample size cases in the 30-item condition. There has been an increase in the standard error values of these items. The increases in the g parameters and standard errors do not show a regular variation on the basis of sample size. In addition, when the sample size is 50 and 100, there has been an increase in the s parameters and standard errors of some of the misspecified items. The increase in the s parameter for 50 participants is higher than for 100 participants.

Table 5. *Parameters estimated through Q1₂ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
I	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s
1	0.0323	0.0310	0.0427	0.0402	0.0209	0.0204	0.0182	0.0176	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.0338	0.0324	0.0398	0.0380	0.0196	0.0198	0.0180	0.0174	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0403	0.0386	0.0335	0.0328	0.0230	0.0228	0.0161	0.0155	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.0412	0.0392	0.0351	0.0345	0.0187	0.0182	0.0203	0.0201	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
5	0.0328	0.0324	**0.5228**	0.1027	0.0206	0.0200	**0.4992**	0.0669	0.0001	0.0001	**0.5183**	0.0043	0.0001	0.0001	**0.5182**	0.0043	0.0000	0.0000	**0.4871**	0.0032
6	0.0323	0.0316	**0.4304**	0.1018	0.0212	0.0209	**0.3749**	0.0652	0.0001	0.0001	**0.4963**	0.0043	0.0001	0.0001	**0.4964**	0.0042	0.0000	0.0000	**0.4553**	0.0032
7	0.0400	0.0382	**0.4797**	0.0920	0.0192	0.0194	**0.5105**	0.0706	0.0001	0.0001	**0.4706**	0.0046	0.0001	0.0001	**0.4705**	0.0045	0.0000	0.0000	**0.5066**	0.0031
8	0.0414	0.0398	**0.5999**	0.0865	0.0233	0.0231	**0.5079**	0.0636	0.0001	0.0001	**0.5312**	0.0045	0.0001	0.0001	**0.5312**	0.0044	0.0000	0.0000	**0.5517**	0.0031
9	0.0349	0.0335	**0.5215**	0.0966	0.0200	0.0197	**0.4629**	0.0680	0.0001	0.0001	**0.4872**	0.0045	0.0001	0.0001	**0.4872**	0.0046	0.0000	0.0000	**0.4480**	0.0031
10	0.0260	0.0254	0.0655	0.0603	0.0131	0.0130	0.0361	0.0342	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001
11	0.0237	0.0230	**0.5794**	0.1353	0.0133	0.0131	**0.3456**	0.0876	0.0001	0.0001	**0.4696**	0.0062	0.0001	0.0001	**0.4699**	0.0061	0.0000	0.0000	**0.5000**	0.0043
12	0.0238	0.0233	**0.4621**	0.1318	0.0135	0.0133	**0.5653**	0.0914	0.0001	0.0001	**0.4335**	0.0064	0.0001	0.0001	**0.4334**	0.0064	0.0000	0.0000	**0.5128**	0.0043
13	0.0386	0.0365	**0.7227**	0.0815	0.0187	0.0181	**0.6854**	0.0651	0.0001	0.0001	**0.7478**	0.0040	0.0001	0.0001	**0.7479**	0.0039	0.0000	0.0000	**0.7431**	0.0027
14	0.0256	0.0254	**0.5323**	0.1263	0.0131	0.0131	**0.4301**	0.0917	0.0001	0.0001	**0.4575**	0.0064	0.0001	0.0001	**0.4576**	0.0064	0.0000	0.0000	**0.5454**	0.0042
15	0.0237	0.0228	**0.7676**	0.1121	0.0136	0.0133	**0.7017**	0.0828	0.0001	0.0001	**0.6830**	0.0059	0.0001	0.0001	**0.6833**	0.0060	0.0000	0.0000	**0.7712**	0.0036
16	0.0324	0.0308	0.0428	0.0411	0.0212	0.0204	0.0178	0.0185	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
17	0.0352	0.0336	0.0402	0.0387	0.0204	0.0198	0.0186	0.0177	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
18	0.0412	0.0401	0.0338	0.0320	0.0242	0.0231	0.0164	0.0163	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
19	0.0403	0.0397	0.0348	0.0338	0.0188	0.0184	0.0192	0.0191	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000
20	0.0242	0.0237	0.0829	0.0758	0.0134	0.0131	0.0344	0.0326	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
21	0.0251	0.0248	0.0719	0.0671	0.0147	0.0147	0.0282	0.0273	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
22	0.0260	0.0256	0.0616	0.0591	0.0128	0.0128	0.0380	0.0367	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001
23	0.0248	0.0243	0.0782	0.0729	0.0135	0.0131	0.0319	0.0307	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001
24	0.0248	0.0241	0.0755	0.0704	0.0134	0.0132	0.0336	0.0324	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001
25	0.0262	0.0255	0.0664	0.0623	0.0132	0.0130	0.0352	0.0334	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001
26	0.0209	0.0204	0.1649	0.1408	0.0118	0.0120	0.0500	0.0483	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0002	0.0002
27	0.0221	0.0219	0.1294	0.1135	0.0108	0.0107	0.0711	0.0647	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0001	0.0001
28	0.0221	0.0217	0.1099	0.1002	0.0117	0.0115	0.0579	0.0552	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0001	0.0001
29	0.0219	0.0213	0.1253	0.1099	0.0115	0.0112	0.0593	0.0554	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0003	0.0003	0.0000	0.0000	0.0002	0.0002
30	0.0204	0.0201	0.2481	0.1953	0.0104	0.0104	0.0981	0.0902	0.0000	0.0000	0.0005	0.0005	0.0000	0.0000	0.0005	0.0005	0.0000	0.0000	0.0003	0.0003

Table 6. *Parameters estimated through the misspecified Q2₂ matrix*

	50 Ps				100 Ps				250 Ps				500 Ps				1000 Ps			
I	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s	g	SE_g	s	SE_s
1	**0.2099**	0.0651	0.0619	0.0580	**0.3977**	0.0557	0.0387	0.0372	**0.3957**	0.0036	0.0002	0.0002	**0.3778**	0.0027	0.0001	0.0001	**0.3219**	0.0024	0.0001	0.0001
2	**0.3918**	0.0713	**0.1214**	0.1061	**0.4560**	0.0534	**0.0700**	0.0649	**0.3843**	0.0033	0.0003	0.0003	**0.3023**	0.0025	0.0001	0.0001	**0.4136**	0.0024	0.0001	0.0002
3	0.0419	0.0400	0.0328	0.0326	0.0235	0.0226	0.0169	0.0164	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	**0.4684**	0.0730	**0.1096**	0.0971	**0.4023**	0.0518	0.0600	0.0560	**0.4045**	0.0033	0.0003	0.0003	**0.3956**	0.0028	0.0001	0.0000	**0.4670**	0.0024	0.0001	0.0001
5	**0.1458**	0.0496	**0.1669**	0.1388	**0.1202**	0.0349	0.0507	0.0484	**0.1442**	0.0024	0.0003	0.0003	**0.1391**	0.0017	0.0001	0.0000	**0.1481**	0.0017	0.0002	0.0002
6	0.0250	0.0239	0.0707	0.0662	0.0145	0.0144	0.0279	0.0265	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
7	**0.1780**	0.0553	0.1089	0.0971	**0.1150**	0.0341	0.0595	0.0556	**0.1501**	0.0024	0.0003	0.0003	**0.1737**	0.0022	0.0000	**0.0001**	**0.1487**	0.0017	0.0001	0.0001
8	0.0244	0.0235	0.0779	0.0718	0.0138	0.0135	0.0317	0.0303	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
9	0.0240	0.0236	0.0752	0.0676	0.0133	0.0131	0.0336	0.0327	0.0001	0.0001	0.0002	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
10	**0.1551**	0.0527	0.1118	0.0985	**0.1372**	0.0367	0.0591	0.0542	**0.1319**	0.0023	0.0003	0.0003	**0.1446**	0.0035	0.0001	0.0001	**0.1580**	0.0018	0.0001	0.0001
11	0.0592	0.0324	**0.2473**	0.1914	**0.1167**	0.0338	0.0989	0.0885	**0.0693**	0.0017	0.0005	0.0005	**0.0713**	0.0045	0.0000	0.0000	**0.0715**	0.0012	0.0003	0.0003
12	**0.0992**	0.0415	**0.2472**	0.1882	0.0530	0.0228	0.1012	0.0909	**0.0650**	0.0016	0.0005	0.0005	**0.0610**	0.0050	0.0000	0.0001	**0.0746**	0.0012	0.0003	0.0003
13	**0.1195**	0.0459	**0.2502**	0.1917	**0.0845**	0.0285	0.1014	0.0905	0.0477	0.0014	0.0005	0.0005	**0.0878**	0.0051	0.0000	0.0001	**0.0821**	0.0013	0.0003	0.0003
14	**0.1008**	0.0425	**0.2501**	0.1916	**0.0851**	0.0291	0.0995	0.0919	**0.0563**	0.0015	0.0005	0.0005	**0.0584**	0.0051	0.0002	0.0001	**0.0671**	0.0011	0.0003	0.0003
15	0.0197	0.0191	0.2473	0.1956	0.0108	0.0106	0.0999	0.0888	0.0000	0.0000	0.0005	0.0005	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0003	0.0003
16	0.0330	0.0314	0.0455	0.0431	0.0231	0.0227	0.0202	0.0198	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
17	0.0378	0.0358	0.0463	0.0431	0.0215	0.0210	0.0201	0.0194	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
18	0.0424	0.0394	0.0331	0.0319	0.0230	0.0227	0.0172	0.0168	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
19	0.0441	0.0418	0.0399	0.0378	0.0207	0.0199	0.0227	0.0215	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0005	0.0001	0.0001	0.0001	0.0000	0.0000
20	0.0241	0.0232	0.0830	0.0746	0.0129	0.0125	0.0348	0.0327	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
21	0.0247	0.0245	0.0702	0.0664	0.0143	0.0138	0.0276	0.0270	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
22	0.0261	0.0255	0.0619	0.0583	0.0130	0.0126	0.0387	0.0365	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001
23	0.0241	0.0239	0.0777	0.0728	0.0142	0.0142	0.0328	0.0322	0.0001	0.0001	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001
24	0.0246	0.0236	0.0766	0.0719	0.0135	0.0131	0.0331	0.0314	0.0001	0.0001	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
25	0.0265	0.0263	0.0663	0.0625	0.0132	0.0129	0.0356	0.0347	0.0001	0.0001	0.0002	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001
26	0.0209	0.0205	0.1670	0.1433	0.0116	0.0116	0.0503	0.0474	0.0000	0.0000	0.0003	0.0001	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0002	0.0002
27	0.0219	0.0217	0.1246	0.1108	0.0113	0.0115	0.0692	0.0639	0.0000	0.0000	0.0003	0.0002	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0002	0.0001
28	0.0219	0.0212	0.1125	0.1005	0.0115	0.0113	0.0596	0.0569	0.0000	0.0000	0.0003	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0001
29	0.0219	0.0209	0.1262	0.1125	0.0113	0.0109	0.0576	0.0541	0.0000	0.0000	0.0003	0.0001	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0002	0.0002
30	0.0203	0.0201	0.2464	0.1906	0.0106	0.0106	0.0992	0.0891	0.0000	0.0000	0.0005	0.0003	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0003	0.0003

RESULT, DISCUSSION AND IMPLICATIONS

This study investigated the effect the Q-matrix Misspecification ($Q1_1$, $Q2_2$, $Q2_1$, and $Q2_2$) on the estimation of parameters s and g in differing sample sizes (50, 100, 250, 500, and 1,000 participants) and test length (15 and 30 items) for the DINA model.

The fact that the parameters g and s are zero in the DINA model points to excellent concordance between data and model, the chosen Q-matrix is correct, all the attributes measured through the measurement tool are correctly specified, and the responses in the dataset fit the model (de la Torre, 2009b; de la Torre & Douglas, 2004; Li, 2008; Rupp & Templin, 2008a). At near-zero values, the model contains incompatible responses. When the parameters are quite a bit higher than zero, there seems to be a problem in determining the measured attributes or defining the Q-matrix. The model concordance is low and the measured attributes are insufficient to explain the participants response patterns. In other words, responding to the items requires a strategy other than the defined attributes (de la Torre & Douglas, 2004).

The values of g and s parameters estimated using the correct Q-matrix are fairly close to zero as they are defined in the generation of the data. However, as the sample size decreases, the parameter estimations (particularly the s parameter) and the standard error values for some items deviate from zero. When the number of the participants increases, this situation disappears, the parameter, and standard error values approach zero. While the estimation values made by the correct Q-matrix for the 30-item condition are quite close to zero for all sample sizes, the parameter values for the 15-item condition deviate from the value in the correct Q-matrix as the number of participants decreases.

The estimations are accurate based on the proximity to zero degree of the parameters and their standard errors (Rupp et al., 2010). All the estimations computed by the correct Q-Matrix are close to zero for all cases. As the sample size and test length expand, the standard error values approach zero.

When the value 1 is transformed to the value 0 (zero) in an item in the Q-matrix, the s parameter is estimated at a higher value than its actual value while the g parameter is correct. That occurs because the item in the Q-matrix is coded to miss an attribute, the items become easier to identify, but the response pattern contains more false answers (Rupp & Templin, 2008a). The underspecified Q-matrices caused the s parameter of items to be higher than zero in the conditions examined in the study.

When the value is replaced with the value 0 in an item in the Q-matrix, the s parameter becomes correct and the g parameter is estimated at a higher level than its actual value. When an item is incorrectly coded to require an additional attribute, since the analysis uses data generated in such a way that an attribute is missing based on the original coding of the Q-matrix, the item was harder coded than usual (Rupp & Templin, 2008a). The overspecification of the Q-matrix caused the g parameters of the items to be higher than zero in the conditions examined in the study.

If one of the g and s parameters is high, that is an indicator that the mode is not well-defined and that the item does not fit well into the data (item-based model discordance) (Rupp & Templin, 2008a). The misspecifications made by the correct Q-matrix caused the relevant item not to fit well with the data. Due to the underspecifications, there has been an increase in the s parameters and standard errors. In the case of overspecification, there has been an increase in the g parameters and standard errors. These increases are regularly not differentiated on the basis of sample size. These parameter values increased in all sample size cases. However, the increase in sample size made it possible to decrease standard errors of the parameters, in other words, to increase the accuracy of the parameter estimations. As the sample size increases, the number of individuals in latent classes that represent different levels of competence at the same rate increases, and the number of individuals who will give wrong answers to the item they do not have the missing attributes, or vice versa. MacDonald (2013) stated that the parameter estimations didn't differentiate on the basis of sample size when the Q-matrix was misspecified. The study by Qin et al. (2015) showed that when the sample size increases, the accuracy also increases as well, and when the sample is 500 and the number of the measured attribute and the number of misspecified items were low, the results were completely compatible with the correct Q-matrix; on the other hand, it is stated that even if the number increases, the concordance maintains.

The increase in the number of items advances the accuracy of parameter estimation (Cassuto, 1996; Henson & Templin, 2006; Kunina-Habenicht et al., 2012; MacDonald, 2013; Ömür-Sünbül & Kan, 2015; Tatsuoka, 1990). The examined cases showed that the parameter estimations which were computed by using the misspecified Q-matrix differed more than those calculated by using the correct Q-matrix when the test was shortened. Contrary to expectation, when the number of items in the measurement tool decreases, the s parameters in the underspecified items, and the g parameters of the items which were not underspecified, increases. In overspecified cases, there has been an increase in the parameters of some items.

In conclusion, the findings of the present study reveal that the parameter estimations are affected by the misspecification of the Q-matrix. The increases in the s parameters of underspecified items are considerably higher than those in the g parameters of overspecified items. The misspecifications under the 30-item condition generally have a regional effect on parameter estimations. The regional effect refers to the increase of the g and s parameters of the misspecified items based on the misspecification. When the number of items are decreased, there has also been an increase in the parameter estimations of the items which are not misspecified. That leads to eliminate the regional effect of misestimating the parameters to differentiate the parameter estimations of the items which were not misspecified. The number of items in the measurement tool affects the influence of the parameter values of items which were not misspecified from the Q-matrix misspecification. The decrease in both sample size and the number of items in the measurement tool increases the error ratio of the parameter estimations.

Based on the findings of the present study, it is recommended that researchers should conduct studies with a sample size of 250 or more, increase the number of items, write more than one item for the same skill pattern, check whether or not they have made any mistake about the identification of the attributes measured with this item when the estimated parameter values become different from zero, present the data for expert review in order to increase the accuracy of the item parameter estimation of the researchers.

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APPENDIX

Appendix I

Correct Q0₁ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	1	0	0
6	1	0	1	0
7	1	0	0	1
8	0	1	1	0
9	0	1	0	1
10	0	0	1	1
11	1	1	1	0
12	1	1	0	1
13	1	0	1	1
14	0	1	1	1
15	1	1	1	1

Appendix II

Misspecified Q1₁ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	0	0	0
6	1	0	1	0
7	1	0	0	1
8	0	1	1	0
9	0	0	0	1
10	0	0	1	0
11	1	1	0	0
12	1	0	0	1
13	1	0	1	1
14	0	1	0	1
15	1	1	1	1

Misspecified Q2₁ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	0
2	0	1	1	0
3	0	1	1	0
4	0	0	0	1
5	1	1	0	0
6	1	0	1	0
7	1	0	0	1
8	0	1	1	0
9	1	1	1	1
10	0	0	1	1
11	1	1	1	1
12	1	1	1	1
13	1	0	1	1
14	0	1	1	1
15	1	1	1	1

Correct Q0₂ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	1	0	0
6	1	0	1	0
7	1	0	0	1
8	0	1	1	0
9	0	1	0	1
10	0	0	1	1
11	1	1	1	0
12	1	1	0	1
13	1	0	1	1
14	0	1	1	1
15	1	1	1	1
16	1	0	0	0
17	0	1	0	0
18	0	0	1	0
19	0	0	0	1
20	1	1	0	0
21	1	0	1	0
22	1	0	0	1
23	0	1	1	0
24	0	1	0	1
25	0	0	1	1
26	1	1	1	0
27	1	1	0	1
28	1	0	1	1
29	0	1	1	1
30	1	1	1	1

Misspecified Q1₂ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1
5	1	0	0	0
6	1	0	0	0
7	0	0	0	1
8	0	0	1	0
9	0	1	0	0
10	0	0	1	1
11	1	1	0	0
12	0	1	0	1
13	0	0	0	1
14	0	0	1	1
15	0	1	0	1
16	1	0	0	0
17	0	1	0	0
18	0	0	1	0
19	0	0	0	1
20	1	1	0	0
21	1	0	1	0
22	1	0	0	1
23	0	1	1	0
24	0	1	0	1
25	0	0	1	1
26	1	1	1	0
27	1	1	0	1
28	1	0	1	1
29	0	1	1	1
30	1	1	1	1

Misspecified Q2₂ Matrix

	Attribute			
Item	1	2	3	4
1	1	0	0	1
2	1	1	0	1
3	0	0	1	0
4	1	0	1	1
5	1	1	1	0
6	1	0	1	0
7	1	0	1	1
8	0	1	1	0
9	0	1	0	1
10	1	0	1	1
11	1	1	1	1
12	1	1	1	1
13	1	1	1	1
14	1	1	1	1
15	1	1	1	1
16	1	0	0	0
17	0	1	0	0
18	0	0	1	0
19	0	0	0	1
20	1	1	0	0
21	1	0	1	0
22	1	0	0	1
23	0	1	1	0
24	0	1	0	1
25	0	0	1	1
26	1	1	1	0
27	1	1	0	1
28	1	0	1	1
29	0	1	1	1
30	1	1	1	1