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Erratum to "On Convolution surfaces in Euclidean spaces"

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Abstract

In the present study we give some corrections for our paper which published in the first volume of this journal.

1. Erratum to "On Convolution surfaces in Euclidean spaces"

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Theorem 3.2. Let $M\star N$ be a convolution surface of a paraboloid M and a translation surface N given with the parametrization (3.4). Then the Gaussian curvature of the convolution surface is

$$K_{M\star N}=\frac{4cf''g''}{(f''+2)\left(g''+2c\right)\left((f')^2+(g')^2+1\right)^2}.$$

Proof. Let $M\star N$ be a convolution surface of a paraboloid M and a translation surface N given with the parametrization (3.4) For simplicity we define $z=x+y$. Then the tangent space of $M\star N$ is spanned by

$$\begin{aligned} z_s &= \frac{1}{2}\left(f''+2,0,f'\left(f''+2\right)\right), \\ z_t &= \frac{1}{2c}\left(0,g''+2c,g'\left(g''+2c\right)\right). \end{aligned}$$

Hence the coefficients of first and second fundamental forms of the convolution surface $M\star N$ are

$$E=\langle z_s,z_s\rangle=\frac{1}{4}\left(\left(f'\right)^2+1\right)\left(f''+2\right)^2,$$

$$F=\langle z_s,z_t\rangle=\frac{f'g'}{4c}\left(f''+2\right)\left(g''+2c\right), \tag{3.5}$$

$$G=\langle z_t,z_t\rangle=\frac{1}{4c^2}\left(\left(g'\right)^2+1\right)\left(g''+2c\right)^2,$$

and

$$e=\frac{\langle z_{ss},z_s\times z_t\rangle}{\sqrt{EG-F^2}}=\frac{f''\left(g''+2c\right)\left(f''+2\right)^2}{8c\sqrt{EG-F^2}},$$

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$$f=\frac{\langle z_{st},z_s\times z_t\rangle}{\sqrt{EG-F^2}}=0, \tag{3.6}$$

$$e=\frac{\langle z_{tt},z_s\times z_t\rangle}{\sqrt{EG-F^2}}=\frac{g''(f''+2)\,(g''+2c)^2}{8c^2\sqrt{EG-F^2}},$$

respectively. By definition the Gaussian curvature of the convolution surface $M\star N$ is given by

$$K_{M\star N}=\frac{eg-f^2}{EG-F^2}. \tag{3.7}$$

So, substituting (3.5) and (3.6) into (3.7) after some calculation we get the result.□

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As a consequence of previous theorem one can get the following results.

Corollary 3.3. Let $M\star N$ be a convolution surface of a paraboloid M and a translation surface (3.4). If the convolution $M\star N$ is a flat surface, then at least one of the following cases occur;

$$f(s)=a_1s+a_2, \text{ or } g(t)=b_1t+b_2,$$

where a_i and b_j are real constants.

Corollary 3.4. The convolution surface $M\star N$ given with the parametrization $f(s)=a_1s+a_2$ and $g(t)=b_1t+b_2$ is a part of a plane.

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Finally,convolution surface $M\star N$ has the parametrization

$$(x+y)(s,t)=(\frac{h'+2ff'}{2f'}\cos t,\frac{h'+2ff'}{2cf'}\sin t,\frac{(h')^2}{4c\,(f')^2}(c\cos^2 t+\sin^2 t)+h(s)). \tag{3.10}$$

Theorem 3.5. Let $M\star N$ be a convolution surface of a paraboloid M and a surface of revolution given with the parametrization (3.8). If $c=1$ then the convolution surface $M\star N$ also a surface of revolution with Gaussian curvature

$$K_{M\star N}=\frac{(\varphi^2+h)'\left\{(\varphi^2+h)''(\varphi+h)'-(\varphi^2+h)'(\varphi+h)''\right\}}{(\varphi+f)\left\{((\varphi^2+h)')^2+((\varphi+h)')^2\right\}^2};\ \ f^{'}\neq 0, \tag{3.11}$$

where $\varphi(s)=\frac{h'(s)}{2f'(s)}$ is a real valued differentiable function different from 1.

Proof. Similar to the proof of Theorem 3.2 we get the result. □

Corollary 3.6. Let $M\star N$ be a convolution surface of a paraboloid M with $c=1$ and a surface of revolution (3.8). If the convolution surface $M\star N$ is a flat surface, then it is either a plane or a surface of revolution satisfying

$$(\varphi^2+h)''(\varphi+h)'-(\varphi^2+h)'(\varphi+h)''=0.$$

Proof. If $M\star N$ is a flat surface, then

$$(\varphi^2+h)'\left\{(\varphi^2+h)''(\varphi+h)'-(\varphi^2+h)'(\varphi+h)''\right\}=0 \tag{3.12}$$

holds. So, we have two possible cases; $\varphi^2+h=const.$, or $(\varphi^2+h)''(\varphi+h)'-(\varphi^2+h)'(\varphi+h)''=0$. For the first case $M\star N$ is a part of a plane □.

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Omit the Equation 3.14.
Finally, the sum $M\star N$ has the parametrization

$$(x+y)(s,t)=\begin{pmatrix}\left(\frac{2tp(s)-z'(s)}{2t}\right)\sin s+(p'(s)+t)\cos s\\ \left(\frac{z'(s)-2ctp(s)}{2ct}\right)\cos s+(p'(s)+t)\sin s\\ z(s)+\left(\frac{z'(s)^2}{4ct^2}\right)\left(c\sin^2 s+\cos^2 s\right)\end{pmatrix},\ t\neq 0. \tag{3.15}$$

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Theorem 3.7. Let $M \star N$ be a convolution surface of a paraboloid M with $c = 1$ and a right helicoid N given with the parametrization (3.17). Then the Gaussian curvature of the convolution surface is

$$K_{M\star N} = -\frac{\psi''\left\{(\psi'(t-k)t + \psi(\psi\psi' + k)\right\}(k-t) - \left\{\psi\psi' + (\psi')^2(k-t) + k\right\}^2}{\left\{(\psi')^2(k-t)^2 + (\psi\psi' + k)^2 + (\psi\psi' + t)^2\right\}^2};\ t \neq 0, \tag{3.18}$$

where

$$\psi(t) = \frac{-k}{2t},$$

is a real valued function.

Proof. Similar to the proof of Theorem 3.2 we get the result. □