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The Quantum Codes over F_q and Quantum Quasi-cyclic Codes over F_p

Yasemin Cengellenmis and Abdullah Dertli*

Abstract
In this paper, the quantum codes over F_q are constructed by using the cyclic codes over the finite ring $R = F_q + vF_q + ... + v^{m-1}F_q$, where p is prime, $q = p^s$, $m-1|p-1$ and $v^m = v$. The parameters of quantum error correcting codes over F_q are obtained. Some examples are given. Morever, the quantum quasi-cyclic codes over F_p are obtained, by using the self dual basis for F_{p^s} over F_p.

Keywords: *Cyclic codes; Quasi-cyclic codes; Quantum codes.*

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****Corresponding author***

1. Introduction

The theory quantum error correcting codes has differences from the theory classical error correcting codes. But Calderbank et al. gave a way to construct quantum error correcting codes from classical error correcting codes in [4].

Many good quantum codes has been constructed by using the classical cyclic codes over F_q with self orthogonal (or dual containing) properties. Recently, some authors have constructed quantum the codes by using the linear codes over some finite ring in [1-4,8-10,12-18].

In 2015, Gao constructed the quantum codes over F_q from the cyclic codes over a finite non chain ring $F_q + vF_q + v^2F_q + v^3F_q$, where $q = p^r$, p is an odd prime, $3|p-1$ and $v^4 = v$ in [8]. In 2016, Sari and Siap constructed the quantum codes over F_p from the cyclic codes of arbitrary length over $F_p + vF_p + ... + v^{p-1}F_p$, where $v^p = v$ and p is a prime in [13].

In [11], Qian et al. gave a method for constructing the self orthogonal quasi-cyclic codes and obtained a large number of new quantum quasi-cyclic codes by CSS construction.

Our aim in this paper is firstly to construct the quantum codes over F_q by using the cyclic codes over the finite ring $R = F_q + vF_q + ... + v^{m-1}F_q$, where p is a prime, $q = p^s$, $m-1|p-1$, $v^m = v$ and later to obtained the parameters of the quantum quasi-cyclic codes over F_p, by using the self dual basis for F_{p^s} over F_p.

This paper is organized as follows. In section 2, some properties of the finite ring R are given. In section 3, a sufficient and necessary condition for the cyclic codes over R that contains its dual is given. The parameters of quantum error correcting codes are obtained from the cyclic codes over R and some examples are given. In section 4, by taking $m = 3$, the parameters of the quantum quasi-cyclic codes over F_p are determined.

2. Preliminaries

In [12], Shi and Yao give the following properties of the finite ring $R = F_q + vF_q + ... + v^{m-1}F_q = F_q[v]/\langle v^m - v\rangle$, where p is a prime $q = p^s$, $m-1|p-1$ and $v^m = v$.

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As $m-1|p-1$, this shows that $v^m - v = v(v-v_1)(v-v_2).....(v-v_{m-1})$ with all v_i's in F_q. Let $f_i = v - v_i$ and $\hat{f}_i = (v^m - v)/f_i$ where $i = 0, ..., m-1$, then there exist $a_i, b_i \in F_q[v]$ such that $a_i f_i + b_i \hat{f}_i = 1$. Let $e_i = b_i \hat{f}_i$, then $e_i^2 = e_i$, and $e_i e_j = 0$, $\sum_{i=0}^{m-1} e_i = 1$, where $i, j = 0, 1, ..., m-1$ and $i \neq j$. So

$$R = e_0 R \oplus e_1 R \oplus ... \oplus e_{m-1} R = e_0 F_q \oplus ... \oplus e_{m-1} F_q$$

and

$$R \cong R/\langle v \rangle \times .... \times R/\langle v - v_{m-1} \rangle \cong F_q \times ... \times F_q$$

They express any $r \in R$ uniquely as

$$r = e_0 r_0 + ... + e_{m-1} r_{m-1},$$

where $r_i \in F_q$ for $i = 0, ..., m-1$ in [12].

Example 2.1. For $q = p = 3$ and $m = 3$, the three idempotents are $e_0 = 1 - v^2, e_1 = 2v^2 + 2v, e_2 = 2v^2 + v$.

A linear code C over R length n is an $R-$submodule of R^n. An element of C is called a codeword.

By defining the set

$$C_i = \{x_i \in F_q^n | \exists x_0, ..., x_{i-1}, x_{i+1}, ..., x_{m-1} \in F_q^n, e_0 x_0 + ... e_{m-1} x_{m-1} \in C\}$$

where $i = 0, 1, ..., m-1$, they represent the linear code C of length n over R as

$$C = e_0 C_0 \oplus .... \oplus e_{m-1} C_{m-1}$$

where C_i are the linear codes over F_q, for $i = 0, ..., m-1$ in [12].

If G is a generator matrix of C over R, then the generator matrix G is expressed as

$$\mathbf{G} = \begin{pmatrix} e_0 G_0 \\ ... \\ e_{m-1} G_{m-1} \end{pmatrix}$$

where $G_0, ..., G_{m-1}$ are the generator matrices of $C_0, ..., C_{m-1}$ in [12].

For any $x = (x_0, x_1, ..., x_{n-1})$, $y = (y_0, y_1, ..., y_{n-1}) \in R^n$, the inner product is defined as

$$x.y = \sum_{i=0}^{n-1} x_i y_i$$

If $x.y = 0$, then x and y are said to be orthogonal. Let C be a linear code of length n over R, the dual code of C

$$C^\perp = \{x : \forall y \in C, x.y = 0\}$$

which is also a linear code over R of length n. A code C is self orthogonal if $C \subseteq C^\perp$ and self dual if $C = C^\perp$.

A code C over R is a linear code with the property that if every $c = (c_0, c_1, ..., c_{n-1}) \in C$, then $\sigma(c) = (c_{n-1}, c_0, ..., c_{n-2}) \in C$. A subset C of R^n is a linear cyclic code of length n iff its polynomial representation is an ideal of $R[x] / \langle x^n - 1 \rangle$.

Proposition 2.1. [12] *Let $C = e_0 C_0 \oplus .... \oplus e_{m-1} C_{m-1}$ be a linear code of length n over R. Then*

$$C^\perp = e_0 C_0^\perp \oplus .... \oplus e_{m-1} C_{m-1}^\perp$$

Morever C is a self dual code over R if and only if $C_0, ..., C_{m-1}$ are all self dual codes over F_q.

In [12], they give a special class of Gray maps, which preserves the property of self dual of linear codes from the ring R to the finite field F_q, by using the group of invertible matrices of size m.

In [12], the Gray map Φ is defined as follows

$$\begin{aligned} \Phi &: R \to F_q^m \\ r = (r_0, ..., r_{m-1}) \mapsto \Phi((r_0, ..., r_{m-1})) &= (r_0, ..., r_{m-1}) M = rM \end{aligned}$$

for any matrix $M \in GL_m(F_q)$, where $GL_m(F_q)$ is the group of invertible matrices of size m and Φ is an F_q-module isomorphism.

The Gray map is extended as follows

$$\begin{aligned} \Phi &: R^n \to F_q^{mn} \\ c = (c_0, ..., c_{m-1}) \mapsto \Phi((c_0, ..., c_{m-1})) &= (c_0 M, ..., c_{m-1} M) \end{aligned}$$

Let C be a code over F_q of length n and $\acute{c} = (\acute{c}_0, \acute{c}_1, ..., \acute{c}_{n-1})$ be a codeword of C. The Hamming weight of $\acute{c}$ is defined as $w_H(\acute{c}) = \sum_{i=0}^{n-1} w_H(\acute{c}_i)$ where $w_H(\acute{c}_i) = 1$ if, $\acute{c}_i \neq 0$ and $w_H(\acute{c}_i) = 0$ if, $\acute{c}_i = 0$. The Hamming distance of C is defined as $d_H(C) = \min d_H(c, \acute{c})$, where for any $\acute{c} \in C$, $c \neq \acute{c}$ and $d_H(c, \acute{c})$ is the Hamming distance between two codewords with $d_H(c, \acute{c}) = w_H(c - \acute{c})$.

In [12],the Gray weight $w_G(r)$ of $r = (r_0, .., r_{m-1}) \in R$ is defined as the Hamming weight of the vector rM. For any vector $c = (c_0, ..., c_{n-1}) \in R^n$, the Gray weight of c is defined to be the rational sum of Gray weight of its components. For any elements $c_1, c_2 \in R^n$, the Gray distance between c_1 and c_2 is given by

$$d_G(c_1, c_2) = w_G(c_1 - c_2)$$

The minimum Gray weight of C is the smallest nonzero Gray weight among all codewords. If C is a linear code, then the minimum Gray distance is the same as the minimum Gray weight.

Lemma 2.1. [12] *If C is a linear code of length n over R, then $\Phi(C)$ is a linear code of length mn over F_q. Morever, the Gray map Φ is a distance-preserving map from C to $\Phi(C)$.*

Proposition 2.2. [12] *Let M be an invertible matrix of size m over F_q, let C be a linear code of length n with the minimum Gray distance d over R. If C has the generator matrix G as above and $|C| = p^{\sum_{i=0}^{m-1} k_i}$, then $\Phi(C)$ is a $[mn, \sum_{i=0}^{m-1} k_i, d]$ linear code over F_q, where k_i's are the respective dimensions of the C_i's.*

Proposition 2.3. [12] *Let C be a linear code of length n over R. Let $M \in GL_m(F_q)$ and $M.M^T = \lambda I_m$, where $\lambda \in F_q \setminus \{0\}$ and I_m be the identity matrix of size m over F_q. If C is a self dual code, then $\Phi(C)$ is a self dual code of length mn over F_q.*

Example 2.2. Let $q = p = 3$ and $m = 3$. By taking

$$\mathbf{M} = \begin{pmatrix} 111 \\ 012 \\ 011 \end{pmatrix}$$

the Gray map can is defined as follows

$$\begin{aligned} \Phi : \quad & F_3 + vF_3 + v^2 F_3 \to F_3^3 \\ a + bv + cv^2 \quad \longmapsto \quad & \Phi(a + bv + cv^2) = (a, a + b + c, a - b + c) \end{aligned}$$

It is easily seen that if C is self dual, so is $\Phi(C)$.

3. Quantum codes from the cyclic codes over R

Theorem 3.1. [5]*(CSS Construction) Let $C_1 = [n, k_1, d_1]_q$ and $C_2 = [n, k_2, d_2]_q$ be linear codes over GF(q) with $C_2 \subseteq C_1$. Then there exists a quantum error-correcting code $C = [[n, k_1 - k_2, min\{d_1, d_2^{\perp}\}]]_q$, where $d_2^{\perp}$ denotes the minimum Hamming distance of the dual code $C_2^{\perp}$ of C_2. Further, if $C_1^{\perp} = C_2$, then there exists a quantum error-correcting code $C = [[n, 2k_1 - n, d_1]]$.*

Proposition 3.1. *Let $C = e_0 C_0 \oplus .... \oplus e_{m-1} C_{m-1}$ be a linear code of length n over R, where C_i are the codes over F_q of length n , for $i = 0, ..., m - 1$. Then C is a cyclic code over R iff C_i are the cyclic codes over F_q, for $i = 0, ..., m - 1$.*

Proof. Let $(a_0^i, ..., a_{n-1}^i) \in C_i$, for $i = 0, 1, ..., m-1$. Assume that $m_j = e_0 a_j^0 + e_1 a_j^1 + ... + e_{m-1} a_j^{m-1}$, for $j = 0, ..., n-1$. Then $(m_0, ..., m_{n-1}) \in C$. Since C is a cyclic code, so $(m_{n-1}, m_0, ..., m_{n-2}) \in C$. Note that $(m_{n-1}, m_0, ..., m_{n-2}) = e_0(a_{n-1}^0, ..., a_{n-2}^0) + e_1(a_{n-1}^1, ..., a_{n-2}^1) + ... + e_{m-1}(a_{n-1}^{m-1}, ..., a_{n-2}^{m-1})$. Hence $(a_{n-1}^i, a_0^i ..., a_{n-2}^i) \in C_i$, for $i = 0, 1, ..., m-1$. So C_i are the cyclic codes over F_q for $i = 0, 1, ..., m - 1$.

Conversely, suppose that C_i are the cyclic codes over F_q, for $i = 0,1,...,m-1$. Let $(m_0,...,m_{n-1}) \in C$, where $m_j = e_0 a_j^0 + e_1 a_j^1 + ... + e_{m-1} a_j^{m-1}$, for $j = 0,...,n-1$. Then $(a_{n-1}^i, a_0^i ..., a_{n-2}^i) \in C_i$, for $i = 0,1,...,m-1$. Note that $(m_{n-1},...,m_{n-2}) = e_0(a_{n-1}^0,...,a_{n-2}^0) + e_1(a_{n-1}^1,...,a_{n-2}^1) + ... + e_{m-1}(a_{n-1}^{m-1},...,a_{n-2}^{m-1}) \in C = e_0 C_0 \oplus ..... \oplus e_{m-1} C_{m-1}$. Hence C is a cyclic code over R.

□

Proposition 3.2. *If $C = e_0 C_0 \oplus e_1 C_1 \oplus e_2 C_2 \oplus ... \oplus e_{m-1} C_{m-1}$ is a cyclic code of length n over R, then*

$$C = < e_0 g_0(x), ..., e_{m-1} g_{m-1}(x) >$$

and $|C| = q^{mn - (\deg g_0(x) + \deg g_1(x) + ... + \deg g_{m-1}(x))}$ where $g_0(x),...,g_{m-1}(x)$ are the generator polynomials of $C_0,...,C_{m-1}$ respectively.

Proposition 3.3. *Let $C = e_0 C_0 \oplus e_1 C_1 \oplus e_2 C_2 \oplus ... \oplus e_{m-1} C_{m-1}$ be a cyclic code of length n over R, then there exists a unique polynomial $g(x)$ such that $C = \langle g(x) \rangle$ and $g(x) \mid x^n - 1$, where $g(x) = e_0 g_0(x) + ... + e_{m-1} g_{m-1}(x)$ and $g_i(x)$ are the generator polynomials of cyclic codes C_i, for $i = 0,1,...,m-1$.*

Lemma 3.1. [5] *A cyclic code C over F_q with generator polynomial $g(x)$ contains its dual code iff*

$$x^n - 1 \equiv 0 \, (mod g(x) g^*(x))$$

where $g(x)^*$ is the reciprocal polynomial of $g(x)$.

Theorem 3.2. *Let $C = e_0 C_0 \oplus e_2 C_2 \oplus ... \oplus e_{m-1} C_{m-1}$ be a cyclic code of length n over R and $C = \langle g(x) \rangle$. Then $C^\perp \subseteq C$ iff*

$$x^n - 1 \equiv 0 \, (mod g_i(x) g_i^*(x))$$

for $i = 0,1,2,3,...,m-1$.

Proof. Let $x^n - 1 \equiv 0 \, (mod g_i(x) g_i^*(x))$ for $i = 0,1,2,3,...,m-1$. From the Lemma 2.1, we have $C_0^\perp \subseteq C_0, C_1^\perp \subseteq C_1, ..., C_{m-1}^\perp \subseteq C_{m-1}$. This shows that $e_i C_i^\perp \subseteq e_i C_i$, for $i = 0,1,..,m-1$. We have $C^\perp = e_0 C_0^\perp \oplus ... \oplus e_{m-1} C_{m-1}^\perp \subseteq C$, by using the Proposition 1.1.

Conversely, if $C^\perp \subseteq C$, then we have $e_i C^\perp = e_i C_i^\perp \subseteq e_i C = e_i C_i$, for any $i = 0,...,m-1$. So $C_i^\perp \subseteq C_i$, for $i = 0,...,m-1$. So from the Lemma 2.1, we get $x^n - 1 \equiv 0 \, (mod g_i(x) g_i^*(x))$, for $i = 0,1,2,3,...,m-1$. □

Theorem 3.3. *Let $C = e_0 C_0 \oplus ... \oplus e_{m-1} C_{m-1}$ be a cyclic code of length n over R and let the parameters of $\Phi(C)$ be $[mn, k, d]$, where d is the minimum Gray distance of C. If $C^\perp \subseteq C$, then there exists a quantum error correcting code with parameter $[[mn, 2k - mn, d]]$ over F_q.*

4. The Quantum Quasi-cyclic codes from the self orthogonal Quasi-cyclic codes over F_{p^s}

In this section, we take m as 3.

In [6], they focus on codes over the finite ring $S = F_q + vF_q + v^2 F_q$, where $v^3 = v$ and q is a prime power. A Gray map ϕ from S^n to F_q^{3n} is defined as follows;

$$\begin{aligned} \phi &: S \to F_q^3 \\ x = a_0 + va_1 + v^2 a_2 \mapsto \phi(x) &= (a_0, a_0 + a_2, a_1) \end{aligned}$$

where $x = a_0 + va_1 + v^2 a_2$, for $a_i \in F_q, i = 0,1,2$.

In [6], the Lee weight of the element of S is defined. They shown that the Gray map is a weight preserving map and if C is a linear code over S, the minimum Lee weight of C is the same as the minimum Hamming weight of $\phi(C)$ and if C is a self orthogonal code, so it $\phi(C)$.

Proposition 4.1. *Let σ be a cyclic shift. Then $\phi\sigma = \sigma^{\otimes 3}\phi$.*

Proof. Let $z = (z_0, z_1, ..., z_{n-1})$ be in S^n. Let $a_i, b_i, c_i, d_i \in F_q$, for $0 \le i \le n-1$ such that $z_i = a_i + b_i v + c_i v^2$. Then, $\sigma(z) = (z_{n-1}, z_0, z_1, ..., z_{n-2})$. From the definition of Gray map, we get $\phi\sigma(z) = (a_{n-1}, a_0, ..., a_{n-2}, a_{n-1} + c_{n-1}, a_0 + c_0, ..., a_{n-2} + c_{n-2}, b_{n-1}, b_0, ..., b_{n-2})$.

On the other hand, since $\phi(z) = (a_0, ..., a_{n-1}, a_0 + c_0, ..., a_{n-1} + c_{n-1}, b_0, ..., b_{n-1})$, by applying $\sigma^{\otimes 3}$, we have $\sigma^{\otimes 3}\phi(z) = (a_{n-1}, a_0, ..., a_{n-2}, a_{n-1} + c_{n-1}, a_0 + c_0, ..., a_{n-2} + c_{n-2}, b_{n-1}, b_0, ..., b_{n-2})$.

□

Theorem 4.1. *If C is a cyclic code of length n over S, then $\phi(C)$ is a quasi-cyclic code of index 3 with length 3n over F_q.*

Proof. Let C be a cyclic code over S. Then $\sigma(C) = C$, so $\phi(\sigma(C)) = \phi(C)$. It follows from the Proposition 3.1, that $\sigma^{\otimes 3}(\phi(C)) = \phi(C)$, which means that $\phi(C)$ is a quasi-cyclic code of index 3 with length 3n over F_q.

□

In [11], they give a sufficient and necessary condition for a one generator l-quasi-cyclic codes over F_q contains its dual. Morever, they give the following theorem.

Theorem 4.2. [11] *Let C be an $[n, k, d]$ quasi-cyclic code over F_q with generator of the form*

$$g(x) = (f_1(x)g_1(x), ..., f_l(x)g_l(x))$$

where $g_i(x)|x^n - 1$ and $(f_i(x), (x^m - 1)/g_i(x)) = 1$ for all $i = 1, 2, ..., l$, and for all $i = 1, 2, ..., l$,

$$x^m - 1 \equiv 0 (mod g_i(x)g_i^*(x))$$

Then $C^\perp \subseteq C$ and there exists a quantum QC code with $[[n, 2k - n, d]]$.

In order to obtain the parameters of the quantum quasi-cyclic codes over F_p via self dual basis, we give necessary some knowledges about self dual basis from [7].

Let p be a prime number and $q = p^s$, where s is a positive integer. The trace $Tr(\alpha)$ over F_p of an element $\alpha \in F_q$ is defined as

$$Tr(\alpha) = \sum_{i=0}^{s-1} \alpha^{p^i}$$

A basis $B = \{\alpha_1, ..., \alpha_s\}$ of F_q over F_p is trace-orthogonal basis if

$$Tr(\alpha_i\alpha_j) = \begin{cases} \text{nonzero}, & i = j \\ 0, & i \neq j \end{cases}$$

A trace-orthogonal basis is called a self dual basis if $Tr(\alpha_i^2)$=1, for $i = 1, ..., s$. In [7], it is shown that a self-dual basis exist if and only if p is even or p and s are both odd.

In this work, we take $q = p^s$, where p and s are both odd.

Let $B = \{\alpha_1, ..., \alpha_s\}$ be a self dual basis of F_{p^s} over F_p. Let C be a quasi-cyclic code over F_{p^s} of index 3 with length $3n$. For any $c = (c_1, ..., c_n) \in C$,

$$\begin{aligned} \psi \quad &: \quad F_{p^s}^{3n} \to F_p^{3ns} \\ c = (c_1, ..., c_n) \mapsto \psi(c) \quad &= \quad (c_{11}, c_{12}, ..., c_{(3n)1}, c_{12}, ...c_{(3n)2}, ..., c_{1s}, ...c_{(3n)s}) \end{aligned}$$

where $c_i = \sum_{j=1}^{s} c_{ij}\alpha_j$ and $c_{ij} \in F_p$, for $i = 1, ..., n$.

Lemma 4.1. *If C is a quasi-cyclic code of index 3 over F_{p^s} of length $3n$, then $\psi(C)$ is a quasi-cyclic code of index $3s$.*

Lemma 4.2. *If C is a self orthogonal code over F_{p^s} of length $3n$, so is $\psi(C)$.*

Theorem 4.3. *If C is a self orthogonal quasi-cyclic code over F_{p^s} with the parameter $[3n, k, d]$, the $\psi(C)$ is also a self orthogonal quasi-cyclic code over F_p with the parameter $[3ns, sk, d' \geq d]$.*

Theorem 4.4. *Let C be a quasi-cyclic code over F_{p^s} with the parameter $[3n,k,d]$ and $C^{\perp} \subseteq C$. Then there exists a quantum quasi-cyclic code with the parameter $[[3ns, 2sk-3ns, d^{'} \geq d]]$ over F_p.*

Example 4.1. Let $n = 10, q = 3$ and $R = F_3 + vF_3 + v^2F_3$. The Gray image of the code is a $[30, 15, 9]$. The $\psi(C)$ is also a self orthogonal quasi-cyclic code over F_3 with the parameter $[30, 15, d^{'} \geq 9]$. Hence, there exists a quantum quasi-cyclic code with the parameter $[[30, 0, d^{'} \geq 9]]$ over F_3.

Example 4.2. Let $n = 28$ and $R = F_5 + vF_5 + ... + v^4F_5$. We have

$$\begin{aligned} x^{28}-1 &= (x+1)(x+2)(x+3)(x+4)(x^6-x^5+x^4-x^3+x^2-x+1) \\ &\quad (x^6-2x^5-x^4-3x^3+x^2-2x-1)(x^6+x^5+x^4+x^3+x^2+x+1) \\ &\quad (x^6-3x^5-x^4-2x^3+x^2-3x-1) \\ &= f_1f_2...f_8 \end{aligned}$$

in F_5. Let $f(x) = e_0f_6 + e_1f_6 + e_2f_6 + e_3f_8 + e_4f_8$ and $C = (f(x))$ be a cyclic code over R. Clearly $x^{28}-1$ is divisible by $f_6f_6^*, f_8f_8^*$. Hence we have $C^{\perp} \subseteq C$. Also, $\Phi(C)$ is a linear code over F_5 with the parameters $[140, 110, 4]$. Then a quantum code with the parameters $[[140, 80, 4]]$ is obtained.

Quantum codes from cyclic codes

n	q	m	$\Phi(C)$	$[[N,K,D]]$
3	19	7	$[21,14,2]$	$[[21,7,2]]$
3	19	10	$[30,20,2]$	$[[30,10,2]]$
11	5	3	$[33,18,7]$	$[[33,3,7]]$
20	9	3	$[60,48,4]$	$[[60,36,4]]$
27	3	3	$[81,63,2]$	$[[81,45,2]]$
30	2	2	$[60,34,6]$	$[[60,8,6]]$
30	5	5	$[150,145,2]$	$[[150,140,2]]$

5. Conclusion

The quantum codes over F_q are constructed by using the cyclic codes over the finite ring $R = F_q + vF_q + ... + v^{m-1}F_q$, where p is a prime, $q = p^s$, $m-1|p-1$ and $v^m = v$. The parameters of quantum error correcting codes over F_q and the quantum quasi-cyclic codes over F_p are obtained.

By finding a Gray map over R which satisfies self orthogonal property and by taking p is even or p and s are both odd, the parameters of quantum QC codes over F_p can be obtained, similarly.

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Affiliations

YASEMIN CENGELLENMIS
ADDRESS: Trakya University, Department of Mathematics, 22000, Edirne-Turkey.
E-MAIL: ycengellenmis@gmail.com
ORCID ID: 0000-0002-8133-9836

ABDULLAH DERTLI
ADDRESS: Ondokuz Mayis University, Department of Mathematics, 55139, Samsun-Turkey.
E-MAIL: abdullah.dertli@gmail.com
ORCID ID: 0000-0001-8687-032X