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ON WEAKLY PRIME FUZZY IDEALS OF COMMUTATIVE RINGS

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Abstract

In this paper, we present a new notion of fuzzy ideals: called weakly prime fuzzy ideal. Let R be a commutative ring with non-zero identity. A nonconstant fuzzy ideal μ of R is called weakly prime fuzzy ideal if $0_t \neq x_r y_s \in \mu$ implies $x_r \in \mu$ or $y_s \in \mu$ for all $t \in (0, \mu(0)]$. We investigate some properties of this notion. Morever, it is established relations between weakly prime ideals and weakly prime fuzzy ideals of commutative rings.

Keywords: Weakly prime fuzzy ideals, prime fuzzy ideals

1. Introduction

The concept of weakly prime ideals of a commutative ring was introduced and studied by D.D. Anderson and E. Smith in [1]. Subsequently, S. E. Atani and F. Farzalipour introduced the notion of weakly primary ideals [2]. These studies brought a new concept to the literature of ring theory. Here we aim to define the concept of weakly prime fuzzy ideals in fuzzy ring theory.

We assume throughout this paper that all rings are commutative with non-zero identity. Unless stated otherwise $L=[0,1]$ will be a complete lattice. $LI(R)$ denotes the set of fuzzy ideals of R. λ_I denotes the characteristic function of I.

Recall from [1] that a proper ideal I of R is called a weakly prime ideal if whenever $0 \neq ab \in I$, then either $a \in I$ or $b \in I$. Note that every prime ideal is a weakly prime ideal. However, the converse is not true. Recall from [7] that a fuzzy ideal μ of R is called fuzzy prime ideal if for any two fuzzy points x_r, y_s of R, $x_r y_s \in \mu$ implies either $x_r \in \mu$ or $y_s \in \mu$. From [7] we know that a nonconstant fuzzy ideal μ is said to be weakly completely prime fuzzy ideal if and only if for any $x, y \in R$, $\mu(xy) = max\{\mu(x), \mu(y)\}$.

Motivated from this concept, in Section 2, we define weakly fuzzy ideals and we investigate many properties of weakly prime fuzzy ideals. A nonconstant fuzzy ideal μ of R is called weakly prime fuzzy ideal if $0_t \neq x_r y_s \in \mu$ implies $x_r \in \mu$ or $y_s \in \mu$ for all $t \in (0, \mu(0)]$. Among many results in this study, it is proved (Example 2.3) that every prime fuzzy ideal is a weakly prime fuzzy ideal but converse is not true. It is shown that how to construct a weakly prime fuzzy ideal by a weakly prime ideal (Theorem 2.6). Also, it is shown (in Theorem 2.7) if $\mu(0) = 1$ then μ is a weakly prime fuzzy ideal if and only if μ_* is a weakly prime ideal and $|Im\mu| = 2$. In theorem 2.9, we show that a nonconstant fuzzy ideal μ is a weakly prime fuzzy ideal if and only if $0_t \neq AB \subseteq \mu$ implies $A \subseteq \mu$ or $B \subseteq \mu$ for any A, B fuzzy ideals and for all $t \in (0, \mu(0)]$. Morever, we investigate to properties of cartesian product of weakly prime fuzzy ideals. Finally, it is defined partial weakly prime fuzzy ideal and associated with partial weakly prime fuzzy ideals and weakly prime ideals.

2. Weakly prime fuzzy ideals

Definition 2.1 A nonconstant fuzzy ideal μ of R is said to be weakly prime if $0_t \neq x_r y_s \in \mu$ implies $x_r \in \mu$ or $y_s \in \mu$ for any fuzzy points x_r, y_s of R and for all $t \in (0, \mu(0)]$.
Note that 0_t is defined by

$$0_t(x) = \begin{cases} t, & x = 0 \\ 0, & \text{otherwise} \end{cases}$$

where $x \in R$ and $t \in (0, \mu(0)]$.
Also we can state the definition as μ is a weakly prime fuzzy ideal if $x_r y_s \in \mu$ such that $x_r \notin \mu$ and $y_s \notin \mu$ implies that $0_t = x_r y_s$ for any $t \in (0, \mu(0)]$.

Theorem 2.2 Every prime fuzzy ideal is a weakly prime fuzzy ideal.

Proof. The result follows by the definition of prime fuzzy ideal.

The following example shows a weakly prime fuzzy ideal needs not be prime fuzzy ideal.

Example 2.3 Let $R = Z_6$. Then the fuzzy ideal 0_1 is a weakly prime fuzzy ideal that not prime fuzzy ideal. Assume that $x_r y_s \in \overline{0}_1$ such that $x_r \notin \overline{0}_1$ and $y_s \notin \overline{0}_1$. Then $r \wedge s \leq \overline{0}_1(xy)$ and $r > \overline{0}_1(x)$, $s > \overline{0}_1(y)$ so $\overline{0}_1(x) \wedge \overline{0}_1(y) < r \wedge s \leq \overline{0}_1(xy)$. Thus we conclude that $0 < r \wedge s$, $x, y \neq 0$, and $xy = 0$. Thus $x_r y_s = xy_{r \wedge s} = 0_t$ for $r \wedge s = t \in (0,1]$. Hence 0_1 is a weakly prime fuzzy ideal. But since $\overline{2}_1 \overline{3}_1 \in \overline{0}_1$ and neither $\overline{2}_1 \in \overline{0}_1$ nor $\overline{3}_1 \in \overline{0}_1$ then 0_1 is not prime fuzzy ideal.

Theorem 2.4 If μ is a weakly prime fuzzy ideal then μ_k is a weakly prime ideal of R for all $k \in (0, \mu(0)]$.

Proof. Assume that $0 \neq xy \in \mu_k$ for all $k \in (0, \mu(0)]$ and $x, y \in R.$ Since $k \neq 0$ and $0 \neq xy$ then $0_k(0) = k \neq 0 = (xy)_k(0)$ so $0_k \neq (xy)_k$. If $xy \in \mu_k$ then $\mu(xy) \geq k$ and $(xy)_k = x_k y_k \in \mu$. Since μ is a weakly prime fuzzy ideal and $0_k \neq (xy)_k = x_k y_k \in \mu$ then $x_k \in \mu$ or $y_k \in \mu$. So we get that $x \in \mu_k$ or $y \in \mu_k$ hence μ_k is a weakly prime ideal.

Note that the converse of the theorem is not true.

Example 2.5 Let $R = Z_4$ and μ be defined by

$$\mu(x) = \begin{cases} \frac{1}{2}, & x \in \{\overline{0}, \overline{2}\} \\ 0, & otherwise \end{cases}.$$

For all $k \in (0, \mu(0)]$, $\mu_k = \{\overline{0}, \overline{2}\}$. Since $\{\overline{0}, \overline{2}\}$ is a prime ideal of R then $\mu_k = \{\overline{0}, \overline{2}\}$ is a weakly prime ideal. Now we show μ is not a weakly prime fuzzy ideal. For fuzzy points $\overline{2}_{\frac{3}{4}}, \overline{3}_{\frac{1}{2}}$ of R, $\overline{2}_{\frac{3}{4}}.\overline{3}_{\frac{1}{2}} \neq 0_t$ for all $t \in (0, \mu(0)]$. Also $0_t \neq \overline{2}_{\frac{3}{4}}.\overline{3}_{\frac{1}{2}} = \overline{2}_{\frac{1}{2}} \in \mu$ because $\mu(\overline{2}) \geq \frac{1}{2}$. But $\overline{2}_{\frac{3}{4}} \notin \mu$ and $\overline{3}_{\frac{1}{2}} \notin \mu$ since $\frac{3}{4} = \overline{2}_{\frac{3}{4}}(\overline{2}) > \mu(\overline{2}) = \frac{1}{2}$ and $\frac{1}{2} = \overline{3}_{\frac{1}{2}}(\overline{3}) > \mu(\overline{3}) = 0$. Hence μ is not weakly prime fuzzy ideal.

Theorem 2.6 Let I be a weakly prime ideal of R and $1 \neq \alpha \in L$. If μ is the fuzzy ideal defined by

$$\mu(x) = \begin{cases} 1 & x \in I \\ \alpha & x \notin I \end{cases} \text{ for all } x \in R$$

then μ is a weakly prime fuzzy ideal of R.

Proof. Assume that $0_t \neq x_r y_s \in \mu$ such that $x_r \notin \mu$ and $y_s \notin \mu$. Then $x_r(x) = r > \mu(x)$ and $y_s(y) = s > \mu(y)$. Then we get that $\mu(x) = \mu(y) = \alpha$ and $x, y \notin I$. Since $0_t \neq x_r y_s$ then there exists an $a \in R$ such that $0_t(a) \neq x_r y_s(a)$.

Case 1 If $a = 0$, then $0_t(a) = t \neq x_r y_s(a)$. For all $t \in (0, \mu(0)]$, it must be $x_r y_s(a) = 0$ so $a = 0 \neq xy.$

Case 2 If $a \neq 0$, then $0_t(a) = 0 \neq x_r y_s(a)$ so $xy = a \neq 0$.

We conclude in all cases that $xy \neq 0$. Since $\mu(x) = \alpha < r$ and $\mu(y) = \alpha < s$ then $\alpha < r \wedge s \leq \mu(xy)$. Thus $\mu(xy) = 1$ and $xy \in I.$But this contradicts that $0 \neq xy$ and $x, y \notin I$ because I is a weakly prime ideal. Hence μ is a weakly prime fuzzy ideal.

Theorem 2.7 Let μ be a nonconstant fuzzy ideal of R and $\mu(0) = 1$. Then μ is a weakly prime fuzzy ideal of R if and only if μ_* is a weakly prime ideal of R and $\left| Im\mu \right| = 2$.

Proof. Let μ_* be a weakly prime ideal and $Im\mu = \{\alpha, 1\}$. We show that μ is a weakly prime fuzzy ideal. By the previous theorem, if we get $I = \mu_*$ then μ is a weakly prime fuzzy ideal. Conversely, let μ is a weakly prime fuzzy ideal. Firstly we show that $|Im\mu| = 2$. Assume that $|Im\mu| \geq 3$. Let $\mu(1) = k$. Since $\mu(0) = 1$ then there exists $0 \neq r \in R$ such that $\mu(r) = m$ and $k < m < 1$.

For fuzzy points $1_m, r_1$ of R, $1_m.r_1 = r_{m\wedge 1} = r_m \in \mu$ because $\mu(r) = m \geq m = r_m(r)$. It is easy to see that $1_m.r_1 \neq 0_t$ for all $t \in (0,1]$. Since μ is a weakly prime fuzzy ideal and $0_t \neq 1_m.r_1 = r_m \in \mu$ then $1_m \in \mu$ or $r_1 \in \mu$. However this contradicts that $1_m(1) = m > k = \mu(1)$ and $r_1(r) = 1 > m = \mu(r)$. Hence $|Im\mu| = 2$.

Now, we prove that μ_* is a weakly prime ideal. Assume that $0 \neq xy \in \mu_*$ and $x, y \notin \mu_*$. Since $\mu(0) = 1$ then for all $t \in (0,1]$, $t \leq \mu(xy) = \mu(0) = 1$, so $0_t \neq (xy)_t \in \mu$. Since μ is a weakly prime fuzzy ideal then $x_t \in \mu$ or $y_t \in \mu$. For all $t \in (0,1]$, $x_t(x) = t \leq \mu(x)$ or $y_t(y) = t \leq \mu(y)$. Especially, for $t = 1$, $\mu(x) = 1$ or $\mu(y) = 1$ so $x \in \mu_*$ or $y \in \mu_*$ which is a contradiction by our assumption. Hence μ_* is a weakly prime ideal.

Lemma 2.8 [6] Let A, B be fuzzy ideals of R then for $t \in L$ $A_t B_t = (AB)_t$.

Theorem 2.9 Let μ be a fuzzy ideals of R. Then μ is a weakly prime fuzzy ideal if and only if $0_t \neq AB \subseteq \mu$ implies $A \subseteq \mu$ or $B \subseteq \mu$ for any A, B fuzzy ideals and for all $t \in (0, \mu(0)]$.

Proof. $\Leftarrow$: Assume that for all $t \in (0, \mu(0)]$, $0_t \neq x_r y_s \in \mu$ where x_r, y_s are any fuzzy points. Let nonconstant fuzzy subsets A, B be defined by

$$A(a) = \begin{cases} r & a \in \langle x \rangle \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad B(b) = \begin{cases} s & b \in \langle y \rangle \\ 0 & \text{otherwise} \end{cases}.$$

It is easy to see that A, B are fuzzy ideals of R.

Case 1 If $z \in \langle x \rangle \langle y \rangle$, then $AB(z) = \vee\{A(a) \wedge B(b) : z = ab\} = r \wedge s \leq \mu(xy) \leq \mu(z)$.

Case 2 If $z \notin \langle x \rangle \langle y \rangle$, then $AB(z) = \vee\{A(a) \wedge B(b) : z = ab\} = 0 \leq \mu(z)$. Then we conclude that in all cases $AB \subseteq \mu$. By our assumption $0_t \neq x_r y_s$ then $xy \neq 0$. Thus $AB(xy) = r \wedge s \neq 0 = 0_t(xy)$ and we get that $0_t \neq AB$. By the hypothesis that $A \subseteq \mu$ or $B \subseteq \mu$ so $x_r(x) = r = A(x) \leq \mu(x)$ and $x_r \in \mu$ or $y_s(y) = s = B(y) \leq \mu(y)$ and $y_s \in \mu$. Hence we conclude that μ is a weakly prime fuzzy ideal.

$\Rightarrow$: Now we show that $0_t \neq AB \subseteq \mu$ implies $A \subseteq \mu$ or $B \subseteq \mu$ for any A, B fuzzy ideals and for all $t \in (0, \mu(0)]$. Let $0_t \neq AB \subseteq \mu$. Then there exists $r \in R$ such that $0_t(r) \neq AB(r)$.

Case 1 If $r = 0$ for all $t \in (0, \mu(0)]$, then $0_t(0) = t \neq AB(0)$ and it must be $AB(0) = 0$. Thus $AB = 0_R$ and $A = 0_R$ or $B = 0_R$ .Hence we have $A \subseteq \mu$ or $B \subseteq \mu$.

Case 2 If $r \neq 0$ for some $t_i \in (0, \mu(0)]$, then $0_{t_i}(r) = 0 \neq AB(r)$. If there exists any $0 \neq r \in R$ such that $0_{t_i}(r) \neq AB(r)$ then we conclude that for all $t \in (0, \mu(0)]$, $0_t(r) = 0 \neq AB(r)$. Let $AB(r) = s \neq 0$ then $(AB)_s \neq \{0\}$. By Theorem 2.4, since μ is weakly prime fuzzy ideal then μ_s is also weakly prime ideal. Thus $0 \neq (AB)_s = A_s B_s \subseteq \mu_s$ and $A_s \subseteq \mu_s$ or $0 \neq (AB)_s = A_s B_s \subseteq \mu_s$. Hence $A \subseteq \mu$ or $B \subseteq \mu$.

Definition 2.10 (1) Let μ and α be two fuzzy ideals of R. The cartesian product of μ and α is defined by $\mu \times \alpha$ such that $\left(\mu \times \alpha\right)(x, y) = \mu(x) \wedge \alpha(y)$ [4].
(2) Let μ and α be two fuzzy ideals of R. If $(x_r, y_s) \in \mu \times \alpha$ for any fuzzy points $\mathrm{x}_r, \mathrm{y}_s$ of R then $x_r \in \mu$ and $y_s \in \alpha$ so $r \wedge s \leq \mu \times \alpha(x, y) = \mu(x) \wedge \alpha(y)$.

Note that if μ and α are fuzzy ideals of R then $\mu \times \alpha$ is a fuzzy ideal of $R \times R$.

Theorem 2.11 Let R_1 and R_2 be rings and μ_1 and μ_2 be weakly prime fuzzy ideals, respectively. Then $\mu_1 \times \lambda_{R_2}$ and $\lambda_{R_1} \times \mu_2$ are weakly prime fuzzy ideals of $R_1 \times R_2$.

Proof. It is easy to see that $\mu_1 \times \lambda_{R_2}$ and $\lambda_{R_1} \times \mu_2$ are fuzzy ideals of $R_1 \times R_2$. Let $(x_r, y_s), (z_k, t_v)$ be fuzzy points of $R_1 \times R_2$ such that $(x_r, y_s)(z_k, t_v) \neq (0_{R_1}, 0_{R_2})$. Then $x_r z_k \neq 0_{R_1}$ and $y_s t_v \neq 0_{R_2}$. $(x_r, y_s)(z_k, t_v) \in \mu_1 \times \lambda_{R_2}$ then $(x_r z_k, y_s t_v) \in \mu_1 \times \lambda_{R_2}$ so $x_r z_k \in \mu_1$ and $y_s t_v \in \lambda_{R_2}$. Since μ_1 is weakly prime fuzzy ideal and $x_r z_k \neq 0_{R_1}$ then $x_r \in \mu_1$ or $z_k \in \mu_1$. Thus $(x_r, y_s) \in \mu_1 \times \lambda_{R_2}$ or $(z_k, t_v) \in \mu_1 \times \lambda_{R_2}$.

Similary, it can be easily shown that $\lambda_{R_1} \times \mu_2$ is a weakly prime fuzzy ideal of $R_1 \times R_2$.

Definition 2.12 Let μ be a nonconstant fuzzy ideal of R. If $\mu(xy) = \mu(x)$ or $\mu(xy) = \mu(y)$ for $xy \neq 0$ then μ is called partial weakly prime fuzzy ideal.

Theorem 2.13 If μ is a weakly prime fuzzy ideal then μ is a partial weakly prime fuzzy ideal.

Proof. Assume that μ is a weakly prime fuzzy ideal and $xy \neq 0$.

If $\mu(xy) = 0$ then $\mu(x) \leq \mu(xy) = 0$ so $\mu(x) = \mu(xy) = 0$.

If $\mu(xy) \neq 0$ then there exists $s \in (0, \mu(0)]$ such that $s = \mu(xy)$. Since $xy \neq 0$ then $(xy)_s \neq 0_t$ for all $t \in (0, \mu(0)]$. Because $0_t \neq (xy)_s \in \mu$ and μ is a weakly prime fuzzy ideal then $x_s \in \mu$ or $y_s \in \mu$. Thus $\mu(xy) = s \leq \mu(x)$ or $\mu(xy) = s \leq \mu(y)$ so $\mu(xy) = \mu(x)$ or $\mu(xy) = \mu(y)$.

The following example shows the converse of the theorem is not true.

Example 2.14 Let $R = Z \times Z$. Define the fuzzy ideal μ of R by

$$\mu(a)=\begin{cases}1, & \text{if } a\in 0\times Z\\ \frac{1}{2}, & \text{if } a\in 2\times Z\setminus 0\times Z\\ 0, & \text{if } a\in Z\times Z\setminus 2\times Z\end{cases}.$$

It is easy to prove that μ is a partial weakly prime fuzzy ideal . However μ is not weakly prime fuzzy ideal. Assume that $a=(2,r)$ and $b=(3,s)$. For fuzzy points of R $a_{\frac{2}{3}}=(2,r)_{\frac{2}{3}}$ and $b_{\frac{1}{3}}=(3,s)_{\frac{1}{3}}$, we get that $a_{\frac{2}{3}}\notin\mu$ and $b_{\frac{1}{3}}\notin\mu$. But $0_t\neq a_{\frac{2}{3}}.b_{\frac{1}{3}}=(ab)_{\frac{1}{3}}=(6,rs)_{\frac{1}{3}}\in\mu$. Hence μ is not weakly prime fuzzy ideal.

Theorem 2.15 Let μ be a nonconstant fuzzy ideal of R. Then μ is a partial weakly prime fuzzy ideal if and only if μ_t is a weakly prime ideal of R for all $t\in(0,\mu(0)]$.

Proof. Assume that μ is a partial weakly prime fuzzy ideal. Let $0\neq xy\in\mu_t$. Then $\mu(xy)\geq t$. Because μ is a partial weakly prime then $\mu(x)=\mu(xy)\geq t$ and $x\in\mu_t$ or $\mu(y)=\mu(xy)\geq t$ and $y\in\mu_t$. Let μ_t be a weakly prime ideal. For $xy\neq 0$, let $\mu(xy)=s$.

Case 1 If $s=0$, then $\mu(x)\leq\mu(xy)=0$ and $\mu(xy)=\mu(x)=0$ or $\mu(y)\leq\mu(xy)=0$ and $\mu(xy)=\mu(y)=0$.

Case2 If $s\neq 0$, then $0\neq xy\in\mu_s$. Since μ_s is weakly prime ideal then $x\in\mu_s$ or $y\in\mu_s$ thus $\mu(x)\geq s=\mu(xy)$ and $\mu(xy)=\mu(x)$ or $\mu(y)\geq s=\mu(xy)$ and $\mu(xy)=\mu(y)$.

Theorem 2.16 Let f be an injective ring homomorphism from R to S . If ξ is a partial weakly prime fuzzy ideal of S then $f^{-1}(\xi)$ is a partial weakly prime fuzzy ideal of R.

Proof. Let $0\neq ru$ where $r,u\in R$. $f^{-1}(\xi)(ru)=\xi(f(ru))=\xi(f(r)f(u))$. Since f is $1-1$ then $f(ru)\neq 0$. Because ξ is partial weakly prime then $\xi(f(ru))=\xi(f(r)f(u))=\xi(f(r))$ so $f^{-1}(\xi)(ru)=f^{-1}(\xi)(r)$ or $\xi(f(ru))=\xi(f(r)f(u))=\xi(f(u))$ so $f^{-1}(\xi)(ru)=f^{-1}(\xi)(u)$. Hence we conclude that $f^{-1}(\xi)$ is a partial weakly prime fuzzy ideal.

Theorem 2.17 Let $f:R\to S$ be a surjective ring homorphism. If μ is a partial weakly prime fuzzy ideal of R which is constant on $Kerf$ then $f(\mu)$ is a partial weakly prime fuzzy ideal of S.

Proof. Assume that $xy\neq 0$ where $x,y\in S$. Since f is an epimorphism then there exist $r,u\in R$ such that $f(r)=x,\ f(u)=s$. Because f is constant on $Kerf$ so

$$f(\mu)(xy)=f(\mu)(f(r)f(u))=f(\mu)(f(ru))=\mu(ru).$$

If $ru=0$ then $f(0)=0=f(ru)=f(r)f(u)=xy$, which is a contradiction. Then $ru\neq 0$. Since μ is a partial weakly prime fuzzy ideal then

$$f(\mu)(xy)=\mu(ru)=\mu(r)=f(\mu)f(r)=f(\mu)(x) \text{ or}$$
$$f(\mu)(xy)=\mu(ru)=\mu(u)=f(\mu)f(u)=f(\mu)(y).$$

Hence we get that $f(\mu)$ is a partial weakly prime fuzzy ideal.

3. Conclusions

In this paper, we have characterized weakly prime fuzzy ideals. Also the notions of partial weakly prime fuzzy ideals and their properties are proposed. Furthermore, we have given generalization of definitions of weakly prime fuzzy ideals.

To extend this study, one could study other algebraic structures and do some further studies of their properties.

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