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Evaluation of Hydrodynamic and Sediment Transport equations and Parameter Sensitivity Analysis Using the SRH_2D Model

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Abstract. One-dimensional (1D) flow models have been routinely used in practical hydraulic applications. Example models include HEC_RAS (Brunner, 2006), MIKE11 (DHI, 2002), CCHE1D (Wu and Vieira, 2002), and SRH_1D (Huang and Greimann, 2007). These 1D models will remain useful, particularly for applications with a long river reach (more than 50 km) or over a long time period (over a year). Their limitation, however, are well known and there are situations where multi-dimensional modeling is needed. For most river flows, water depth is shallow relative to width and vertical acceleration is negligible in comparison with gravity. So the two-dimensional (2D) depth-averaged model provides the next level of modeling accuracy for many practical open channel flows. Indeed, time has been ripe that 2D models may be routinely used for river projects on a personal computer. A range of 2D models have been developed and applied to a wide range of problems since the work of chow and Ben-Zvi (1973). The ability of a 2D model to solve open channel flows with complex geometries has always been a thrust for improvement as it is relevant to practical applications. In this report hydrodynamic and Sediment Transport Equations and Sensitivity Parameter Using the SRH_2D Model are introduced.

Keywords: Depth-Averaged Model, Hybrid Mesh, Unstructured Mesh, SRH_2D Model

1. INTRODUCTION

The Sedimentation and River Hydraulics two-dimensional model (SRH-1D) is a two-dimensional hydraulic, sediment, temperature, and vegetation model for river systems under development at the U.S. Bureau of Reclamation.

SRH-2D solves the 2D dynamic wave equations, the depth-averaged St. Venant equations. In terms of modeling capabilities, SRH-2D is comparable to many existing models such as RMA-2 (US Army Corps of Engineers 1996) and MIKE21 (DHI software 1996). Comparison with the model Hydro As-2D has been done on the hydrodynamic modelling by tollosa (2008) has proven the model's comparable capacities for hydraulic calculation.

SRH-2D does possess a few salient features:

1. SRH-2D uses hybrid unstructured meshes which may contain arbitrarily shaped cells. In practice, a hybrid mesh of quadrilateral and triangular cells is recommended though purely quadrilateral or triangular elements may also be used. A hybrid mesh may achieve the best compromise between solution accuracy and computational demand.

2. SRH-2D adopts very robust and stable numerical schemes with seamless wetting-drying algorithms. The outcome is that the model is very stable, and few tuning parameters are needed to obtain reliable solutions. The model has been verified, validated, and successfully applied to numerous flow cases.

SRH-2D is also developed with the objective that a 2D model does not have to be too complex to use. With SRH-2D, users do not have to memorize many commands; they are guided by a

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pre-processor, an interactive user interface, through the partial-interface mode. Most user input errors may be automatically detected by the pre-processor so errors may be removed before carrying out the final analysis (SRH-2D user's manual 2008).

The model has three modules the flow module, morph module and the Mobile module. These modules are applicable to their respective uses: for hydraulic computation (flow module) and additional sediment transport computations to the flow are incorporated in the remaining two modules.

2. NUMERICAL METHODS

In terms of numerical methods, flow and sediment transport models are categorized as finite difference, finite volume, and finite element models. Since each of these numerical methods have their advantages and disadvantages, numerical models based on all of them exist in the literature. The choice of a specific model depends on the nature of the problem, the experience of the modeller, and the capacity of the computer being used. In most 2D sediment transport problems, the mathematical modelling of morphodynamics relies on a system of four equations; three equations (shallow water equation) describing flow continuity and momentum conservation with shallow water assumptions, and one equation (Exner equation) expressing the sediment continuity.

3. HYDRODYNAMIC EQUATIONS

SRH-2D is based on shallow water equations for the hydrodynamics and a sediment transport equation for the morphodynamics. The shallow water equations are obtained by depth averaging the Euler equations under the assumptions that the vertical component of acceleration has a negligible effect on the fluid pressure. In two space dimensions, the shallow water equations are:

$$\frac{\partial h}{\partial t} + \frac{\partial hU}{\partial x} + \frac{\partial hV}{\partial y} = e$$

$$\frac{\partial hU}{\partial t} + \frac{\partial hUU}{\partial x} + \frac{\partial hUV}{\partial y} = \frac{\partial hT_{xx}}{\partial x} + \frac{\partial hT_{xy}}{\partial y} - gh\frac{\partial z}{\partial y} - \frac{T_{bx}}{\rho} + D_{xx} + D_{xy}$$

$$\frac{\partial hV}{\partial t} + \frac{\partial hUV}{\partial x} + \frac{\partial hVV}{\partial y} = \frac{\partial hT_{xy}}{\partial x} + \frac{\partial hT_{yy}}{\partial y} - gh\frac{\partial z}{\partial y} - \frac{T_{by}}{\rho} + D_{yx} + D_{yy} \qquad (1)$$

In the above equations, t is time, x and y are horizontal Cartesian coordinate, h is water depth, U and V are depth-averaged velocity components in x and y directions, respectively, e is excess rainfall rate, g is gravitational acceleration. The turbulent stress terms, which are usually important in rivers with complex geometry, are calculated with the aid of the depth averaged k-model. T_{xx}, T_{xy} and T_{yy} are depth-averaged turbulent stresses given by:

$$T_{xx} = 2(\vartheta + \vartheta_t)\frac{\partial U}{\partial x} - \frac{2}{3}k$$

$$T_{xy} = (\vartheta + \vartheta_t)(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x})$$

$$T_{yy} = 2(\vartheta + \vartheta_t)\frac{\partial V}{\partial y} - \frac{2}{3}k \qquad (2)$$

Where ϑ is the kinematic viscosity of water; ϑ_t is the turbulent eddy viscosity; and k is turbulent kinetic energy. A turbulence model is used to compute the turbulent eddy viscosity. Two turbulence models may be used (Rodi 1993): the depth-averaged parabolic model and the two equations $k-\varepsilon$ model. With the parabolic model, in which U_* is the bed frictional velocity. The model constant c_t ranges from 0.3 to 1.0.

$$\vartheta_t = c_t U_* h \tag{3}$$

The bed shear stresses are calculated as follows using the Manning's roughness equation:

$$(\tau_{bx}, \tau_{by}) = \rho c_f (U, V)\sqrt{U^2 + V^2}\ ; \ \ c_f = \frac{gn^2}{h^{1/3}} \tag{4}$$

In SRH-2D the governing equations are discretized using the finite-volume approach, following the work of Lai (1997, 2000) and Lai et al. the solution domain is covered with an unstructured mesh with each mesh element assuming arbitrarily shaped polygons. Most commonly used polygons are triangle and quadrilaterals. All dependent variables are stored at the geometric center of a polygon. The governing equations are integrated over a polygon using the Gauss theorem.

4. TWO DIMENSIONAL SEDIMENT TRANSPORT EQUATIONS

In SRH-2D, it is assumed that sediment transport takes place such that the bed level depends on time as well. This requires an additional equation for its evolution. Mathematically, the morphological evolution of the bed is described by the so called sediment continuity. This equation states that the time rate of change of the bed elevation is equal to the divergence of the sediment flux, which can be expressed in terms of the local flow properties through the use of an empirical sediment transport formula. The sediments can be assumed to be uniform or non-uniform and are divided into a number of sediment size classes. Each size class (k) obeys the following mass conservation equation providing the non-equilibrium transport:

$$\frac{\partial hC_k}{\partial t} + \frac{\partial hUC_k}{\partial x} + \frac{\partial hVC_k}{\partial y} = \frac{1}{L_b}\left(q_t^* - \sqrt{U^2 + V^2}\, hC_k\right) \tag{5}$$

Where h is water depth; t is time; C_k is depth-averaged sediment concentration by volume for k^{th} sediment size class; U and V are depth-averaged velocity components respectively in x and y direction; L_b is the non-equilibrium adaptation length and q_t^* is the equilibrium fractional sediment transport capacity for the k^{th} size class.

Four different sediment transport equations are available to compute the fractional sediment transport capacity (q_t^*).

1. Engelund and Hansen (1972)

2. Parker (1990)

3. Wilcox and Crowe (2003)

4. Mayer-peter and Muller (1948) modified by Parker and Wong (2006)

Bed sediment dynamics and interaction with the bed material load transport were also simulated. In reality, the bed elevation is changing due to net erosion or deposition of the bed. The bed elevation z_b change due to sediment size class k calculated, as follow:

$$(1-p_b)\left(\frac{\partial z_b}{\partial t}\right)_k = -\frac{1}{L_t}\left(q_t^* - \sqrt{U^2+V^2}\,hC_k\right) \quad (6)$$

In SRH-2D the bed is divided into two layers to account for the bed sediment dynamics:

1. The active layer

2. The sub-surface layer(s)

The active layer is the top bed layer participating in the sediment exchange between the bed and bed load as shown in figure 1; the sub-surface layer number can be divided in to a number of layers based on sediment composition of the sub-layer. The sub-surface layer provides sediment supply to the active layer once eroded. The gradations of both layers may change over time and are calculated. The active layer gradation equation is given by:

$$\frac{\partial \delta_a p_{ak}}{\partial t} = \left(\frac{\partial z_b}{\partial t}\right) + p_{ak}^*\left(\frac{\partial \delta_a}{\partial t} - \frac{\partial z_b}{\partial t}\right) \quad (7)$$

Where $\delta_a$the thickness of the active is layer; p_{ak} is the active layer volumetric fraction of sediment size class k;

$$p_{ak}^* = p_{ak} \quad if \ldots \left(\frac{\partial \delta_a}{\partial t} - \frac{\partial z_b}{\partial t}\right) < 0$$

p_{ak}^* is the sub-surface fraction of sediment size class if… $\left(\frac{\partial \delta_a}{\partial t} - \frac{\partial z_b}{\partial t}\right) > 0$.

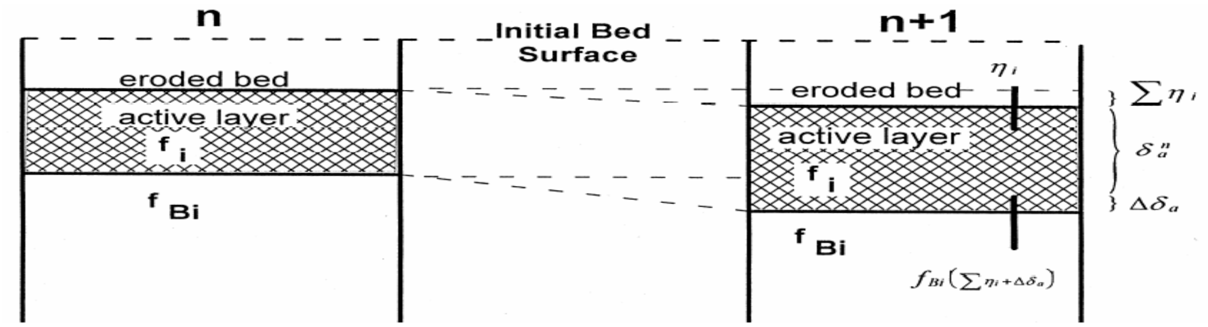

Figure 1. Active layer definition sketch.

5. SEDIMENT TRANSPORT FUNCTIONS IN SRH-2D

Literature contains many sediment transport functions. Usually, each transport function was developed for a certain range of sediment size and flow conditions. Computed results based on different transport functions can differ significantly from each other and from measurements. No universal function exists which can be applied with accuracy to all sediment and flow conditions. SRH-2D employs four sediment transport functions. Many transport formulas were developed assuming uniform size gradations. In these cases, the transport capacity is modified by the fraction of the size class in the active layer according to the following equation,

$$Q_i = P_i Q_i^T \quad (8)$$

Where P_i is the mass fraction of that size class in the active layer and Q_i^T is the transport capacity predicted by the formula assuming uniform size.

5.1. Engelund and Hansen (1972) Function

Engelund and Hansen (1972) proposed the following transport function for use in primarily sand bed rivers. Engelund and Hansen's equation is based on the stream power approach. The rate of energy used in transporting materials should be related to the rate of materials being transported

$$f'\emptyset = 0.1\theta^{5/2}$$

$$f' = \frac{2gSD}{V^2} \;, \; \emptyset = q_t \,/\, \sqrt{(s-1)gd^3} \;, \; \theta = \frac{\tau}{(\gamma_s - \gamma)d} \quad (9)$$

Where g is the gravitational acceleration; S is the energy slope; V is the average flow velocity; q_t is the total sediment discharge by volume per unit width; s is the specific gravity of sediment; γ_s and γ are specific weight of sediment and water, respectively; d is the median particle diameter; D is the mean water depth; and τ_b is the shear stress along the bed.

5.2. Parker (1990) Function

Parker (1990) developed an empirical gravel transport function based on the equal mobility concept and field data; since all grain sizes are assumed to have approximately equal mobility, only one grain size, the sub-pavement size d_{50}, is used to characterize the bed load transport rate.

$$\frac{q_{bi}\,g(s-1)}{P_i(\tau_b/\rho)^{1.5}} = 11.93\, f(\emptyset_i) \quad (10)$$

$$\emptyset_i = \theta_i \,/\, (\xi_i \theta_r) \;, \; \theta_i = \tau_g \,/\, (\gamma(s-1)d_i) \;, \; \frac{U}{\sqrt{\tau_g/\rho}} = 2.5\ln\left(\frac{12.27R'}{k_s}\right) \quad (11)$$

τ_g Is the grain shear stress. U is the cross sectional average velocity, R' is the hydraulic radius due grain shear stress $(\tau_g = \gamma R' S_f)$. The parameter k_s is the grain roughness height computed as $k_s = 2d_{90}$ as suggested by parker (1990). The parameter ξ_i is the exposure factor, which accounts for the reduction in the critical shear stress for relatively large particles and the increase in the critical shear stress for relatively small particles:

$$\xi_i = (d_i \,/\, d_{50})^{-\alpha} \quad (12)$$

Where α is a constant? The function, $f(\emptyset_i)$ was fit to field data And is:

$$f(\emptyset) = (1 - 0.853/\emptyset)^{4.5} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\; \emptyset > 1.59$$

$f(\emptyset) = 0.000183\exp\left(14.2(\emptyset - 1) - 9.28(\emptyset - 1)^2\right) \ldots\ldots\ldots\ldots 1 < \emptyset \leq 1.59$

$f(\emptyset) = 0.0001830^{14.2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . \emptyset \leq 1$ (13)

5.3. WILCOX AND CROWE (2003) FUNCTION

The Wilcox and Crowe formula is similar to the Parker (1990) in that it is a bed load formula and written in a similar form:

$$\frac{q_{bi} g(s-1)}{F_i (\tau_b/\rho)^{1.5}} = 14 f(\emptyset_i) \quad (14)$$

Where the variable definition is the same as in the parker equation? The roughness height used to compute the grain shear stress is$k_s = 2d_{65}$. The function f is computed as:

$f(\emptyset) = (1 - 0.894/\sqrt{\emptyset})^{4.5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \emptyset \geq 1.35$

$f(\emptyset) = 0.000143\emptyset^{7.5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . \emptyset \leq 1.35$ (15)

Where $\emptyset_i = \theta_i / (\xi_i \theta_c)$, $\theta_c = 0.21 + 0.015\exp(-20F_s)$, $\xi_i = (d_i / d_m)^{-\alpha}$ (16)

Where d_m is the mean diameter? Notice that the mean diameter is used in the above equation and not the median. The parameter α is specified as:

$$\alpha = 1 - 0.67[1 + \exp(1.5 - d_i / d_m)]^{-1} \quad (17)$$

Where d_m is the mean particle diameter in the bed? The above equation has the behavior of approaching 0.33 for large d_i / d_m and approaching 0.88 for smalld_i / d_m.

5.4. Meyer-Peter And Muller's Formula (1948) Modified By Wong And Parker (2006) Function

The Meyer-Peter and Muller's Formula is a bed load formula for gravel or coarse materials. The original equation by Meyer-Peter-Muller's in 1948 was given by:

$$\frac{q_b}{\sqrt{g(s-1)d^3}} = 8\left(\frac{(K_s/K_r)^{3/2}\gamma RS}{(\gamma_s-\gamma)d} - 0.047\right)^{1.5} \quad (18)$$

Where γ and γ_s specific weights of water and sediment, respectively; R hydraulic radius; S energy slope; d mean particle diameter; s specific gravity of sediment; q_b volume bed load transport per unit width; The value of K_s and Kr can be computed from:

$$K_s = \frac{V}{C_m R^{2/3} S^{1/2}} , Kr = \frac{26}{d_{90}^{1/6}} \quad (19)$$

Where d_{90} the size of sediment for which 90 percent of the material is finer and is in meters. Wong and Parker (2006) reanalyzed the data used by Meyer-Peter and Muller and found that the energy slope correction in the equation is unnecessary. The modified formula suggested by them is:

$$\frac{q_{bi}}{\sqrt{g(s-1)d^3}} = 3.97\left(\frac{RS}{(s-1)d} - 0.0495\right)^{1.5} \quad (20)$$

The range of application of the four equations is summarized in Table 1.

Table 1. Sediment transport equations applicability.

Equation	Main development area	Recommendation
Engelund and Hansen	Sandy Bed	Total Load
Parker	Mixed Gravel Beds	Bed Load
Wilcox – Crow	Mixed Gravel Beds	Bed Load
Meyer – Peter - Muller	Gravel Beds	Bed Load

5.5. Modeling Steps

Three programs are needed for a complete analysis using SRH-2D:

1. Pre-processor (a mesh generation program)

2. The SRH-2D package

3. A post-processing graphical program

5.6. Pre-processing

One of the salient features of SRH-2D is the use of the hybrid mesh, which is based on the arbitrarily shaped element method of Lai (1997, 2000) for geometry representation. This unstructured hybrid meshing strategy is flexible that facilitates the implementation of the zonal modeling concept. SRH-2D essentially allows the use of most existing meshing methods available, such as the structured curvilinear mesh (pure quadrilaterals), conventional finite element mesh (purely triangles), Cartesian mesh (purely rectangular or square mesh), and the hybrid mixed element mesh. Typical meshes used by SRH-2D are the hybrid meshes.

In general, a combination of quadrilaterals and triangles is the most common mesh type used by SRH-2D. SMS, the surface-water modelling System, is the mesh generator supported by SRH-2D at present. SMS is a pre- and post-processor for surface water modelling and design. SRH-2D uses the resulting 2DM mesh file from SMS.

The SRH-2D release package consists of two programs: srhpre and srh2d.

Srhpre is a text-based interactive user interface that guides a user to set up the SRH-2D simulation in an easy-to-understand manner. It may be interpreted as a pre-processor to obtain an input file to run srh2d. the interface is designed such that a user does not need to memorize many input commands. The interface has the error checking capability so that errors may be detected before running SRH-2D program. The pre-processor stage made ready the next running stage by taking the necessary modelling input, mesh in. 2DM format, boundary conditions, initial conditions, Manning’s roughness, model time-steps, turbulence models and a number of model parameters in a simple user interface mode.

5.7. Boundary and Initial Conditions

SRH-2D offers assigning of different alternative direct model boundary conditions for the respective model reach. For initial conditions the model offers three alternatives:

1. DRY a dry bed with flow from all the boundaries,

2. AUTO a dry bed with flow only from upstream,

3. RST a hot start files which uses results from previous model run

For each river in the network, the energy equation is used as the downstream boundary condition. The model can directly account the following types of hydraulic boundary conditions:

Table2. Boundary conditions in SRH-2D.

Description	Model ID
A sub critical upstream inlet boundary, (flow discharge)	INLET-Q
A sub critical downstream exit boundary, (water surface elevation)	EXIT-H
A known flow discharge at exit boundary, (mostly as a secondary exit boundary)	EXIT-Q
A super- critical up stream upstream inlet boundary,(both discharge and water surface elevation are required)	INLET-SC
A super- critical exit boundary, (neither discharge or water surface elevation are required)	EXIT- EX
A no slip solid boundary on which velocity is zero	WALL

Sediment entering at the upstream of the reach should be specified for each size class. The model uses two options to account for these:

1. Inlet-q_s: A sediment rating curve is used. The total sediment discharge is divided into fractional sediment discharge using a table of water discharge and fraction of total sediment load for each size fraction. Or the total sediment load is specified as a time series; the amount of sediment entering the reach as a function of time. The total sediment discharge is divided into the sediment size fractions.

2. Capacity: An equilibrium sediment load can be assumed. If this option is chosen, then the sediment load coming into the reach is calculated based on the bed material and the sediment transport equations specified. The hydraulics of the most upstream section is used in the transport equation.

5.8. Post Processing and SRH-2D Package

Srh2d is the main solver that reads the input data generated by srhpre, carries out the simulation, and outputs the simulated results to data files in a format accessible to graphical post-processing packages. The output data files contain the final results and may be viewed and processed using the selected graphic packages such as SMS, TECPLOT, or ArcGIS.

The model results include, water surface elevation, velocity, water depth, bed-shear stress erosion-deposition depths, mean sediment diameter, sediment concentration, bed elevation, and the Froude number at a specified model interval for the whole reach in a single file and sediment gradation curve at a specified monitoring point in another one.

5.9. Sensitivity Analysis

The dependence of the calibration results against the variation of the different sediment transport parameters was carried out. The parameters used to check the model sensitivity are:

1. Active layer thickness

2. Use of different adaptation length approaches

3. Manning's roughness

4. Averaging of similar sediment classes

6. CONCLUSIONS AND RECOMMENDATIONS

The major advantage of applying a 2d model is the ability to sufficiently reproduce spatial variations that a 1d model cannot adequately capture. If the study reach is too long, 1D numerical model are often used in the entire reach; they can provide boundary conditions for 2D and 3D numerical models for detailed analyses in important sub-reaches. A 2D model is applicable in areas such as flows with in-stream structures, through bends, with perched rivers, with side channels and with braided channel systems where the flow is more of two dimensional. A 2D model may also be needed if one is interested in local flow velocities, eddy pattern, flow recirculation, lateral velocity variation, and flow over banks and levees. But the later ones can be very well represented only in a 3D model. To solve a real-life engineering problem correctly and effectively, the integration of field investigation, physical modeling, and computational simulation is needed. Field investigation is the first step in gaining a comprehensive understanding of the problem. It provides the necessary hydraulic, hydrologic and sediment information on the study domain and boundary conditions, which are required in both physical modelling and computational simulation. It also provides data to calibrate physical and computational models. Due the presence of dry cells in the model prediction around flow periphery, the use of fine mesh is recommended. However, for all upcoming modelling work it remains clear that every simulation can only be as good as its input. An important step in performing a sediment transport modelling is determining the necessary and relevant flow and sediment parameters of the study and what type of model is needed to obtain this information. The sediment transport results obtained through the different equations are relatively consistent to each other and are comparable to the measured data. As the model results proved, applying an equation which is developed for a specific bed nature to a different bed is simply not worth while. The Engelund and Hansen approach is recommended for sandy river beds. Many investigators such as Yang (1996) have also compared bed-load transport formulas. The conclusions are usually different because different data have been used. However, it has been shown that almost all existing formulas have better predictions for flume data than for field data (Wu 2007). The reasons are that the bed load transport is more complex and the measurement instruments are less efficient in natural rivers. As recognized by many researchers, it is hardly possible to predict the bed-load transport rate with accuracy less than a factor of 2. Perhaps this remark is useful for sediment engineers to judge the prediction capability of the existing sediment formulas.

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