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Pre-service Teachers' Noticing of 7th Grade Students' Errors and Misconceptions about the Subject of Equations*

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Abstract. Teacher noticing is one of the issues that math educators have given importance recently. The noticing-raising skills that require a professional vision beyond their use in daily life requires a series of activities within the framework of prior learning by the teacher, then interpreting and providing appropriate feedback. Investigating the level of awareness in pre-service teacher trainees will provide a foresight for future education. In the field of algebra learning, it has been identified in previous studies that primary school students experience various misconceptions and have a trouble. Therefore, the aim of this study is to determine the misconceptions and errors of 7th grade students about equations and to observe their noticing of these errors and misconceptions. Participants of the study, in which qualitative research methods have been adopted, are 3 student and 3 mathematics teacher candidates with medium and high academic achievement. As a means of data collection, in the scale of error and misconceptions about equations was utilized and teacher candidates' awareness of these responses was investigated. At the end of the study, it was found that the success of the attending step was not sufficient for the other steps.

Keywords: Pre-service math teacher, noticing, algebra teaching, equations.

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1. INTRODUCTION

The algebra learning area has an important place in mathematics education. There are many studies in the literature about this learning area (Dede, 2004; MacGregor and Stacey, 1997; Masal & Sert-Çelik, 2018; van den Kieboom, Magiera, & Moyer, 2017; Yaman, Toluk, & Olkun; 2003).These studies show that students have difficulties in many subjects such as establishing and solving equations, the use of algebraic expressions and problem solving.A significant part of the studies reported in the relevant literature focus on students' difficulties and misconceptions about the concepts of variable and equality (Dede, 2004; Mac Gregor, & Stacey, 1997; Masal, & Sert-Çelik, 2018; van den Kieboom, Magiera, & Moyer, 2017; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005).

In mathematics teaching, it is necessary to attach importance to not only the operational understanding but also the conceptual understanding of subjects. When this is taken into consideration, it would be more likely to determine and eliminate misconceptions. The current study, within the general framework of the misconceptions in algebra, is directed towards examining the concept of equality in particular. When the approaches adopted by students towards the concept of equality are examined in the literature, it is seen that the approaches are concentrated in two main categories as relational and operational thinking (Knuth et al., 2005; van den Kieboom, Magiera, & Moyer, 2017). Students who think operationally believe that equality should only produce results, while relational thinkers think that the equal sign is a symbol that separates the same quantities on both sides (van den Kieboom et al., 2017). In fact, recognizing this distinction will help the elimination of the misconception "equality only produces a result". Therefore, it is stated in the literature that it is important for teachers to realize the thoughts of the students who see equal sign "as a symbol separating equal amounts" (Carpenter et al., 2005; Matthews et al., 2012). Teachers' noticing students' thoughts about equality and conducting a number of activities to ensure transition from operational thinking to relational thinking will break this resistance in the student (van den Kieboom et. al., 2017). To this end, it would be important for pre-service teachers to properly observe and interpret students' mathematical approaches before they start their professional career because the teacher's knowledge and the use of this knowledge are closely interrelated (Llinares, & Krainer, 2006). Teacher knowledge is a special type of knowledge that goes beyond the teacher or pre-service teacher's knowing the basic concepts and subjects in the field (Shulman, 1987; Ball, Thames, & Phelps, 2008). Within this knowledge included estimating the prior knowledge to be possessed by the student, determining the difficulties and misconceptions that the student has encountered or is to encounter and taking precautions against these (Shulman, 1987). Therefore, how the mathematics teacher uses his / her knowledge in the classroom environment has become one of the important topics in mathematics education (Ponte, & Chapman 2006). At this point, it is an important step for the teacher or pre-service teacherto make sense of student works. This act of making sense, which is called "noticing" in the literature, includes the ability of defining and interpreting student works to make correct decisions

(Jacobs, Lamb, & Philipp, 2010; Sherin, Jacobs, & Philipp, 2011). Here, it is assumed that through "noticing", the teacher defines the elements belonging to the math problem to be solved by his/her student. Familiarity with student responses will make the teacher advantageous in interpreting student knowledge and taking necessary training decisions (Sa´nchez-Matamoros, Ferna´ndez, & Llinares, 2015). In order to understand and analyze the student's mathematical thinking, it is important to infer and restructure what the student understands from what he/she writes, says and does. The teacher's ability to recognize his/her student's mathematical thinking requires more than seeing the answers to be true or false (Callejo, & Zapatera, 2017). In fact, it is necessary to discuss what students' answers mean and whether they are meaningful or not in terms of mathematical learning (Hines, & McMahon, 2005; Wilson, Mojica, & Confrey, 2013).

In the literature, there are many studies investigating teachers and pre-service teachers' noticing of student works and student's thinking process (Schack, Fisher, Thimas, Eisenhardt, Tassell, & Yoder, 2013; Bartell, Webel, Bowen and Dyson, 2013; van den Kieboom, Magiera, & Moyer, 2017; Callejo, & Zapatera, 2017; Fernández, Llinares, & Valls, 2012, Fernández, Llinares, & Valls, 2012). For example, Bartell, Webel, Bowen, & Dyson (2013) examined pre-service teachers' ability to recognize the evidence of children's conceptual understanding and the role of pre-service teachers' subject-area knowledge in identifying students' mathematical understanding. As a result, they reported that pre-service teachers are not adequate in analyzing student knowledge. Schack, Fisher, Thimas, Eisenhardt, Tassell and Yoder (2013) have studied the development of professional noticing skills of pre-service teachers (remembering students' strategies, interpreting students' mathematical understanding and deciding how to help students understand). The development of the pre-service teachers in these three components was investigated with pretest and posttest design. The findings have revealed that the pre-service teachers are better at remembering students' strategies than interpreting students' mathematical thinking both before and after the application. In addition, significant improvements were observed in the pre-service teachers in three of the components. This study is important in terms of contributing to the measures and decisions to be taken through the analysis of student works by pre-service teachers. Before starting their professional career, pre-service teachers should be prepared for educational settings they will encounter in the future.Although the field of algebra is of great importance for the student, the research shows that students' success in algebra both in Turkey and abroad is low (Erbaş, Çetinkaya, & Ersoy, 2009; Kenney, & Silver, 1997; Kieran, 1992).Reducing these difficulties will only be possible by accurately identifying and eliminating the difficulties that students face.Numerous and multidimensional studies have been conducted on algebra, one of the main learning areas of mathematics and gaining the importance it deserves in recent years as a results of updated curriculums (English, & Halford, 1995; MacGregor, & Stacey, 1993; Perso, 1992; Wagner, 1983). In some of these studies, as in the different fields of mathematics, different difficulties and misconceptions faced by students in the algebra learning have been elicited and various suggestions have been made for the elimination of these problems.

When concepts are not learned meaningfully, misconceptions may occur. Due to the fact that any mathematical misconception is probably a misconception considered to be correct by the student for a long time, it can be really difficult to change it although it contradicts with mathematical realities (Erbaş et al., 2009). Error in mathematics occurs as a result of erroneous use of mathematical expressions (Erbaş et al., 2009). The existing research shows that there are various difficulties and misconceptions concerning the algebraic concepts such as equality, equation, algebraic expressions, variables (English, & Halford, 1995; MacGregor, & Stacey, 1993; Perso, 1992; Wagner, 1983). According to Stacey and MacGregor (1997), the reasons behind the misconceptions in algebra are students' not having adequate arithmetical experience, the difference between the use of letters in algebra and the use of letters in other parts of life, existence of a language and rules specific to algebra. In order to overcome these misconceptions and difficulties that students have in the field of algebra, it is necessary for the teacher to determine them correctly and to conduct activities to improve them. Therefore, it is important that a teacher or a pre-service teacher understands and makes sense of student answers. For this process whose theoretical framework was elaborated by Jacobs et al., (2010) and which is called as "noticing" in the literature to be effective, the things to be done are listed by the researchers as follows: The teacher should be familiar with the strategies, approaches and solutions used by students, should define them correctly, should interpret what is understood from the student and should offer the guidance for the student to arrive at the correct answer.

Purpose

The concept of noticing is believed to be a concept new to Turkish education research. A concept similar to the concept of noticing was encountered in studies focusing on the collective lesson model in the national literature (Özdemir-Baki, & Işık, 2018; Güner & Akyüz, 2017). In their study, Özdemir-Baki and Işık (2018) only investigated the math teacher's understanding and interpretation of student responses yet didn't look at the stage of responding. In this regard, the current study is believed to expand their study. Another difference of the current study from their study is that the participants are pre-service teachers while in their study the participants were active teachers. On the other hand, Güner and Akyüz (2017) focused on the pre-service teachers' noticing in the stages of lesson planning and delivery. Thus, the current study is also different from their study, as it is built on the pre-service teachers' interactions with students.

In fact, the concept of noticing, which is directly or indirectly used by every experienced teacher in the class, basically relies on the teacher's reading and interpreting the incidences in the class correctly and developing action plans in relation to them. The participants of the current study are pre-service teachers and the study was conducted to investigate their noticing levels. In this connection, the purpose of the current study is to determine the noticing levels of three pre-service teachers attending the department of elementary school math teacher education. To this, end, the theoretical framework developed by Jacobs et al. (2010) was drawn on.

Problem

The problem statement of the current study is expressed as follows; "What are the pre-service teachers' levels of noticing students in the field of algebra?"

2. METHOD

Research Model

In the current study aiming to determine the pre-service teachers' levels of noticing on the basis of student responses, qualitative research methods and the case study design were used in the collection, analysis and interpretation of the collected data. The case study design is used to define and see the details making up an incidence, develop possible explanations of an incidence and evaluate an incidence (Gall, Borg, & Gall, 1996).

Participants of the Research

The selection of the participants for the research took place in two stages. The criterion sampling method, one of the purposive sampling methods, was used in the selection of the study groups (Yıldırım, & Şimşek, 2003). The criterion used in the selection of the participants was that the secondary school students had to study the subject of equations and that the pre-service teachers had to take the course of "basic algebraic concepts and instructional approaches" so that they could have known the basic concepts of algebra.

The application was conducted in the middle of the fall term. In the current study conducted within the context of the "private working" elective course, first the pre-service teachers were asked to determine students' misconceptions reported in the literature in relation to algebra learning and teaching. The pre-service teachers made readings from books and articles and then reported the information they had internalized. Here the purpose was to enable the pre-service teachers to revise their basic knowledge (in the second year of their undergraduate education, they took the course of algebraic concepts and instructional approaches and in this course, they conducted a research on this subject and wrote a report) and to introduce them to the subject. In the second phase, on the basis of the principle of easy accessibility, one 7th grade student was selected for each pre-service teacher and the pre-service teachers directed the questions in the data collection tool to their students.The voice recordings of these one-to-one interviews were listened in the faculty classes.As a requirement of the undergraduate course, a total of 6 pre-service teachers worked on this issue and prepared a report.In the next stage of the study, volunteerism was taken as the basis and opinions of 3 out of these 6 pre-service teachers were consulted. A great care was taken to select the participating secondary school students from among the students with medium or high academic achievement and volunteer students. These students were selected with the references of the teachers working in the schools where the pre-service teachers were doing their teaching practicum and the interviews were conducted in the class.For the sake of confidentiality, codes such as S1, S2,… were used for the names of students and PST1, PST2,... were used for the names of pre-service teachers.

Data Collection Tool

As the data collection tool in the current study, the “Scale of Determining the Errors and Misconceptions in the Subject of Equation” consisted of 7 questions was used to test the errors and misconceptions in the subject of equations (Erdem, 2013). This scale is a scale whose reliability and validity studies were conducted by Erdem (2013). In the current study, employing qualitative research methods, the tape recordings of the interviews conducted by the pre-service teachers with the students were then transcribed. Then, the pre-service teachers were asked to make sense of and interpret the responses of the students they had interviewed. The student interview form interpreted by the pre-service teachers is as follows:

Error and Misconception Determination Scale in Equations

Q.1. The number of girls in a class is three times the number of boys. K: number of girls in class; E: Since it represents the number of men in the class, which of the following statements is true?

a) E + K=3 **b)** E=3K

c) K=3E **d)** E + K=4

Describe the reason for choosing this option.

Q.2. Which of the following is the the first degree 1 unknown equation?

I. 2m+13=-66 **II.** $x^2 + 8x - 13 = 0$

III. 102=45+x **IV.** $\frac{2x+3}{5}=11$

a) I, II, III and IV **b)** I, III and IV

c) II and III **d)** I and III

Describe the reason for choosing this option.

Q.3. 176 = ☐ - 128 which number should be instead of ☐ ?

a) 304 **b)** 176 **c)** 128 **d)** 48

Explain why you choose this option. Perform the process.

Q.4. $-4 + \frac{7}{6}a = \frac{a}{3} + 6$

I. I equate the denominators of unknown terms. Then add up.

II. I divide both sides of equality by 5.

III. I collect unknown terms on the one side of the equation, known terms on the other side of the equation.

IV. I multiply both sides of the equation by 6.

Which of the following is the procedure for solving the equation?

a) III-I-IV-II **b)** II-III-IV-I **c)** I-II-III-IV **d)** II-IV-III-I

Explain your reason.

Q.5. How to save the following equation from the denominator?

$$\frac{x+6}{5}=\frac{9}{3}$$

Answer :………………………………………………………

Do you know the reason for this process?…………………………………………………………………

Is there another way to save the denominator?……………………………………………………………

If it is possible describe and dissolve the equation.

Q.6. 7(x+2)-12 = 2x+27 Find the solution set of the equation. Explain each one process.

Q.7.4x + 6 = 18 solve this equation by modeling on the balance.

In the second stage of the data collection process, each pre-service teacher was asked to analyze the transcribed data for their students. For the pre-service teachers to conduct this analysis, an interview form was administered to them. In this interview form, the pre-service teachers were asked to answer these questions:

1- Indicate whether the student response is correct or not with a reason. What did the student do and which strategies did he/she use in his/her solution? If you know the names of these strategies, please explain them; if you don't, explain them in mathematical terms you know.

2- Please explain why the student may have given such an answer and to which mathematical concepts you know it is related (Interpret).

3- If you were the teacher of this student, how would you respond to his/her answer (solution)? How would you direct the student to the correct solution?

The pre-service teachers were asked to prepare a worksheet for the third question after they had responded the first two questions in the interview form because the positive effect of worksheets in the elimination of misconceptions is known (Akkaya & Durmuş, 2010).

Data Analysis

On the basis of the students' responses, a scoring was made; and the responses were scored as follows; "strong=3", "limited=2" and "deficient=1". These terms were the terms used by Jacobs et al. (2010). In order to establish the reliability of the scoring, evaluations were made by the same researcher at different times and the means of the scores obtained from two different evaluations were taken. In order to understand what these scores mean, the explanations developed by Barnhart and Van Es (2015) were examined:

Table 1

Explanation for scoring

	Underdeveloped	Moderately Developed	Highly Developed
Attending	Focuses on class occurrences, teacher pedagogy, teacher behaviours, and / or classroom atmosphere. Does not care about the student's opinions.	Elicits the student's opinions by collecting data (focuses on the scientific process)	Elicits the student's opinions by collecting data, conducting analysis and making interpretation (focuses on scientific conceptuality)
Interpretation	Emphasizes occurrences that are of little or no importance. Does not explain interactions and classroom activities in detail. Use of evidence to support claims is little or no at all.	Starts making sense of important events. Uses some evidence to support claims.	Starts continuously making sense of important events. Frequently uses evidence to support claims.
Responding	Cannot detect and define a special idea of the student throughout the lesson; makes incoherent and ambiguous suggestions about what should be done different next time.	Determines and defines a special idea of the student throughout the lesson; makes suggestions about what should be done different next time.	Determines and defines a special idea of the student throughout the lesson; makes suggestions about what should be done different next time on the basis of evidence. Establishes logical connections between learning and teaching.

This structure explained in Table 1 is the theoretical framework of the scoring criteria. For instance, the part from which the pre-service teacher gets low mean score corresponds to the part in the underdeveloped section "Focuses on class occurrences, teacher pedagogy, teacher behaviours, and / or classroom atmosphere. Does not care about the student's opinions". Construction of such a framework; as stated by Franke and Kazemi (2001), will help us make sense of the pre-service's level of noticing. The rubric prepared on the basis of this framework is scored as follows:

Table 2

Rubric

	Underdeveloped	Moderately Developed	Highly Developed
Attending	Defines the existing state. Does not care about the student's opinions.	Elicits the student's opinions by collecting data (focuses on the scientific process)	Elicits the student's opinions by collecting data, conducting analysis and making interpretation (focuses on scientific conceptuality)
Score	1	2	3
Interpretation	Emphasizes occurrences that are of little or no importance. Does not explain interactions and classroom activities in detail. Use of evidence to support claims is little or no at all.	Starts making sense of important events. Uses some evidence to support claims.	Starts continuously making sense of important events. Frequently uses evidence to support claims.
Score	1	2	3
Responding	Cannot detect and define a special idea of the student throughout the lesson; makes incoherent and ambiguous suggestions about what should be done different next time	Determines and defines a special idea of the student throughout the lesson; makes suggestions about what should be done different next time.	Determines and defines a special idea of the student throughout the lesson; makes suggestions about what should be done different next time on the basis of evidence. Establishes logical connections between learning and teaching.
Score	1	2	3

For each student, each question was scored. The findings obtained in this way are presented below:

3. FINDINGS

Table 3

Scores for explanations

	PST1			PST2			PST3		
	Attending	Interpretation	Responding	Attending	Interpretation	Responding	Attending	Interpretation	Responding
1	3	3		3	2		3	3	
2	3	3		3	2		3	3	
3	3	3		3	2		3	2	
4	3	2		3	3		3	2	
5	3	2		3	3		3	2	
6	3	2		3	1		3	3	
7	3	3	3	3	3	3	3	3	3

In Table 3, the scores gained by the pre-service teachers for each of their explanations are shown. For the "response" stage, the pre-service teachers were asked to explain only the last question in the "Scale of Determining Misconceptions" because as a result of the interviews conducted with the pre-service teachers, it was concluded that the most comprehensive explanations would be made by the students for this question.

The lowest score to be taken from the rubric is 0 point and the highest score to be taken from the scale is 3 points. Thus, it is possible to perform a grading as the one given in Table 4:

Table 4

Mean scores

Deficient	Limited	Strong
0-1	1.1-2	2.1-3

Thus, the participants' mean scores taken from their responses for the first two stages are given in Table 5:

Table 5

Mean scores

	Attending	Interpretation
S1	3	2.5
S2	3	2.1
S3	3	2.5

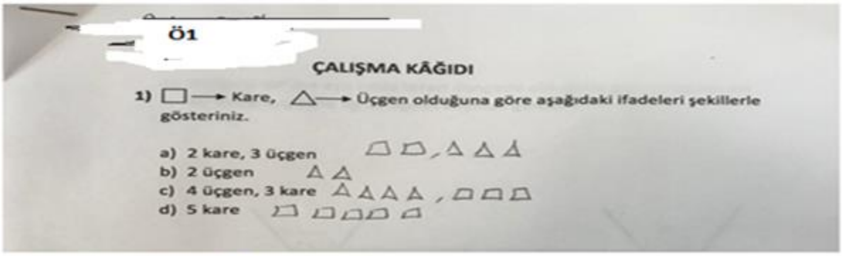
From the mean scores, it is seen that the "attending" ability of each participant is highly developed. In the explanations about how scores were produced, direct quotations were made from the pre-service teachers' explanations in the interview form. These explanations and their corresponding scores are given below:

- PST1-question 1 (attending): The student understood that the number of female students is three times higher than that of the male students yet while expressing this in algebraic terms, he/she says "E=3K". →3points
- PST1-question 2 (interpretation): As the number of female students is three times higher than that of the male student, the student directly put 3 as multiplier in front of the number of female students. In fact, he/she does not know what he/she has written means. That is, he/she cannot convert a verbal term into an algebraic term. →3points
- PST1-question 7 (attending, interpretation): "Sefa correctly solved the equations and reached the correct result. He got stuck in the subject of modelling in the scales. He said there should be 4x on the one scale and 6 on the other; thus, thought that they must be equal to each other and their sum should be 18. Even he added the constant term and the unknown and found the result as 18. Then he said it would not be possible, which showed that he did not have the misconception that + and – always entail a result. The student made an error in the modelling related to the scales. In this equation, he thought that the "=" sign determines the result, which is a misconception. In the previous questions, he did not make such an explanation. When he encountered a question type he was not familiar with, he made such explanations. The student could not establish a link between the "equality" in the equation and the scales. As he had probably not encountered such modellings in the education process, he could not produce any idea". →both are 3 points.

For the "responding" stage, the pre-service teachers were asked to select a question which all the students solved wrong and to prepare a worksheet to eliminate this misconception. As a result of the interviews conducted with the pre-service teachers and the analysis of the students' responses, it was determined that the students made errors in the question 7 in the "Scale of Determining the Misconceptions". Thus, the pre-service teachers were asked to offer guidance to the students for this question. The worksheet prepared to eliminate the errors seen in the 7th question by PST1 is given below:

Worksheet

I went next to the student and gave her the worksheet I prepared. She read the first question and began to draw shapes. She solved the question correctly.


Ö1

ÇALIŞMA KÂĞIDI

1) □ → Kare, △ → Üçgen olduğuna göre aşağıdaki ifadeleri şekillerle gösteriniz.

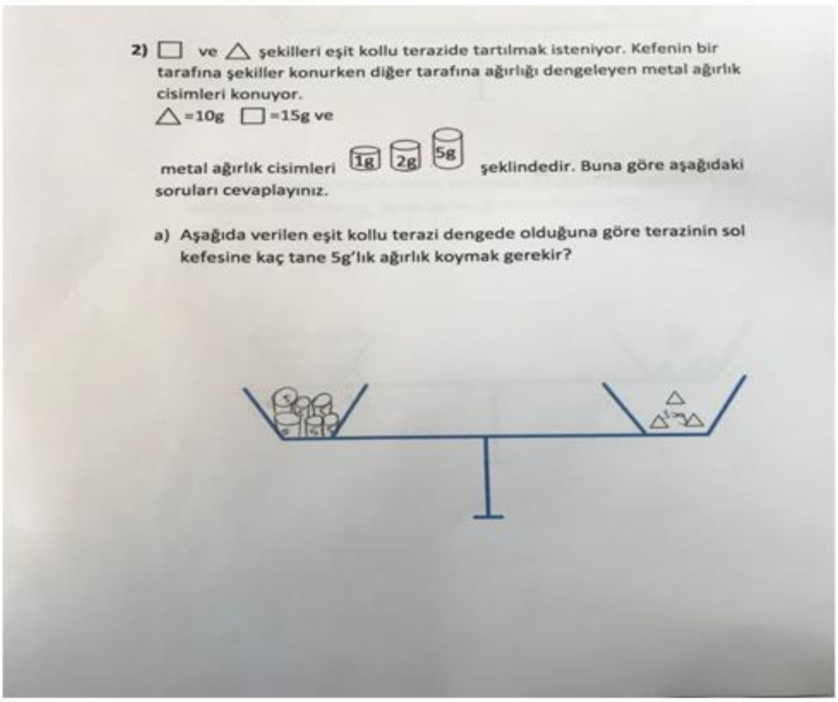
a) 2 kare, 3 üçgen

b) 2 üçgen

c) 4 üçgen, 3 kare

d) 5 kare

Then, when she moved to question 2, she calculated the total weight of the objects on the right side of the scale. And she found 30g and wrote it into the shroud. In order to keep the balance on the balance, the left shroud should have the same weight. In order to figure out how many weights should be 5g, she did 30/5 and drew six.


2) □ ve △ şekilleri eşit kollu terazide tartılmak isteniyor. Kefenin bir tarafına şekiller konurken diğer tarafına ağırlığı dengeleyen metal ağırlık cisimleri konuyor.

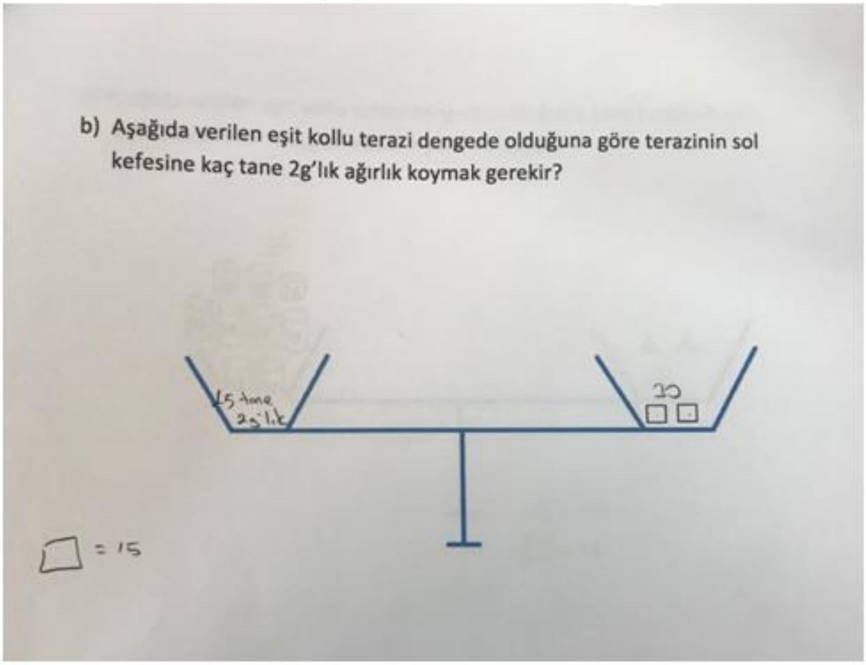
△=10g □=15g ve

metal ağırlık cisimleri 1g 2g 5g şeklindedir. Buna göre aşağıdaki soruları cevaplayınız.

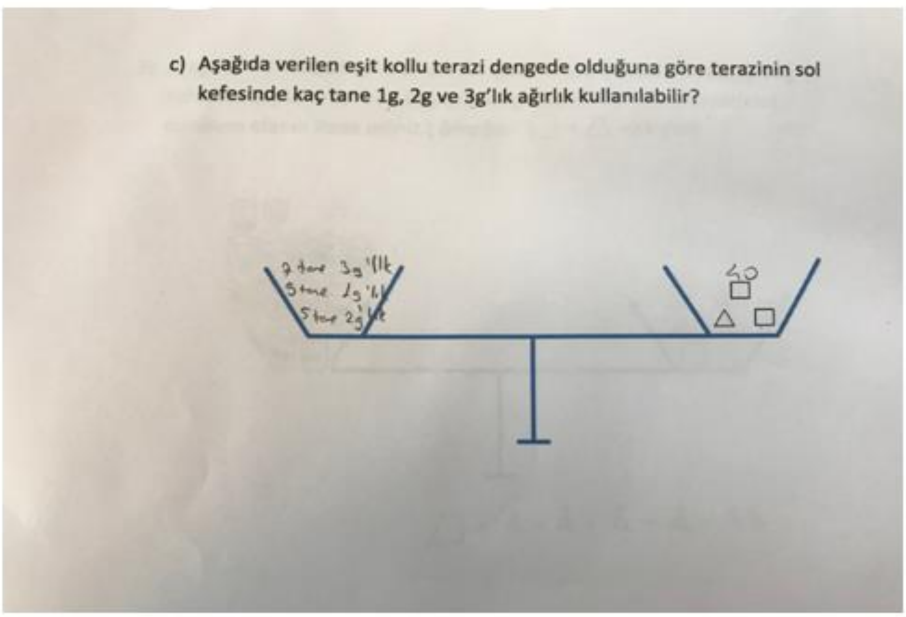
a) Aşağıda verilen eşit kollu terazi dengede olduğuna göre terazinin sol kefesine kaç tane 5g'lık ağırlık koymak gerekir?

When he came to question B, he followed the same steps. First, calculate the total weight of the objects on the right scaffold should be the same weight in the left-side of the process, 30/2 to do the process and should have 15 2g of weight weights, he said.

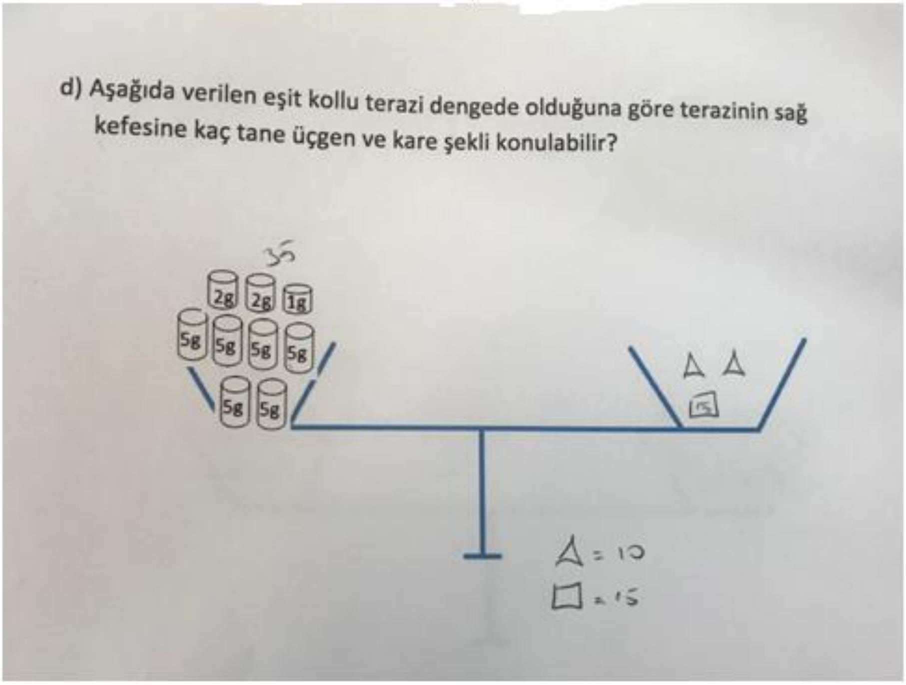
S



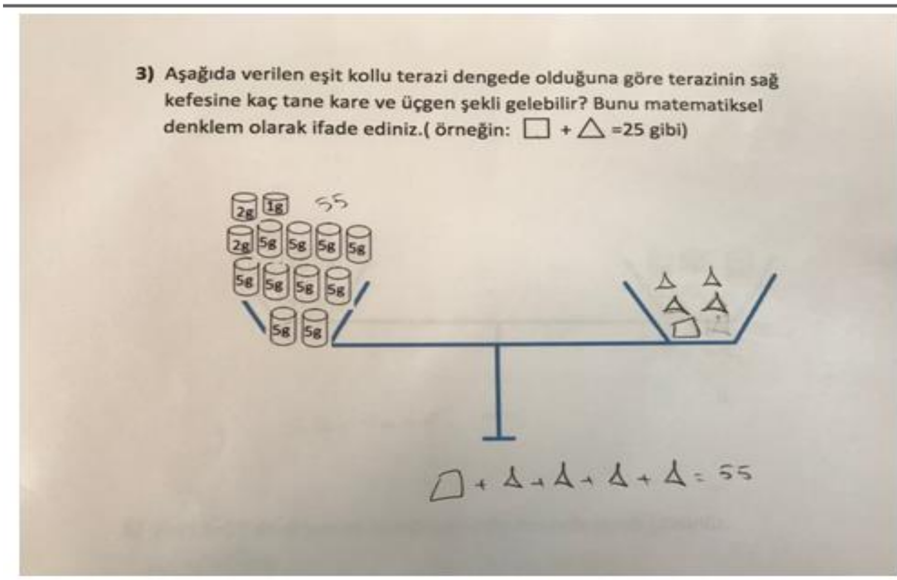
When she came to question C, she calculated the weights of the given objects on the right side of the scale and then calculated how many she could put in the weights of 1, 2 and 3 g. In doing so, it was primarily based on weights of 5g. Then 2g and a few of the last weights in the balance of how many can be found.



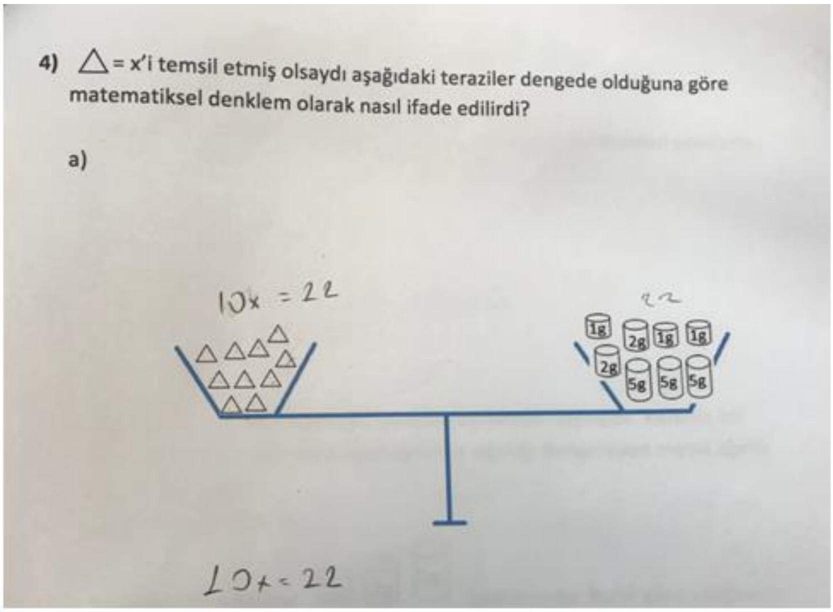
In D, this time he calculated how many grams of metal weights, and not the weights of the objects. And he found out how many triangular and square objects he had to draw to maintain balance.



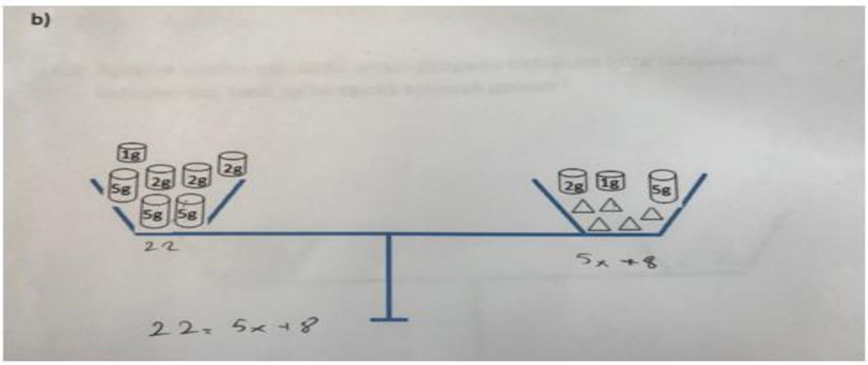
When it comes to question 3, she calculated the metal weights as before. She then figured out how many squares and triangles she can draw. And the most recent result is expressed in the form of equations.



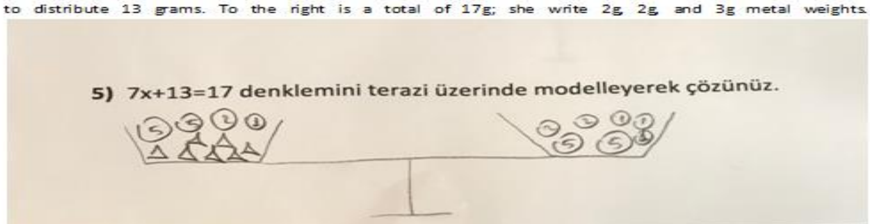
In question 4, there was now x representing the triangle, not the triangle. And since the balance of the scale was in balance, she thought how to express it mathematically. First, she calculated the metal weights on the right side of the scale. She then calculated how many triangular shapes were on the left. She said 10x because she had 10 triangles. And since the balance was achieved, 10x = 22 gave the correct answer.



In question B, there was a metal weight on the right side of the scale beside the x's. The student first calculated the metal weights on the left. She then found how many x's were on the right screen and how many grams of metal weights were. The equation was written as 22 = 5x + 8.



In the 5th question, she drew 7 x on the left side of the scale. She write 2 times 5g, 1time 2g, and 1 time 1 gram to distribute 13 grams. To the right is a total of 17g; she write 2g, 2g, and 3g metal weights.



<u>PST1-question7 (response):</u> *In the "Scale of Determining the Errors and Misconceptions in the Subject of Equations" I administered to the student, I saw that the student had a misconception in the 7th question. First, the student could not make sense of what the sign "=" means. After reading the question, the student drew a square and named it as "x". Then, he drew a circle and named it as "+6". After reading the equation "4x+6=18", he/she divided 18 into 6 and found 3 and then he/she tried to divide 18 into 4 but couldn't and then he/she concluded it could not be solved and then tried another way. As there is the*

term "4x" in the equation, he/she drew 4 squares on the left scale. Then though he/she said that +6 in the form of 1 circle, he/she drew 6 circles. However, according to the operation he/she conducted, he/she should have drawn 1 circle. Then, he/she thought that "I need to draw 18 figures in total on the right scale of the scales. That is, the total number of squares and circles must be 18". Yet, as he/she did not know how to determine the number of squares and circles, he drew them in a random number. Yet, he arranged it so that the total number of the figures would be 18.

I prepared a worksheet to eliminate the misconceptions the student had in this question. My priority while developing the worksheet was sequencing my questions from the easiest one to the most difficult one. Moreover, while developing my questions, I planned to guide him/her towards the last question of the scales step by step. My next question was always built on the former question. My questions were not independent of each other. I developed interconnected questions. In fact, the student would be led to the question of the scales step by step from the first question. That is, instead of directly asking the question of the scales, I developed a separate question to get him/her familiar with each step required for the solution of the question of the scales. In this way, while solving each question, the student would have solved each step involved in the question of the scales. When he/she arrived at the last question of the scales, he/she would not experience any difficulty because he/she would have already solved each step involved in the question of the scales. There is a total of 5 questions in the worksheets. In fact, the first four questions represent the steps of the 5th question. That is, the first 4 questions constitute the fifth question. →3points

The pre-service teacher determined that the student had experienced difficulties in relation to the concepts of equation and variable and prepared a worksheet directed to the elimination of them. As can be seen from the explanations of the pre-service teacher, the prepared worksheet serves the purpose of eliminating the difficulties experienced by the student.

4. RESULTS, DISCUSSIONS AND CONCLUSION

This current study includes the pre-service teachers' monitoring and developing some student activities so that they can form the proper view of the math teaching. This means the pre-service teachers' participating in remarkable incidences and then based on their observations, making instructional decisions (Van Es, 2010). In the current study, the pre-service teachers' levels of noticing students' works were investigated. It was observed that all the pre-service teachers got high scores in the "Attending" stage. In the interpretation stage, a decrease was observed in the scores. The students produced more general expressions rather than associating students' responses with the concepts and terms they had previously known. Unlike the former studies focusing on pre-service teachers (Ferna´ndez, Llinares, & Valls, 2012), they were found to be successful in the attending stage, which can be explained by the fact that the pre-service teachers took the course of algebraic concepts and instructional approaches, that they were highly successful in this course and underwent an intense preparation about the subject of equations. However, the pre-service teachers were not as successful in the interpreting stage as they were in the former stage. This result concurs with the findings reported by the previous studies (Barnhart, & Van Es, 2015; Choy, 2016; Doerr, 2006; Jacobs et al., 2010; Santagata, & Yeh, 2016). When the relationships between the skills are examined, it becomes clear that high level analysis of and responding to student thoughts is only possible through high level attending student thoughts. On the other hand, high level attending student thoughts does not guarantee high level analysis and responding (Barnhart, & Van Es, 2015). In addition, it is pleasing that none of the pre-service teachers were found to be weak in any of the categories. This might be because of the contribution of the pre-service teachers to the process.

For the last stage of responding, a worksheet developed by a pre-service teacher who had a high score in this stage was presented as an example. The pre-service teacher lived the experience of eliminating the difficulties and problems of his/her student. The pre-service teacher stated that through this work, the student understood the meaning of "equality" in an equation. What is meant by the pre-service teacher in fact is that the student was directed from the operational thinking to the relational thinking. The pre-service teacher stating that he/she received logical answers to the questions he/she asked later; thus, the work of this pre-service teacher was considered to be successful and this sample work was discussed.

The results obtained in the current study, as different from the previous studies (Ferna´ndez et al., 2012), show that the pre-service teachers' explanations about their noticing students are highly promising for the first stage. This might be explained by the intense preparation process the pre-service teachers were engaged in. Thus, though it is known that pre-service teachers' subject-area knowledge is not enough for noticing (Barnhart, & van Es, 2015), its necessity has been once more revealed. This is proved by the pre-service teachers' focusing on the subject-area knowledge throughout the preliminary works. In the suggestions section of the comprehensive study previously conducted by Sánchez-Matamoros, Fernández and Llinares (2019), the issue of completing the prior learning of pre-service teachers was emphasized. With its approach complying with this suggestion, the current study can help fill this void in the literature.

The pre-service teacher having undergone a detailed analysis was able to make sense of the student's responses, to determine the student's shortcomings and to develop a step-by-step plan to compensate for these shortcomings. This pre-service teacher's having a high level of noticing is one of the visible outcomes of the current research; yet, through the detailed analysis of the reasons behind this result, a framework can be formed. For example, while three of the participating pre-service teachers have the similar academic achievement and prior knowledge, what can be the elements that lead to difference? (there are pre-service teachers having lower scores from the stage of interpreting). Future research may seek answers to this question.

This current research designed as limited to the subject of equations aimed to determine the pre-service teachers' levels of noticing, who conducted one-to-one interviews with the students. By going beyond the goals of the current study, studies can be conducted on the development of the noticing ability of pre-service teachers. In the literature, there is some evidence showing that the ability of noticing can be developed with the help of some activities (Güner, & Akyüz, 2017; Özdemir-Baki, & Işık, 2018; Star, & Strickland, 2008; van Es, & Sherin, 2002), this evidence can shed some light on the future research. In the national literature, there is a limited amount of research on the noticing level of pre-service teachers and its development (Özdemir-Baki, & Işık, 2018; Güner, & Akyüz, 2017), which confirms the need for such research.

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Appendix

Work Sheet

1) □ → square, △ → triangle. According to this, show the following statements with figures.

a) 2 square, 3 triangle
b) 2 triangle
c) 4 triangle, 3 square
d) 5 square

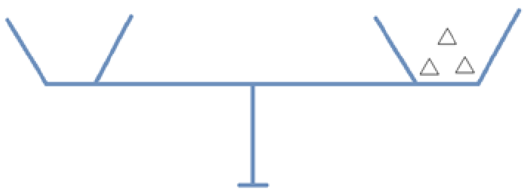
2) □ and △ you have. You will weigh the shape on an even arm balance. On one side of the scaffold is placed shapes, while the other side is weighted by metal weights.

△ =10g □ = 15g and

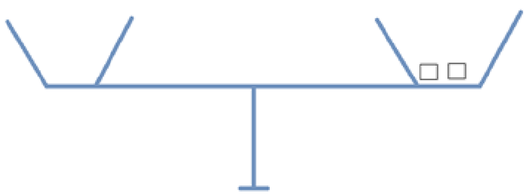
Metal weights is: 

Please answer the following questions.

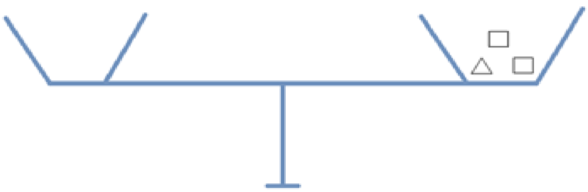
a) The following is equal to the equal arm balance. how many 5g weights should be put on the left side of the scale?



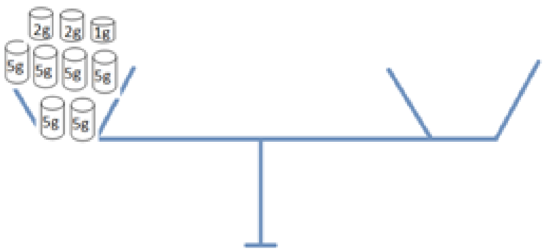
b) The following is equal to the equal arm balance. How many 2g weights should be put on the left side of the scale?



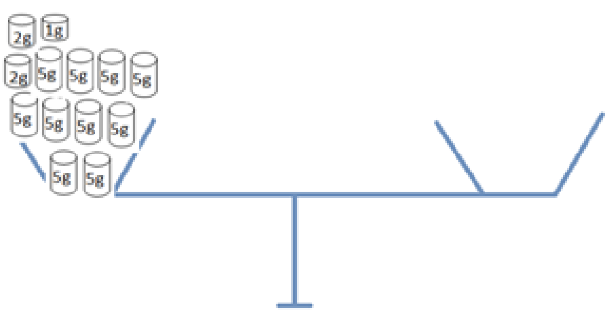
c) The following is equal to the equal arm balance. How many 1g, 2g and 3g weights should be put on the left side of the scale?



d) How many triangles and squares can be placed on the right side of the scale for balance?

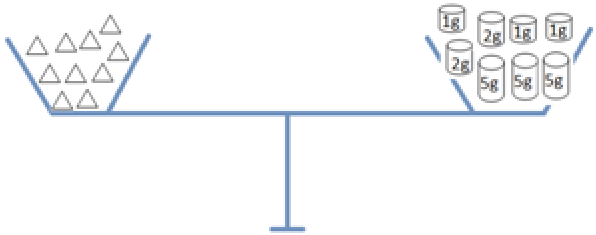


3) A How many squared and triangular shapes can be found on the right side of the scale, since the equal arm scale below is in balance? Express this as a mathematical equation. (For example: □ + △ =25)

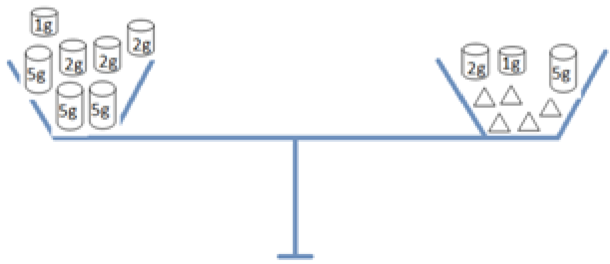


4) If △= x, and how the following balances are expressed in terms of mathematical equations?

a)



b)



5) 7x+13=17 solve the equation by modeling on the balance.