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Implementation of Deterministic Inventory Models with Backorders and Lost Sales in a Retail Company

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Abstract: A study is carried out on inventory planning and control for a retail company, which sells electronics and computer parts, to find answers to the questions of how much to order and when to order. In this paper, after describing the details of the problem, the methodology for finding optimal order quantities and the obtained results are explained. Deterministic inventory models are used with a consideration of backorders and lost sales. In stockout, some sales of the company are completely backordered, some are completely lost, and some are partially backordered while the rest is lost. Each of these cases are analyzed separately with the aim of determining minimum cost order quantities.

Keywords: Inventory, EOQ, Backorder, Lost sales, Partial backordering.

1. Introduction

In this study, inventory planning and control problem of a retail company and use of deterministic inventory models to compute economic order quantity (EOQ) under various stockout situations are considered. The Company has two types of customers and when shortages occur they behave differently yielding three cases: pure backorders, pure lost sales, and mixture of backorders and lost sales. We make use of the work of Montgomery et al. [1] in which they develop a model and solution methodology for deterministic and stochastic inventory problems with backorders and lost sales.

The first EOQ formula is developed by Harris (2013) [2]. In his study, Harris presents the well-known square root EOQ formula, that minimizes the cost. The optimal order size is the same as the EOQ. The EOQ calculations are the most acceptable analysis and yields important results in any inventory management.

Later, some extensions of EOQ formula have been derived which are used to determine optimal order quantities under various situations. A group of these studies are related to allowing shortages which yield backorders and/or lost sales. Formulation of complete backorders and complete lost sales cases are available in several texts such as Zipkin [3] and Waters [4]. Montgomery et al. [1] formulate a cost equation including a mixture of backordering and lost sales where the fraction of backorders and lost sales are denoted by b and $1-b$ respectively. They use order quantity (Q) and total demand during stockout period (S) as the decision variables, transform them into two new variables which yields a reformulation of cost equation, and then and develop a two-step methodology which yields optimal solution. Rosenberg [5], follows a similar approach for partial backordering. He starts with the same variables, Q and S, but then replaces them with two variables which are the cycle length (T) and a fictitious demand rate (X), reformulates the cost equation and develops a method to obtain an optimal solution. Park [6], assuming a fraction β of the demand is backordered and the remaining fraction $1-\beta$ is lost during stockout, defines a time proportional backorder cost and a fixed penalty cost per unit lost and formulates the average annual cost as a convex objective function in his deterministic inventory model that yields optimal values of the variables. Pentico and Drake [7] introduce a new approach for deterministic EOQ with partial backordering. They use cycle length (T) and fraction of demand filled from stock, i.e. fill rate (F) as their decision variables and formulate average profit per year. They show that the average profit per year is maximized by the T, F values that minimize the average cost, and develop a procedure for finding the optimal values which is relatively easier to solve when compared the previous works.

There are also additional studies handling the partial backordering problem from different aspects. San José et al. [8-11], use several approaches related to customers' behavior in characterizing the backlogging and develop models for partial backordering. Wee [12], Yang et al. [13], Sarkar and Sarkar [14] make studies concerning partial backordering for deteriorating items.

In this study, we have the opportunity of implementing inventory models with a mixture of backorders and lost sales, as well as pure backorders and pure lost sales cases. The modeling and solution approach of Montgomery et al. [1] is used to find the optimal values of policy variables that minimizes total cost, because their approach gives optimal solutions for the three situations (pure backorders, pure lost sales, and mixture of backorders and lost sales) that the Company experiences in case of stockout. Also, they assume that a constant fraction of shortages is backordered, and this fits the state in our case.

This paper is organized as follows. In Section 2, after giving the assumptions and notation, we describe the approach of Montgomery et al. [1]. Section 3 involves description of the three cases, implementation of the model on these cases, and the numerical results. Finally, a conclusion is given in Section 4 which addresses possible extensions.

2. Inventory model with backorders and lost sales

When a customer demands an item that is out of stock two things may happen; either the customer waits to receive the item from the next replenishment which yields backorder, or he simply moves to another supplier that results in a lost sale. The Company considered in this study experiences both situations. Optimal order quantities and shortages for a group of products are determined under three cases. In the following part after giving our assumptions and the notation, we explain the modeling and solution approach.

2.1. Assumptions and notation

The assumptions concerning our study are as follows.

- The system is a single echelon system
- Single item is considered in each case
- Demand is deterministic, continuous and known; and its rate is constant
- Demands for different items are independent
- Year is considered as the planning period
- All costs and selling prices are known and constant
- Lead time is zero

The following notation is used in modeling and analyzing the cases.

D: demand per year

S: total demand per cycle during the stock out period

Q: order quantity

TC: total cost per year

C: unit cost of each item (purchasing price)

I: interest rate

K: fixed ordering cost per inventory cycle

γ_s: shortage cost per unit period

γ_b: backorder cost per unit per year

γ_l: lost sales cost per unit per year (profit per unit)

b: fraction of unmet demand which is backordered during stockout period

2.2. Modeling and solution approach

In this part the work of Montgomery et al. [1] is explained. For the deterministic model, they develop a formulation and a solution methodology for single-echelon, single-item systems that yields the optimal solution. They assume that a fraction b of demand is backordered whereas the remaining fraction $1-b$ is lost. They describe the inventory geometry for this system as in Figure 1.

The notation used in Figure 1 is as follows.

U = total demand during cycle

V = on-hand inventory at the beginning of cycle

S = total demand per cycle during stockout

Q = order quantity

T = cycle length

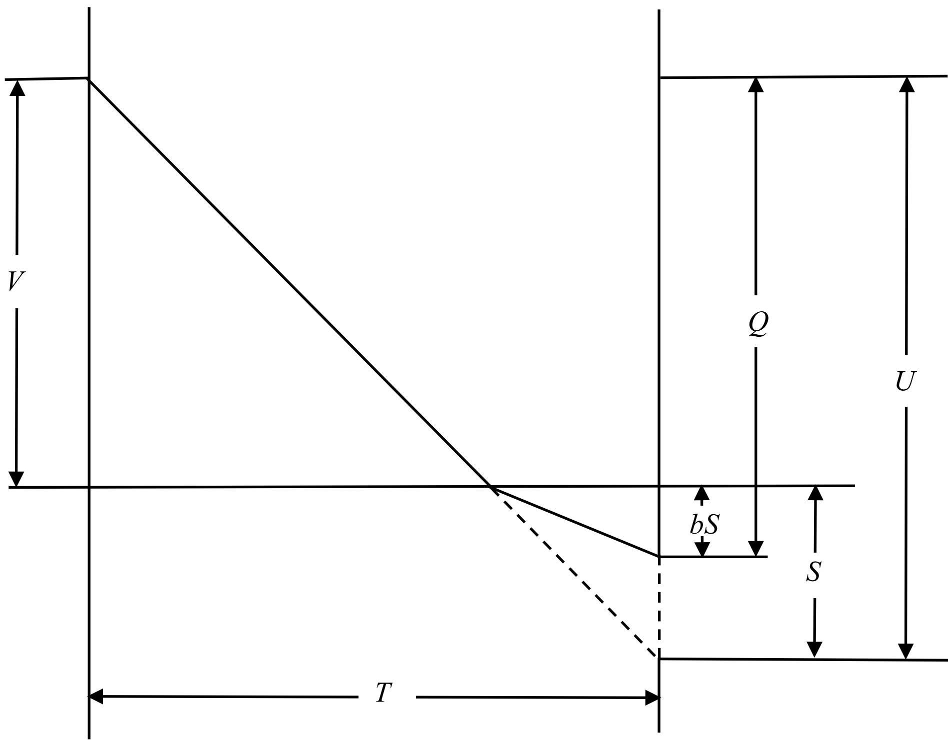

FIGURE 1. An inventory cycle with backorders and lost sales

Based on the geometry given in Figure 1 and the assumption that mixture of backorders and lost sales is constant, they obtain a total cost function that gives average annual cost. Using our notation, the cost function is as follows.

$$TC(Q,S) = \frac{KD}{Q+S(1-b)} + \frac{IC(Q-bS)^2}{2[Q+S(1-b)]} + \frac{\gamma_s SD}{Q+S(1-b)} + \frac{\gamma_b\, bS^2}{2[Q+S(1-b)]} + \frac{\gamma_l SD(1-b)}{Q+S(1-b)} \quad (1)$$

Right side of Equation (1) includes five cost components which are ordering, carrying, stockout penalty, backorder, and lost sale costs respectively. Ordering cost is obtained by multiplying by the fixed cost per order (K) by the number of cycles (orders) in a year. Carrying or holding cost is based on the cost of capital tied in inventory and therefore computed by multiplying unit purchasing cost (C), interest rate (I) and average inventory hold during year. Costs related to shortages can be explained as follows. Stockout penalty cost is obtained from the multiplication of stockout penalty per unit short (γ_s), the number of units which are short during a cycle (S), and the number of cycles in a year. Cost of backordering is unit backordering cost (γ_b) multiplied by number of units backordered and number of cycles in a year. Finally, cost of lost sales is obtained from the product of unit lost sales cost (γ_l), number of units lost during cycle and number of cycles in a year, and it represents the profit lost due to lost sales.

Pointing out that the cost function is not convex and necessary conditions ($\partial TC/\partial Q = 0$ and $\partial TC/\partial S = 0$) yield two simultaneous nonlinear equations which are hard to solve and may not yield global minimum of TC, they develop a solution methodology that guarantees the global minimum. They make the following transformation (Equations (2) and (3)) and obtain the total cost function in terms of U and V as given in Equation (4).

$$U = Q + S(1-b) \quad (2)$$

$$V = Q - bS \quad (3)$$

$$\widehat{TC}(U,V) = \frac{1}{U}[a_1 + a_2(U-V) + a_3(U-V)^2 + a_4V^2] \quad (4)$$

where,

$a_1 = KD$

$a_2 = \gamma_s D + \gamma_l D(1-b)$

$a_3 = \gamma_b b/2$

$a_4 = IC/2$

They first find the optimal U and V values (U^* and V^*) that minimizes $\widehat{TC}$, and indicate that optimal Q and S values (Q^* and S^*) minimizing TC could be obtained from the following inverse transformation (Equations (5) and (6)).

$$Q^* = bU^* + (1-b)V^* \tag{5}$$

$$S^* = U^* - V^* \tag{6}$$

Later in their analysis they define and use the following additional parameters.

$$a_5 = 4a_1a_3/a_2^2$$

$$a_6 = 4a_1a_4/a_2^2 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2}$$

Finally, they describe the optimal solution based on two conditions depending on the value of a_6 as explained below.

Condition 1:

If $a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l(1-b)]^2} \leq 1$,

then no shortages are allowed in the optimal solution and total demand during cycle is equal to on-hand inventory at the beginning of cycle (U^*=V^*).

Then, from Equations (5) and (6), optimal order quantity becomes equal to total demand during cycle (Q^*=U^*) whereas demand during stockout period is zero (S^*=0) since no shortages are allowed. Therefore, optimal order quantity is computed using basic EOQ formula.

$$Q^* = U^* = \sqrt{\frac{2KD}{IC}} \tag{7}$$

Hence, the total cost function becomes

$$TC\,(Q, S) = \frac{KD}{Q} + \frac{ICQ}{2} \tag{8}$$

Condition 2:

If $a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l(1-b)]^2} > 1$,

then U^* and V^* are obtained from the following equations.

$$\beta^{*}=\frac{a_5}{a_5+a_6}+\frac{1}{a_5+a_6}\sqrt{\frac{a_5a_6}{a_5+a_6-1}} \tag{9}$$

$$U^{*}=\sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2+IC\beta^2}} \tag{10}$$

$$V^{*}=\beta^{*}U^{*} \tag{11}$$

Finally, optimal order quantity and total demand during stockout period is computed from inverse transformation ($Q^{*}=bU^{*}+(1-b)V^{*}$ and $S^{*}=U^{*}-V^{*}$) given by Equations (5) and (6).

Interested readers may see the details of their work in [1]. In the following section, we explain the determination of optimal order quantities for the three cases using this solution procedure.

3. Cases and Numerical Study

The Company buys computer and electronics parts from suppliers and sells them to two types of customers: dealers and individual customers. Some products are demanded by both types of customers, whereas the others are purchased by one type of customer only (either by dealers or by individuals). In stockout situations, that is, when demands are not covered sufficiently, three different cases arise depending on behavior of customers. The shortages for sales through individual customers are assumed to be lost sales, whereas the shortages resulting from sales to dealers are backordered. Additionally, for the products sold to both customer types a mixture or backorders (for dealers) and lost sales (for individuals) are considered. Therefore, related to stockout situations the following three cases are considered.

(i) pure backorders (sales to dealers),

(ii) pure lost sales (sales to individual customers),

(iii) mixture of backorders and lost sales (sales to dealers and individual customers)

For the items demanded by both customer types, sales data shows that the percentage of sales to dealers changes between 80% and 95%.

Justification of constant demand assumption

Ten products are selected from each category (total 30 items) and examining the annual sales data for five years, from 2013 through 2017, it is observed that demands for these products are not too much spread. However, to make sure that whether demands are sufficiently regular, and therefore to justify the constant demand assumption such that we can use EOQ models, variability coefficients for the demands are computed. Then, the procedure recommended by Peterson and Silver [15], and Winston [16] is applied as explained below.

Let D_i be the demand in year i.

Step 1. Compute the average demand ($\overline{D}$) per year using

$$\overline{D} = \frac{1}{5}\sum_{i=1}^{5} D_i$$

Step 2. Compute the variance of annual demand (var (D)) using

$$\text{var}\ (D) = \frac{1}{5}\sum_{i=1}^{5} D_i^2 - \overline{D}^{\,2}$$

Step 3. Compute the variability coefficient (VC) using

$$VC = \frac{\text{var}\ (D)}{\overline{D}^{\,2}}$$

Although demand is known and variable, if variability coefficient is small, then variability can be neglected and demand can be assumed as constant, and hence EOQ model can be applied instead of a variable (and known) demand model. Research on this issue suggests that the EOQ can be used if $VC < 0.20$. (See Winston (2004). [16])

To illustrate this, three items are selected from each category (items sold to dealers, items sold to individual customers, items sold to both) and variability coefficient calculations are performed. For example, considering item 1, which is one of the products sold to dealers only, demand during the five years are 5214, 5020, 4400, 4945, and 5423. Then,

$$\overline{D} = \frac{(5214+5020+4400+4945+5423)}{5} = 5000.40$$

$$\text{var}\ (D) = \frac{1}{5}(5214^2+5020^2+4400^2+4945^2+5423^2) - 5000.40^{\,2} = 117{,}629.84$$

$$\text{VC} = \frac{117{,}629.84}{5000.40^{\,2}} = 0.0047$$

Demands and variability coefficients for the 9 items are given in Table 1. In the Table, items 1-3 are the selected products for case I, whereas items 11-13 and 21-23 are the selected products for case II and case III respectively. Since all variability coefficients are small (<0.20) constant demand assumption is justified and EOQ models can be used for those items.

TABLE 1. Demands and variability coefficients

Item	Annual demand					Average demand	Estimated variance	Variability Coefficient
	2013	2014	2015	2016	2017			
1	5214	5020	4400	4945	5423	5000.40	117,629.84	0.0047
2	3190	3685	3569	4850	3708	3800.40	309,929.84	0.0215
3	3920	3486	3845	3032	3615	3579.60	99,237.84	0.0077
11	1000	1350	860	795	993	999.60	36,834.64	0.0369
12	865	769	899	969	1250	950.40	26,589.44	0.0294
13	825	685	637	702	650	699.80	4464.56	0.0091
21	1295	1410	1568	1695	1478	1489.20	18,534.96	0.0084
22	1048	1470	1205	1359	1232	1262.80	20,522.96	0.0129
23	900	1150	1024	965	1100	1027.80	8087.36	0.0077

Based on the information obtained from the Company the data used in the calculations are determined as explained below.

- Annual demand, D, is obtained from the average of the available 5-year annual sales data.
- Unit cost, C, of each item is obtained from company records.
- Ordering cost, K, is assumed as 50₺ per order.
- Shortage cost, γ_s, due to loss of goodwill, is considered as 0.08₺ per unit short for cases I and II, and it is assumed to be 0.10₺ for case III.
- Backorder cost, γ_b, is 0.20₺ per unit backordered.
- Lost sale cost per unit, γ_l, is computed as 20% of the unit cost of each item.
- Interest rate is assumed as 10% per year, therefore holding cost is $h = IC = 0.10C$.

The following sections include our analysis and numerical results separately for each of the three cases.

3.1. Case I: All Shortages are Backordered

When a dealer demands a product, which is out of stock, it usually agrees to wait to receive the item from the next delivery from suppliers. Therefore, it is assumed that in case of stockout the products sold to dealers are backordered. Setting b=1 (the fraction of shortages backordered), the problem becomes pure backorder problem as shown in Figure 2. Now, the cost function (given by Equation (1)) can be written as in Equation (12) below.

$$TC(Q,S) = \frac{KD}{Q} + \frac{IC(Q-S)^2}{2Q} + \frac{\gamma_s SD}{Q} + \frac{\gamma_b S^2}{2Q} \tag{12}$$

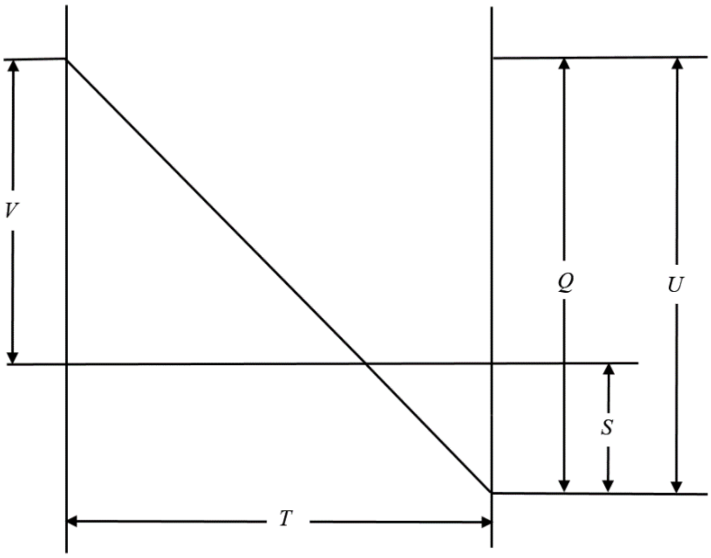

FIGURE 2. An inventory cycle with pure backorders

Now, the total cost consists of four components, which are ordering, carrying, stockout penalty, and backorder costs. The selected 10 items, which are sold to dealers and subject to backordering, are numbered as 1, 2, …, 10. Order quantities, number of cycles per year, and the total inventory cost are computed for these items using the procedure explained in Section 2. Input data and solution results are summarized in Table 2. Data consists of demand (D), purchasing cost (C), ordering cost (K), interest rate (I), stockout penalty (γ_s), and backorder cost per unit short (γ_b). Results include optimal order quantity (Q), amount of shortage (S), total cost (TC) and number of cycles per year.

TABLE 2. Input data and results for case I (pure backorders)

	Data						Results				
Item	D	C	K	I	γ_s	γ_b	a_6	Q	S	TC	**Cycles/ year**
1	5000	3.93	50	0.1	0.08	0.2	1.23	1317.82	198.82	439.76	3.79
2	3800	1.43	50	0.1	0.08	0.2	0.59	1630.14	0	233.11	2.33
3	3580	1.26	50	0.1	0.08	0.2	0.55	1685.61	0	212.39	2.12
4	3200	2.80	50	0.1	0.08	0.2	1.37	1254.02	198.18	295.64	2.55
5	3180	1.29	50	0.1	0.08	0.2	0.63	1570.07	0	202.54	2.03
6	3160	1.26	50	0.1	0.08	0.2	0.62	1583.65	0	199.54	2.00
7	3155	1.62	50	0.1	0.08	0.2	0.80	1395.54	0	226.08	2.26
8	3000	1.47	50	0.1	0.08	0.2	0.77	1428.57	0	210.00	2.10
9	2800	1.87	50	0.1	0.08	0.2	1.04	1247.29	23.88	228.78	2.24
10	2700	1.00	50	0.1	0.08	0.2	0.58	1643.17	0	164.32	1.64

Detailed calculations for two selected items, 1 and 2, are shown below.

Item 1

Annual demand for this item is 5000 units, purchasing cost is 3.93₺ per unit, ordering cost is 50₺ per order, whereas stockout penalty and backorder cost per unit per unit are taken as 0.08₺ and 0.20₺ respectively. Interest rate is 10% per year. First, value of a_6 is computed as follows.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\left(1\text{-}b\right)]^2} = \frac{2(0.1)(3.93)(50)}{5000[0.08 + 0]^2} = 1.23$$

Since $a_6 > 1$, the procedure defined in condition 2 is applied. Necessary calculations are shown below.

$$a_1 = KD = (50)(5000) = 250{,}000$$
$$a_2 = (\gamma_s D + \gamma_l D(1 - b\ ) = (0.08)(5{,}000) + 0 = 400$$
$$a_3 = \gamma_b b \,/\, 2 = (0.2)(1) \,/\, 2 = 0.1$$

and

$$a_5 = 4a_1a_3 \,/\, a_2^2 = 4(250{,}000)(0.1) \,/\, 400^2 = 0.625$$
$$\beta^* = \frac{a_5}{a_5 + a_6} + \frac{1}{a_5 + a_6}\sqrt{\frac{a_5a_6}{a_5 + a_6 - 1}}$$
$$= \frac{0.625}{0.625 + 1.228} + \frac{1}{0.625 + 1.228}\sqrt{\frac{(0.625)(1.228)}{0.625 + 1.228\text{-}1}} = 0.849$$

then,

$$U^* = \sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2 + IC\beta^2}} = \sqrt{\frac{2(50)(5000)}{(0.2)(1)(1-0.849)^2 + 0.1(3.93)(0.849)^2}} = 1317.99$$
$$V^* = \beta^* U^* = (0.849)(1317.82) = 1118.98$$

Finally, optimal order quantity and backordered units (demand during stockout) are obtained using Equations (5) and (6).

$$Q^* = bU^* + (1 - b)V^* = \ (1)(1317.99) + 0 = 1317.99$$
$$S^* = U^* - V^* = 1317.99 - 1118.98 = 199.01$$

When these values are substituted into Equation (12), minimum total cost is found as 439.76₺. Finally, number of orders per year is obtained as $D \,/\, Q^* = 5000 \,/\, 1317.99 = 3.79$.

Item 2

This item has an annual demand of 3800 units and a purchasing cost of 1.43₺ per unit. Other cost data is the same as in Item 1. First, value of a_6 is computed as

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(1.43)(50)}{3800[0.08 + 0]^2} = 0.59$$

Since $a_6 < 1$, condition 1 is realized. Hence, no shortages are allowed and the optimal order quantity is obtained as from EOQ formula as follows.

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(3800)}{(0.1)(1.43)}} = 1630.14$$

Using Equation (12) minimum cost is computed as 233.11₺. Number of cycles in a year is $D/Q^* = 3800 / 1630.14 = 2.33$.

According to these solutions, the Company allows backorders for three items (1, 4, 9) which have relatively high unit purchasing costs. For the other items, all demand would be met on time and hence no shortages occur.

3.2. Case II: All Shortages are Lost

If shortages occur, then individual customers usually do not wait for backorders to arrive and prefer another supplier. Therefore, the sales demanded by only individual customers are assumed to be completely lost and this can be modeled as a pure lost sales problem.

Setting the fraction of shortages backordered equal to zero ($b = 0$) yields the problem with lost sales as demonstrated in Figure 3. The cost function is given in Equation (13) below.

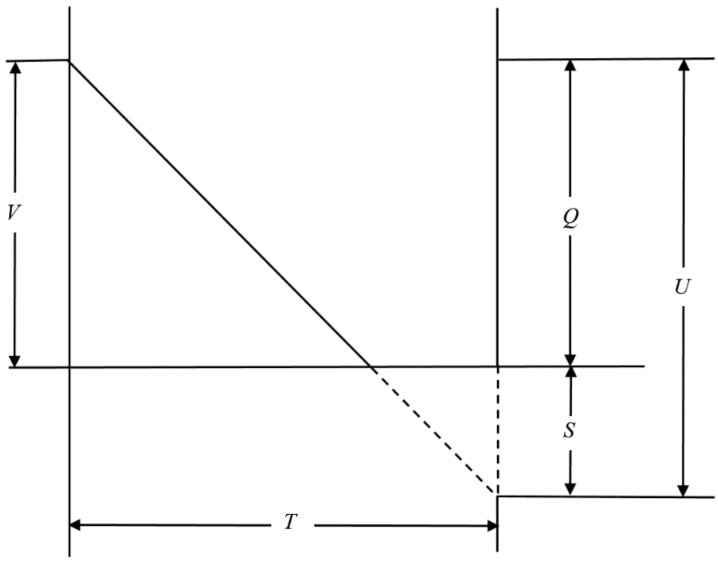

FIGURE 3. An inventory cycle with pure lost sales

$$TC(Q,S) = \frac{KD}{Q+S} + \frac{IC(Q)^2}{2(Q+S)} + \frac{\gamma_s SD}{Q+S} + \frac{\gamma_l SD}{Q+S} \tag{13}$$

Among the products which are sold to individuals only, 10 items are selected and named as 11, 12, …, 20. Input data and solution results are given in Table 3. It is assumed that the Company makes a profit of 20% of purchasing cost. It is used as lost sale cost per unit and therefore $\gamma_l = 0.20C$.

If all demand during stockout period is lost, then it is shown that optimal solution is either to meet all demands (have no shortages) or to have all shortages (do not stock at all) (see Waters [4]). As seen in the Table 3, $a_6 < 1$ for all items. Therefore, condition 1 is applied and basic EOQ formula is used to obtain the order quantities. That is, for the given items optimal solutions yield no stockout (S=0).

TABLE 3. Input data and results for case II (pure lost sales)

Item	Data						Results				
	D	C	K	I	γ_s	γ_l	a_6	Q	S	TC	Cycles/year
11	1000	2.53	50	0.1	0.08	0.51	0.07	628.69	0.00	159.06	1.59
12	950	3.42	50	0.1	0.08	0.68	0.06	527.05	0.00	180.25	1.80
13	700	3.16	50	0.1	0.08	0.63	0.08	470.66	0.00	148.73	1.49
14	600	2.07	50	0.1	0.08	0.41	0.13	538.38	0.00	111.45	1.11
15	890	2.10	50	0.1	0.08	0.42	0.09	651.01	0.00	136.71	1.37
16	750	3.34	50	0.1	0.08	0.67	0.08	473.87	0.00	158.27	1.58
17	580	2.40	50	0.1	0.08	0.48	0.12	491.60	0.00	117.98	1.18
18	900	1.42	50	0.1	0.08	0.28	0.11	796.12	0.00	113.05	1.13
19	1000	1.51	50	0.1	0.08	0.30	0.09	813.79	0.00	122.88	1.23
20	960	2.39	50	0.1	0.08	0.48	0.58	633.78	0.00	151.47	1.51

As an example, some calculations for product 11 are given below.

$\gamma_l = 0.20C = 0.20(2.53) = 0.51$₺

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l(1-b)]^2} = \frac{2(0.1)(2.53)(50)}{1000[0.1 + 0.51]^2} = 0.068 < 1$$

Then, optimal order quantity is

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(1000)}{(0.1)(2.53)}} = 628.69$$

This yields a total cost of 159.06₺. Then number of cycles in a year is obtained from D/Q^*=1000 / 628.69 = 1.59.

3.3. Case III: Mixture of Backorders and Lost Sales

Some products of the Company are demanded by both dealers and individuals. In case of shortages, considering the attitudes of different types of customers, it is assumed that demands from dealers are backordered and demands from individual customers are lost. Hence, the model that allows both backorders and lost sales are used where a fraction b of shortages is backordered whereas the rest (1–b) is lost. Since the percentage of sales to dealers changes between 80% and 95% for these kinds of products, 90% of shortages are assumed to be backordered and 10% are lost. So, the fraction of backorders is set as b = 0.9 for all products.

Ten items of this type are considered and named from 21 to 30. Problem data related to these items and solution results are presented in Table 4.

TABLE 4. Input data and results for case III (mixture of backorders and lost sales)

Item	Data							Results				
	D	***C***	***K***	***I***	γ_s	γ_b	γ_l	a_6	***Q***	***S***	***TC***	**Cycles/ year**
21	1489	4.53	50	0.1	0.1	0.2	0.91	0.84	573.32	0	259.71	2.60
22	1263	3.42	50	0.1	0.1	0.2	0.68	0.95	607.70	0	207.83	2.08
23	1028	3.27	50	0.1	0.1	0.2	0.65	1.16	620.98	69.64	182.57	1.66
24	884	2.05	50	0.1	0.1	0.2	0.41	1.17	702.70	53.25	134.23	1.26
25	1200	2.03	50	0.1	0.1	0.2	0.41	0.86	768.85	0	156.08	1.56
26	500	3.22	50	0.1	0.1	0.2	0.64	2.38	542.85	197.10	117.68	0.92
27	3000	0.50	50	0.1	0.1	0.2	0.10	0.14	2449.49	0	122.47	1.22
28	2920	0.45	50	0.1	0.1	0.2	0.09	0.13	2547.33	0	114.63	1.15
29	2500	0.48	50	0.1	0.1	0.2	0.10	0.16	2282.18	0	109.54	1.10
30	2400	0.49	50	0.1	0.1	0.2	0.10	0.17	2213.13	0	108.44	1.08

Example calculations for items 21 and 23 are as follows.

Item 21

This item has an annual demand of 1489 units and purchasing cost of 4.53₺ per unit. Therefore, lost sale cost per unit is

$\gamma_l = 0.20C = 0.20(4.53) = 0.91$₺.

Then, value of a_6 is computed as follows.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(4.53)(50)}{1489[0.1 + 0.91(1-0.9)]^2} = 0.84 < 1$$

Since $a_6 < 1$, we follow condition 1. No shortages occur and the optimal order quantity is obtained from basic EOQ formula as follows.

$$Q^* = \sqrt{\frac{2KD}{IC}} = \sqrt{\frac{2(50)(1489)}{(0.1)(4.53)}} = 573.32$$

Using the total cost formula, minimum cost is computed as 259.71₺. Number of cycles for this item is $D/Q^* = 1489 / 573.32 = 2.60$.

Item 23

Annual demand for this item is 1028 units and purchasing cost is 3.27₺ per unit. Hence,

$\gamma_l = 0.20C = 0.20(3.27) = 0.65$₺.

Next, the value of a_6 is computed.

$$a_6 = \frac{2ICK}{D[\gamma_s + \gamma_l\,(1-b)]^2} = \frac{2(0.1)(3.27)(50)}{1028[0.1 + 0.65(1-0.9)]^2} = 1.16 > 1$$

Since $a_6 > 1$, the procedure in condition 2 is applied. Necessary calculations are shown below.

$a_1 = KD = (50)(1028) = 51400$

$a_2 = \gamma_s D + \gamma_l D(1-b) = (0.1)(1028) + 0.65(1028)(1 - 0.9) = 170.00$

$a_3 = \gamma_b b/2 = (0.2)(0.9)/2 = 0.09$

and

$a_5 = 4a_1a_3/a_2^2 = 4(51400)(0.09) / 170^2 = 0.640$

$$\beta^* = \frac{a_5}{a_5 + a_6} + \frac{1}{a_5 + a_6}\sqrt{\frac{a_5a_6}{a_5 + a_6 - 1}}$$

$$= \frac{0.640}{0.640 + 1.16} + \frac{1}{0.640 + 1.16}\sqrt{\frac{(0.640)(1.16)}{0.640 + 1.16 - 1}} = 0.889$$

$$U^* = \sqrt{\frac{2KD}{\gamma_b b(1-\beta)^2 + IC\beta^2}} = \sqrt{\frac{2(50)(1028)}{(0.2)(0.9)(1-0.889)^2 + 0.1(3.27)(0.889)^2}} = 627.94$$

$V^* = \beta^* U^* = (0.889)(627.94) = 558.30$

Then optimal order quantity and shortages (demand during stockout) are obtained using Equations (5) and (6).

$Q^* = bU^* + (1-b)V^* = 0.9(627.94) + 0.1(558.30) = 620.98$

$S^* = U^* - V^* = 627.94 - 558.30 = 69.64$

Finally, total cost is obtained as 182.57₺, and number of cycles is found as D/Q^*=1028/620.98 = 1.66. For this item, the Company has a shortage of 69.64 ≅ 70 units. 90% of these items (63 units) are backordered whereas 10% of them (7 units) are lost. According to the results shown in Table 5, the Company has shortages in three items (23, 24, 26) and meets all demands from stock for the other seven items.

3.4. Calculations with Different Backorder Fractions

Regarding case III, mixture of backorders and lost sales, order quantities are computed using four different backorder percentages which are 80%, 85%, 90% and 95%. These values are selected based on the observation that the percentage of sales to dealers varies between 80% and 95%. Table 5 shows the resulting values of order quantity (Q), amount of shortage (*S*), and total cost (*TC*) for all the 10 items used in case III under different backorder fractions, *b*.

TABLE 5. Comparison with different backorder fractions for case III

Item	*b*=0.80			*b*=0.85			*b*=0.90			*b*=0.95		
	Q	*S*	*TC*	*Q*	*S*	*TC*	*Q*	*S*	*TC*	*Q*	*S*	*TC*
21	573.3	0	259.7	573.3	0	259.7	573.3	0	259.7	744.3	194.7	253.4
22	607.7	0	207.8	607.7	0	207.8	607.7	0	207.8	760.6	176.0	202.9
23	560.7	0	183.4	560.7	0	183.4	621.0	69.6	182.6	735.2	207.7	175.9
24	656.7	0	134.6	656.7	0	134.6	702.7	53.3	134.2	771.2	134.1	132.0
25	768.9	0	156.1	768.9	0	156.1	768.9	0	156.1	823.1	59.4	155.6
26	448.0	71.5	125.8	501.1	142.1	122.5	542.9	197.1	117.7	577.0	241.4	112.0
27	2449.5	0	122.5	2449.5	0	122.5	2449.5	0	122.5	2449.5	0	122.5
28	2547.3	0	114.6	2547.3	0	114.6	2547.3	0	114.6	2547.3	0	114.6
29	2282.2	0	109.5	2282.2	0	109.5	2282.2	0	109.5	2282.2	0	109.5
30	2213.1	0	108.4	2213.1	0	108.4	2213.1	0	108.4	2213.1	0	108.4
		Total cost:	1522.5			1519.1			1513.2			1486.9

Considering the problem data used in the calculations, the results shown in Table 5 demonstrate the expected behavior. Comparing the results presented in the table, it is observed that as fraction of backorders increases number of items for which shortages are allowed and the amount of shortages increase, whereas the total costs decrease. For example, for 80% and 85% only one item (item 26) has shortages, whereas at 95% level shortages are allowed for six items (items 21-26).

4. Conclusions

In this study an implementation of a deterministic inventory model is presented. The approach developed by Montgomery et al. [1] is used for inventory planning decisions of a retail company which sells electronics and computer parts. The company has two different types customer as retailers and individuals. Three cases are defined based on the customers' attitudes in case of stockout; sales to dealers only are completely backordered (case I), sales to individuals only are completely lost (case II), and therefore sales to both customer types are partially backordered and partially lost (case III). Since demands for most of the products of the Company have low variability, EOQ models are used to find optimal order quantities for the selected items.

The following issues are identified as possible extensions of our study. A rationing policy can be taken into consideration to determine the critical stock level where, for example, the Company stops meeting the demands of individuals while continuing to meet the demand of dealers. The demands of both customer types (dealers and individuals) for some products can be backordered or lost, with different backordering or lost sale costs for each. For items with high demand variability use of methods such as Wagner-Whitin algorithm and Silver-Meal heuristics can be considered. Also, for items whose demands cannot be assumed deterministic, Stochastic inventory models can be used.

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